ERF Working Papers Series

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ECONOMIC RESEARCH FORUM
منتدى البحوث الاقتصادية

Working Paper No. 1820
December 2025

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An early version of this work presented in the ERF 31st Annual Conference 2025, Cairo, Egypt. This document is part of the third chapter in the thesis: Informality, Unemployment, and Monetary Policy: Further Dynamics and Effect. The authors acknowledge the valuable guidance from Nooman Rebei and Hafedh Bouakez.

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First published in 2025 by
The Economic Research Forum (ERF)
21 Al-Sad Al-Aaly Street
Dokki, Giza
Egypt
www.erf.org.eg

Abstract

We evaluate the transmission of monetary policy in an economy characterized by heterogeneous households and a sizable informal sector. We construct a consumption-based measure of informality at the household level which we use to estimate the Informality Engel Curve. The results are then reproduced endogenously in a dual-sector Heterogeneous Agent New Keynesian (HANK) Model. We test the effects of informality across different model specifications and at different informal sector sizes and then estimate the model's dynamics for the Tunisian economy using the Bayesian method in a novel framework. Our results reveal that: (i) Monetary shocks from our HANK model are stronger and more effective, in terms of sacrifice ratio, than in other specifications, but within our model, the prevalence of informality dampens transmission and increases its cost. (ii) Accounting for informality doesn't appear to undermine the transmission of monetary shocks in Tunisia but restrictive policy favors the expansion of the informal sector and affects informal workers the least. (iii) Wealth remains the primary factor influencing household responses to monetary shocks, but employment status is particularly significant among lower-wealth households.

Keywords: Monetary policy, Heterogeneous Agent New Keynesian model, Informal sector, Informal Engel Curve, Bayesian estimation
JEL Classifications: E12, E26, E31, E52

ملخص

نقوم بتقييم انتقال السياسة النقدية في اقتصاد يتميز بالأسر غير المتجانسة وقطاع غير رسمي كبير الحجم. نقوم ببناء مقياس قائم على الاستهلاك للنشاط غير الرسمي على مستوى الأسرة والذي نستخدمه لتقدير منحنى إنجل للنشاط غير الرسمي. يتم بعد ذلك إعادة إنتاج النتائج داخليًا في نموذج كينزي جديد للعامل غير المتجانس ثنائي القطاع (HANK). نقوم باختبار آثار القطاع غير الرسمي عبر مواصفات نموذجية مختلفة وفي أحجام مختلفة من القطاع غير الرسمي، ثم نقوم بتقدير ديناميكيات النموذج للاقتصاد التونسي باستخدام الطريقة البايزية في إطار جديد. تكشف نتائجنا ما يلي: (1) الصدمات النقدية من نموذج HANK الخاص بنا أقوى وأكثر فعالية، من حيث نسبة التضحية، مقارنة بالمواصفات الأخرى، ولكن ضمن نموذجنا، يؤدي انتشار الطابع غير الرسمي إلى إضعاف انتقال العدوى وزيادة تكلفتها. (ii) لا يبدو أن المحاسبة عن القطاع غير الرسمي تقوض انتقال الصدمات النقدية في تونس، ولكن السياسة التقييدية تفضل توسع القطاع غير الرسمي وتؤثر على العمال غير الرسميين أقل من غيرهم. (iii) تظل الثروة العامل الأساسي الذي يؤثر على استجابات الأسر للصدمات النقدية، ولكن الوضع الوظيفي له أهمية خاصة بين الأسر ذات الثروة المنخفضة.

1. Introduction

In recent years, central banks' monetary policy quickly went from accommodating economic recovery following the COVID-19 pandemic to a restrictive stance in response to a worldwide surge in inflation. The effects of this transition were not felt equally among households. Indeed, monetary policy presents itself as a systematic action that is often based on the evolution of macroeconomic aggregates but affects a wide selection of heterogeneous economic agents and in the process generates asymmetric outcomes, i.e. winners and losers. In the literature, this is often called the "redistributive effect" and recently, a growing body of literature is expanding the theoretical and empirical understanding of its importance for the transmission of monetary policy. Naturally, these works were conducted in the context of advanced economies which puts into question their adoption in emerging markets and developing economies. One aspect that is always missing is a representation of an informal sector which is especially relevant since it represents a sizable share of output and employment in the latter countries.

The informal economy, defined in this paper as economic activities that are hidden from official authorities mainly to avoid taxation, can have different implications on the transmission of monetary policy and its redistributive effects. On the one hand, the informal sector can amplify monetary transmission. The prominence of informal labor can introduce additional sources of employment insecurity and, by extension, increase the probability of uninsurable income shocks, especially at lower wealth levels. These income shocks shape the marginal propensity to consume (MPC) across households, which is at the heart of the redistributive effect of monetary policy (Kaplan et al. (2018)). Moreover, informality can strengthen the earnings heterogeneity channel of monetary policy (Auclert (2019)) since income gain/losses cannot only be equal across the wealth distribution but also between different worker types (unemployed, formal, informal) at the same wealth level. On the other hand, the informal sector can soften the effects of monetary shocks by acting as a "buffer" (Castillo and Montoro (2012), Alberola and Urrutia (2020)), especially when it comes to household consumption, which could weaken transmission and its redistributive effects. To test these mechanisms in a more realistic setup, we also chose to introduce heterogeneity in household exposure to the informal sector represented by the budget share used for informal purchases. This share is not constant among households as documented by the existence of an Informality Engel Curve (IEC) in a large set of countries (Bachas et al. (2023)). Since formal and informal sectors react differently to monetary shocks, the existence of the IEC can create additional distortions to policy transmission at the household level.

We split our analysis into three parts. In the first part, we follow the methodology used in Bachas et al. (2023) to construct a consumption-based measure of informality at the household level using data from the 2021 National Survey on Household Budget, Consumption and Standard of Living (ENBCNV). We propose a baseline naive identification and two calibrated ones to identify different measures of the informal spending share based on the classification of the place of purchase. The processed data is used to estimate the slope of the IEC. In the second part, we develop a one-asset dual-sector Heterogeneous Agent New Keynesian (HANK) model. One sector in the model assumes the role of the informal sector and includes features like lower productivity, tax evasion, and lower friction. The model is solved and the generated monetary shocks transmission is compared with the results from 3 alternative specifications representing the cases of a standard HANK, a dual-sector Representative Agent New Keynesian (RANK), and a stranded RANK. We also test the model at different sizes of the informal sector. This preliminary analysis serves as an initial step to validate our approach. By testing alternative model specifications and assessing the roles of heterogeneity and informality, we ensure that the core structure of our model is appropriate for our study. Finally, we expand our dual-sector HANK model by introducing additional dynamics and shocks. The main feature of this augmented model is its ability to endogenously reproduce the IEC curve as a result of household consumption optimization. We estimate a set of key parameters for friction and shocks using a novel framework that we developed. Our approach combines the methodologies from three different papers to perform the Bayesian estimation of a continuous-time HANK model using discrete-time data. We use the toolbox developed in Ahn et al. (2017) to obtain the rational expectations solution of our HANK model in continuous time. From this solution, we are able to obtain the exact discrete-time state-space representation of our model by following Christensen et al. (2024), and with some modifications, we are able to perform Bayesian inference using the framework developed in Liu and Plagborg-Møller (2023).

From the first part of our analysis, we found a statistically significant negative ICE slope for our 3 identification strategies. This confirms the existence of a negative relationship between the share of informal spending and consumption expenditure, and by proxy wealth, a result that we recreate later in our theoretical model. From the second part, we found a more active "buffering effect" in our dual-sector model when comparing transmission with those of alternative specifications. This is coupled with stronger and more persistent deviation from consumption inflation thus resulting in a significantly lower sacrifice ratio. Once we account for heterogeneity, however, the prevalence of informality seems to dampen policy transmission and increase its cumulative output cost per unit of inflation stabilization.

This suggests that the "buffering effect" overcomes the additional labor income risk and heterogeneity introduced by a larger informal sector. Finally, from our estimated model, we found typical reaction functions for transmission to output and inflation that don't suggest a weakening in the effects of monetary shocks in Tunisia when we consider the effects of informality. This result supports the use of policy rate shocks by the central bank to pursue price stability objectives. Nevertheless, we found that a restrictive monetary shock favors the expansion of the informal sector in the short term. Further investigation of household consumption reveals that the restrictive monetary shock slightly reduces consumption inequality while having the strongest effects among the unemployed workers group and the least on informal ones. Yet, work status is only relevant for the reaction of households to the monetary shock at low wealth as it remains the driving factor.

The paper will proceed as follows. In section 2, we identify the share of household spending on informal goods in the Tunisian economy and estimate the slope of the IEC. In section 3, we develop a dual-sector HANK model and compare the resulting transmission of a monetary shock with alternative model specifications and at different levels of informality. In section 4, we propose some extensions to our HANK model to endogenously recreate the IEC, perform a Bayesian estimation using data from the Tunisian economy, and report policy transmission to aggregates and consumption by household groups. Finally, we conclude and provide some policy recommendations.

2. Informality, Heterogeneity, and Household Consumption

In this section, we explore heterogeneity in informal consumption at the household level and estimate the slope of the Informal Engel Curve. We start by showcasing the prevalence of informality in Tunisia and presenting the problem of its measurement. Then, we investigate consumption behavior across households in Tunisia from microdata and propose three identification strategies, one naive and two calibrated, to identify the share of informal spending. Finally, we estimate the slope of the IEC from the processed data and report our results.

2.1. Informality In Tunisia

Informality is a complex and multifaceted phenomenon especially present in EMDEs. Despite its role in shaping the economies of these countries, it is not well explored in the

macroeconomic literature. The lack of timely and reliable data is a major contributor to the status quo since, by nature, macroeconomic indicators for informal activity can only be inferred. For estimates of the size of the informal economy, methods such as national account discrepancies, general equilibrium simulations, and the Multiple-Indicators and Multiple-Causes (MIMIC) are often used. Notable works attempting in this field include Medina et al. (2018),Medina and Schneider (2019), Elgin et al. (2021), and more recently Asllani et al. (2024). Even then, these estimates present a fuzzy picture of the prevalence of informality. Taking Tunisia as an example, Figure 1 presents the evolution of multiple informal size estimates for Tunisia across the years. Values range from 17% of GDP using the national account estimate to as close to 40%of GDP using the MIMIC method. For its part, the Tunisian National Institute of Statistics (INS) includes an estimate of informal sector activity in their official GDP figures putting it at $27,4\%$ of GDP in 2015[1]. Similarly, informal labor estimates are hard to obtain since they rely mostly on direct methods like labor market surveys. ILO (2018) provide an estimate of 58.8% for the share of informal employment in Tunisia using the 2014 Labour Market Panel Survey while the INS estimates using the National Population and Employment Survey 2019 puts it at 44.8%[2] of the working force.

2.2. Heterogeneity and Household Informal Spending.

The prevalence of informality can also be registered as a sizable share of household spending on the consumption of informal goods. Measures of this share are not observed to be equal across households. Bachas et al. (2023) documents the existence and significance of this type of heterogeneity represented by a downward-sloping Informality Engel Curve (IEC); the informal budget share declines as a function of household total consumption spending. To obtain their results, the authors construct a rough measure of the share of informal spending using data from household consumption surveys for a set of emerging and advanced economies. In their process, data entries for consumption transactions are classified using the type of store where purchases took place into ones conducted in traditional-type or modern-type stores. The share of informal spending is then identified based on the likelihood of store type compliance with taxation, which is the same criterion for informality that we use in our work. Overall, modern stores are, larger, employ more workers, and are better equipped to maintain inventory and accounting records. Also, fiscal authorities are more likely to inspect these larger stores making them more likely to comply with regulations.

[1]Source (documentation is in French): Les Comptes Nationaux changent de base
[2]Source (documentation in Arabic): Indicateurs sur l'emploi informel 2019

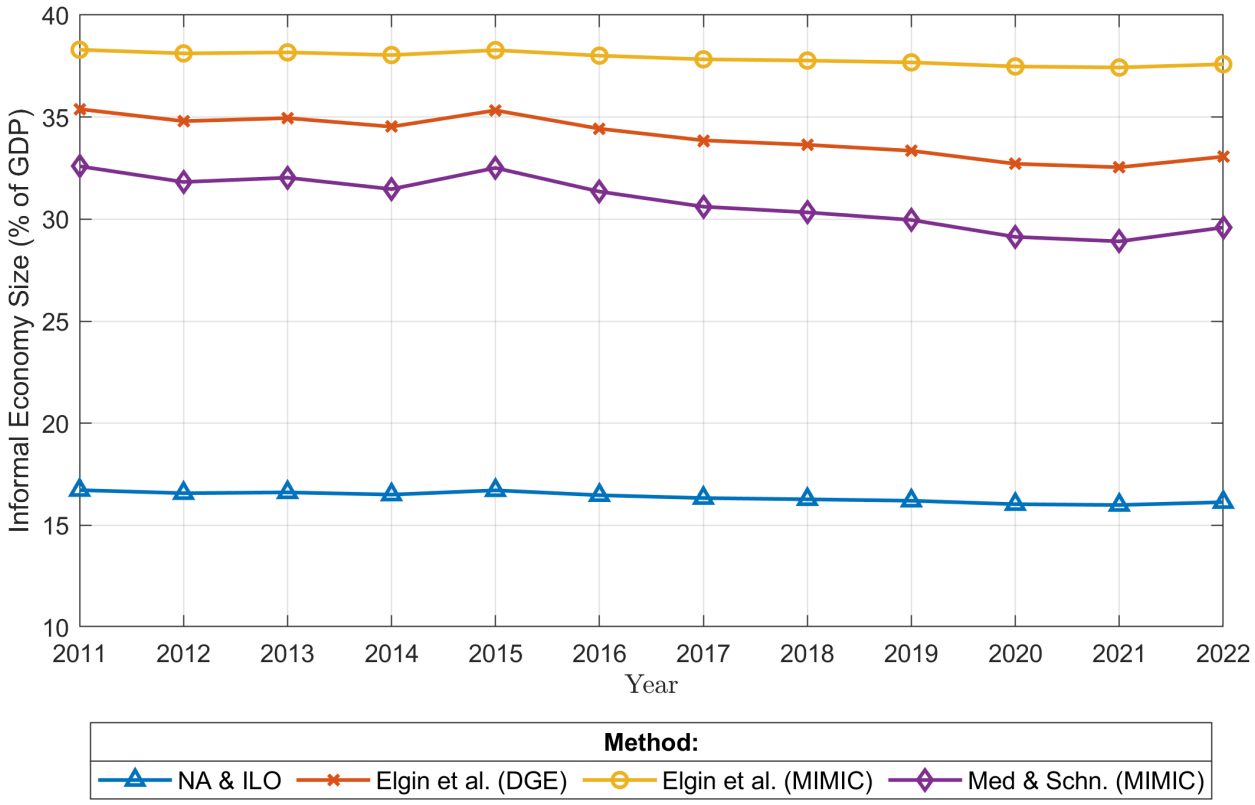


Figure 1: Asllani et al. (2024): Estimates for the informal economy size in Tunisia

In our work, we apply this methodology on the 2021 National Survey on Household Budget, Consumption and Standard of Living(ENBCNV)[3] to construct a consumption-based measure of informality at the household level[4]. In the survey, the "place of purchase" variable takes on the following modalities: (1) "Private shop", (2) "Supermarket", (3) "Fixed market", (4) "Weekly market", (5) "Exhibition", (6) "Online", (7), "Other acquisition place", (8) "Not declared", and (9) "Missing". Inspecting the volume of transactions by store type (Table 1), we find that "Private shop" and "Missing" categories account for roughly 90% of transaction volume (61.58% and 28.53% respectively). Classifying transactions by nature into "good purchase" or —service purchase" [5]. We find that the majority of good purchase transactions took place in the "Private shop" category while the majority of service

[3]The 2021 ENBCNV survey follows the 3 conditions mentioned in Bachas et al. (2023): (1) National representative, (2) Open diary consumption dairy, (3) Reporting the place of purchase of each product.

[4]It should be stressed here that we are not attempting to provide a measure of the aggregate size of the informal sector rather we estimate the potential share of spending allocated to informal products in the household budget.

[5]We classify transactions to "good purchase" or "service purchase" based on the product code provided in the survey.

purchases are classified in the "Missing" category.

Place of purchase	Goods (%)	Services (%)	Total (%)
Private shop	46.254%	15.332%	61.586%
Supermarket	1.842%	0.223%	2.065%
Fixed market	1.474%	0.007%	1.481%
Weekly market	4.008%	0.026%	4.033%
Exhibition	0.017%	0.001%	0.017%
Online	0.062%	0.013%	0.076%
Other acqu.	1.566%	0.114%	1.679%
Not declared	0.182%	0.343%	0.525%
Missing	3.841%	24.697%	28.538%
Total	59.244%	40.756%	100.000%

Table 1: Share of expenditure by type of store and nature of transaction

Investigating the relation between the share of spending at a specific store-type and log total household expenditure (Figure 2) reveals that, ceteris paribus, across seven out of the nine categories, there exists a statistically significant[6]. We find positive slope coefficients for categories "Private shop", "Supermarket", and "Online" and negative slope coefficients for the rest. In particular, we find the steepest slope coefficients for the "Private shop" and the "Missing" categories (5.74 and -4.37 respectively). These results support the presence of heterogeneity in where people shop, an observation that can be explained by household characteristics, store accessibility, and non-homothetic preferences, and is the main reason behind the emergence of the ICE in empirical data.

In their work, Bachas et al. (2023) also combined data sets from the World Bank and Euromonitor International to find that, on average, 75% to 90% of modern stores escape taxation while around 5% to 15% of traditional stores comply with fiscal norms. They also found that these results are fairly consistent between countries at different levels of development. For Tunisia, the INS estimates that, in 2015, 40% of the service sector is conducted in the informal sector [7]. To account for the additional information, we propose three identification strategies; one naive baseline identification and two calibrated strategies.
Baseline identification: The simplest method to identify the share of the informal budget is to follow strict traditional/modern store-type categories. We assign the "Private shop", "Supermarket", and "Online" categories as modern stores and the "Fixed market", "Weekly

[6]p-value $(P > |t|) \leq 0.05$
[7]Source (in French): Les Comptes Nationaux changent de base

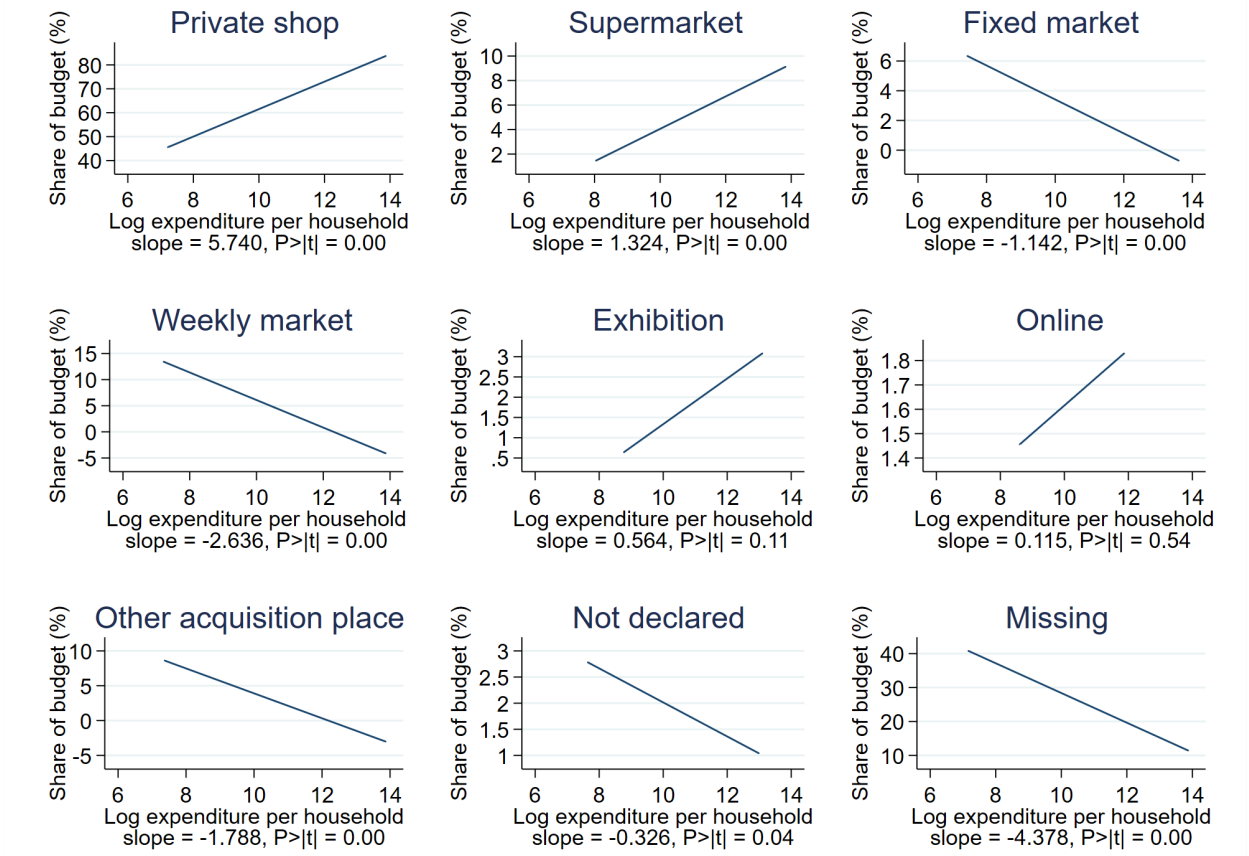


Figure 2: The relation between the share of total consumption and household expenditure

market", and "Exhibition" categories as traditional stores. We classify "Other acquisition place" as traditional stores since it is mostly goods and "Not declared" and "Missing" as modern stores since they consist mostly of services.

Calibrate identification 1: We set the share of informal activity in modern stores and the share of formal activity in traditional stores both to 5%. For both "Not declared" and "Missing" categories, we apply an informality rate of 40% to account for informality in services.

Calibrate identification 2: We set the share of informal activity in modern stores and the share of formal activity to 10%. For the "Not declared" category, we apply an informality rate of 40%. For the "Missing" category, we choose an informality rate of 65%. We base the latter choice on the inspection of the composition of this category.

For these three identification strategies, we determine the share of informal spending in

each household and report the summary statistics in Table 2. In the baseline identification, we get an average share of 9.423%. It is easy to see how these results do not reflect the true scale of informal consumption in Tunisia. The calibrated identifications yield more possible results. On the lower end, we get an average share of 22.686% in the first case and 33.467% in the second. In what follows, we focus on the second calibrated identification, but report results for all three.

Variable	Mean	Std. Dev.	Min	Max
Log Consumption per household	9.678	0.599	7.168	13.866
Share of informal budget: Baseline	9.423	9.210	0	97.679
Share of informal budget: Calibration 1	22.686	7.316	5.1271	93.292
Share of informal budget: Calibration 2	33.467	8.569	10.194	88.742

Table 2: Summary Statistics

2.3. Estimation of the IEC

Using the generated informal sector, we estimate the slope of the IEC for each of our cases using the following regression:

$$Informalshare_i = \beta \ln(consumption_i) + \Gamma X_i + \epsilon_i \tag{1}$$

X_i is a vector of control variables related to the household; size, head's sex, head's age, head's education level, and the urban state. We report the results in Table 3. Given household characteristics, we find a statistically significant slope coefficient and constant for all cases. For shares identified in the second calibrated strategy, we get a β equals -5.423 implying, a reduction of informal budget share by one as household consumption expenditure increases by roughly 20%. Figure 3 showcases the downward trend of the IEC in Tunisia.

3. Monetary Policy in a Heterogeneous Agent New Keynesian Model with Informality

Since we are, to the best of our knowledge, the first to investigate monetary policy in the presence of informality using a HANK model, we begin with a preliminary analysis to validate our approach. We develop a dual-sector HANK model with informality and compare the generated monetary shocks transmission results from 3 alternative specifications. To complete this step, we also investigate the effect of informal prevalence on households' MCP and policy transmission in our model.

	Baseline	Calibration 1	Calibration 2
Log household expenditure	-2.311***	-4.239***	-5.423***
	(0.121)	(0.100)	(0.119)
Household size	0.726***	0.273***	-0.0366
	(0.0465)	(0.0385)	(0.0458)
Household head size	0.00586	0.0126**	0.0157**
	(0.00509)	(0.00421)	(0.00501)
1.Household head sex	-	-	-
	(.)	(.)	(.)
2.Household head sex	-0.914***	-0.173	0.105
	(0.177)	(0.146)	(0.174)
1.Household head eduction	-	-	-
	(.)	(.)	(.)
2.Household head eduction	-0.997***	-0.235	0.0335
	(0.187)	(0.155)	(0.184)
3.Household head eduction	-1.723***	0.0287	0.656**
	(0.214)	(0.177)	(0.210)
4.Household head eduction	-2.451***	1.169***	2.778***
	(0.273)	(0.226)	(0.269)
1.Urban	0	0	0
	(.)	(.)	(.)
2.Rural	5.172***	1.216***	-0.187
	(0.144)	(0.119)	(0.142)
_cons	27.61***	61.31***	84.65***
	(1.145)	(0.947)	(1.127)
N	17118	17118	17118

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Slope of the IEC

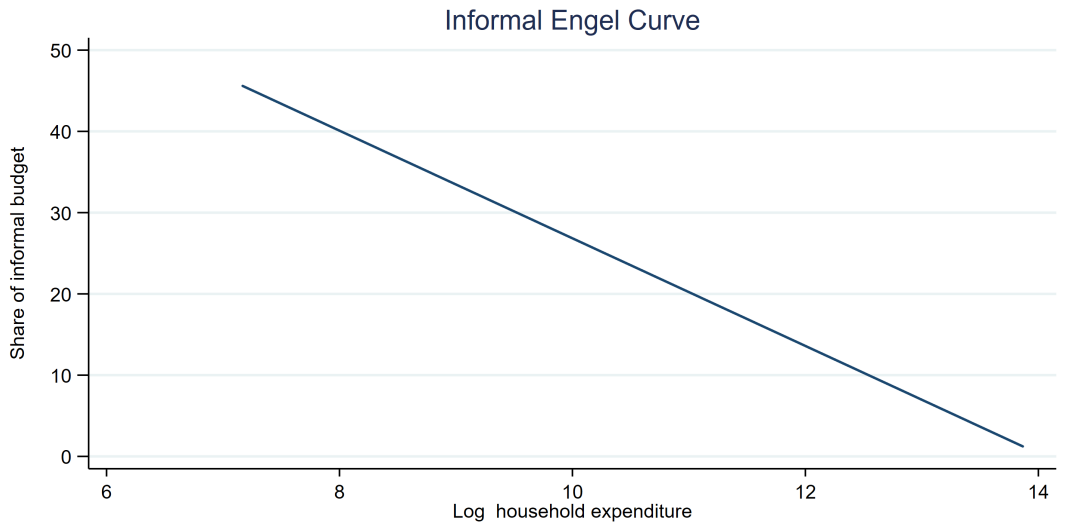


Figure 3: Informality Engel Curve (IEC) for Tunisia

3.1. A Dual-Sector HANK Model with Informality

We develop an illustrative Heterogeneous Agent model. The model is a one-asset HANK model, in the spirit of Kaplan et al. (2018), and augmented with a dual-sector structure analogous to works like Anand and Khera (2016), Moez and Nooman (2019), and Colombo et al. (2019). At this stage, we retain only the core parts necessary for our analysis which allows for the recreation of key stylized facts from the informal economy.

3.1.1. Households

The main feature of the HANK models is the explicit inclusion of heterogeneous states between households. Heterogeneity can be summarized by two idiosyncratic states (a, z). Here, $a \in \mathbb{R}^+$ is the individual wealth of the household, and $z \in \{z_u, z_i, z_f\}$ is the employment status and productivity of the household. Here, $z_t = z_u$ indicates the unemployment state, while z_i and z_f indicate employment in the informal and formal sectors respectively. We make the assumption that the employment state also contains information on the average productivity of workers in their respective groups. As such, we set $z_u = 0$ and choose $z^f > z^i$. We explain the latter choice by the existence of a productivity difference between formal and informal firms (Porta and Shleifer (2008), La Porta and Shleifer (2014)) that we attribute in part to differences in workers' productivity. For simplicity, we assume that the labor status follows a Markov chain where the transition from state i to state $j; j \neq i$ is an exponential process with arrival probability $\lambda_{i,j}; \quad i \in \{u, i, f\}, j \neq i$. We note by $\mu_t(da, dz)$ the joint

distribution for the state of the economy. Households receive utility u from consumption c and disutility from labor l. Preferences are conditional on savings and the future discount rate $\rho \geq 0$. Households maximize:

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t, l_t) dt. \tag{2}$$

We assume that household utility takes the form of a constant relative risk aversion (CRRA) function:

$$u(c_t, l_t) = \frac{(c_t)^{1-\sigma}}{1-\sigma} - \Phi \frac{(l_t)^{1+\phi}}{1+\phi} \tag{3}$$

with σ controlling the elasticity of consumption, ϕ is the inverse Frish elasticity of labor, and Φ are scaling parameters. Household's asset holdings evolve according to:

$$\dot{a}_t = w_t(z_t) z_t l_t + r_t a_t + T_t + \Pi_t - p_t^c c_t \tag{4}$$

Where p_t^c is the real consumption price and $w_t(z_t)$ is the net real wage level associated with the state z_t. r_t is the real return on assets and T_t is a universal government transfer to all households. Combined net profits from formal and informal firms are distributed to households as a dividend, Π_t, based on their productivity-level [8].

3.1.2. Goods Producers

In the economy, there are two sectors of activity; formal, indexed by f, and informal, indexed by i.

Final-Goods Producers:

In each sector $s; s \in \{f, i\}$, a competitive final-good producer aggregates a continuum of intermediate inputs indexed by $j_s \in [0, 1]$ to produce the final good Y_t^s

$$Y_t^s = \left[\int_0^1 \left(y_{j_s,t}^s \right)^{\frac{\xi_s - 1}{\xi_s}} dj_s \right]^{\frac{\xi_s}{\xi_s - 1}} ; \quad s \in \{f, i\} \tag{5}$$

Where ξ_s is the elasticity of substitution across inputs in the sector s. From cost minimization, we obtain the demand functions for the input j_s:

$$y_{tj_s,t}^s = \left(\frac{P_{j_s,t}^s}{P_t^s} \right)^{-\xi_s} Y_t^s \tag{6}$$

[8] As explained in Kaplan et al. (2018), the distribution process of assets plays a critical role in shaping the resulting distribution of wealth.

With the sector-specific price P_t^s:

$$P_t^s = \left[\int_0^1 \left(P_{j_s,t}^s\right)^{1-\xi_s} dj_s\right]^{\frac{1}{1-\xi_s}}; \quad s \in \{f, i\} \tag{7}$$

Intermediate Goods Producers:
There is a continuum of intermediate-good producers, indexed by j_s, operating in each sector $s; s \in \{f, i\}$ and using the production function:

$$y_{j_f,t}^s = Z_t^s n_{j_s,t}^s \tag{8}$$

Here, Z_t^s is the aggregate productivity shocks associated with the sector s. Both shocks follow the Ornstein-Uhlenbeck process:

$$dZ_t^s = -\theta_{z_s} Z_t^s dt + \sigma_{z_s} dW_t^{z_s}; \quad s \in \{f, i\} \tag{9}$$

Firms are subject to sector-specific quadratic price adjustment costs (Rotemberg (1982)) using the following cost function:

$$\Theta_t^s\left(\frac{\dot{P}_t^s}{P_t^s}\right) = \frac{\Omega_s}{2}\left(\frac{\dot{P}_t^s}{P_t^s}\right)^2 y_t^s \tag{10}$$

Where P_t^s is the price of final goods and Ω_s is the price adjustment cost parameter in sector s. From the firm's optimization problem, we can derive the following New Keynesian Phillips curves:

$$\begin{aligned}\left(r_t - \frac{\dot{Y}_t^f}{Y_t^f}\right)\pi_t^f &= \frac{\xi_f}{\Omega_f}\left(m_t^f - p_t^f\frac{\xi_f - 1}{\xi_f}\right) + \dot{\pi}_t^f \\ \left(r_t - \frac{\dot{Y}_t^i}{Y_t^i}\right)\pi_t^i &= \frac{\xi_i}{\Omega_i}\left(m_t^i - p_t^i\frac{\xi_i - 1}{\xi_i}\right) + \dot{\pi}_t^i\end{aligned} \tag{11}$$

Where $\pi_t^f = \frac{\dot{P}_t^f}{P_t^f}$ is the inflation rate of formal goods and $\pi_t^i = \frac{\dot{P}_t^i}{P_t^i}$ is the inflation rate of informal goods, while m_t^s and p_t^s are the real marginal cost and real price in sector $s; s \in \{f, i\}$ respectively.

Consumption Goods Producer:
A competitive consumption goods producer combines formal and informal goods C_t^f and C_t^i respectively to produce the final consumption good C_t following the CES function:

$$C_t = \left[\alpha_c^{\frac{1}{\eta_c}}\left(C_t^f\right)^{\frac{\eta_c-1}{\eta_c}} + (1-\alpha_c)^{\frac{1}{\eta_c}}\left(C_t^i\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} \tag{12}$$

where α_c is the share of the formal good in final consumption goods and η_c is the elasticity of substitution between the two goods. We can express the consumption price, P_t^c, as follows:

$$P_t^c = \left[\alpha_c (P_t^f)^{1-\eta_c} + (1-\alpha_c)(P_t^i)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \tag{13}$$

The aggregate consumption C_t is :

$$C_t = \int c_t(a,z) d\mu_t \tag{14}$$

3.1.3. Monetary Policy

Monetary policy is set using the following Taylor rule:

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y (GDP_t - G\bar{D}P) + \varepsilon_t^{mp} \tag{15}$$

ε_t^{MP} follows the Ornstein-Uhlenbeck process:

$$d\varepsilon_t^{mp} = -\theta_{mp}\varepsilon_t^{mp} + \sigma_{mp}.dW_t^{mp} \tag{16}$$

where dW_t^{mp} is the innovation to a standard Brownian motion, θ_{mp} is the rate of mean reversion, and σ_{mp} captures the size of innovations.

3.1.4. Government

The government budget satisfies the constraint given by:

$$\dot{B}_t^g + p_t^f G_t + T_t = \tau_w w_t^f L_t^f + r_t B_t^g \tag{17}$$

Where L_t^f is aggregate formal labor, τ_w is the income tax collected only on formal workers, and G_t is government expenditure assumed to follow the Ornstein-Uhlenbeck process:

$$dG_t = -\theta_g G_t dt + \sigma_g dW_t^g \tag{18}$$

Finally, to maintain the budget balance and assume that the government uses transfers, T_t, as its policy instrument[9].

[9]The choice of fiscal instrument has an important role in the transmission of shocks as showcased in Kaplan et al. (2018). Other specifications could use taxes, government expenditures, or government debt. Exploring these alternative cases is left out of our analysis.

3.1.5. Equilibrium

The model has 5 markets that should verify equilibrium; the bond market, the formal and informal labor markets, and the formal and informal goods markets. The bond market clearing:

$$B_t^g = \int a d\mu_t \tag{19}$$

Clearing labor markets for the formal and informal sectors:

$$\begin{aligned} L_t^f &= \int z_f l_t(a, z_f) \mu_t(a, z_f) da \\ L_t^i &= \int z_i l_t(a, z_i) \mu_t(a, z_i) da \end{aligned} \tag{20}$$

The formal goods market clearing condition:

$$Y_t^f = C_t^f + G_t + \Theta_t^f \tag{21}$$

And finally clearing the informal goods market

$$Y_t^i = C_t^i + \Theta_t^i \tag{22}$$

We define real GDP as:

$$GDP_t = p_t^f Y_t^f + p_t^i Y_t^i \tag{23}$$

And define the share of the informal sector to GDP as:

$$share_t^y = \frac{p_t^i Y_t^i}{GDP_t} \tag{24}$$

3.2. Alternative Models

To complement our analysis, we also consider three alternative specifications alongside our main dual-sector HANK model.

Standard HANK Model: In this model, we abstract from the hypothesis of dual-sector and treat the economy as a single formal sector. We are left with a standard baseline HANK model encompassing the core features used in the absolute majority of literature available on heterogeneous agent models.

RANK Model with Informality: In this model, we abstract from the assumption of household heterogeneity and retain that of informality. We are left with a dual-sector Representative Agent New Keynesian (RANK) model. This representation is similar to those

often used in the literature on monetary policy in the presence of informality, or on the macroeconomics of informality in general.

Standard RANK Model: This model is representative of the core New Keynesian model and is akin to a textbook example.

3.3. Calibration process

To evaluate the transmission of monetary policy generated by each case, we solve our models using calibrated parameters. We attempt, to the best of our ability, to maintain key values and the resulting steady-state equilibrium between models as close as possible. The values we chose do not follow a specific case but are representative of an emerging market with a sizable informal sector. We present our choice in Table 4:

Symbol	Parameter	$HANK_I$	$HANK_F$	$RANK_I$	$RANK_F$
σ	CRRA parameter	1	1	1	1
ϕ	Inverse Frish elast.	2	2	2	2
Φ	Labor disutility	20.25	20.25	20.25	17.8
ρ	Discount rate, Annualized	1.5%	1.5%	$\bar{r}$	$\bar{r}$
ξ_f	Elast. , Formal	6	6	6	6
ξ_i	Elast. , informal	11	-	11	-
Ω_f	Adj. cost, Formal	100	100	100	100
Ω_i	Adj. cost Informal	75	-	75	-
η_c	Elast,	1.2	-	1.2	-
α_c	Share of formal goods	0.8150	-	0.8153	-
ϕ_π	Taylor rule, inflation	1.25	1.25	1.25	1.25
τ_w	Labor tax	20%	20%	20%	20%
θ_{mp}	Mon. pol. reversion rate	0.25	0.25	0.25	0.25
σ_{mp}	Mon. pol. innovation size	0.2236	0.2236	0.2236	0.2236
$[z_u, z_i, z_f]/[z_u, z_f]$	Relative prod, vector	[0,0.75,1]	[0,1]	-	-
$\bar{r}$	Real rate, SS, Annualized	1.38%	1.28%	1.38%	1.28%
$\bar{\pi}$	Inflation, SS	0	0	0	0
$\frac{\bar{p}^i \bar{Y}^i}{\bar{GDP}}$	Informal sector size, SS	25%	-	25%	-
$\frac{\bar{B}}{\bar{GDP}}$	Asset size to GDP, SS	10	-	10	-
$\frac{\bar{T}}{\bar{GDP}}$	Gov. transfers to GDP, SS	7%	-	7%	-

Table 4: Parameter calibration by model

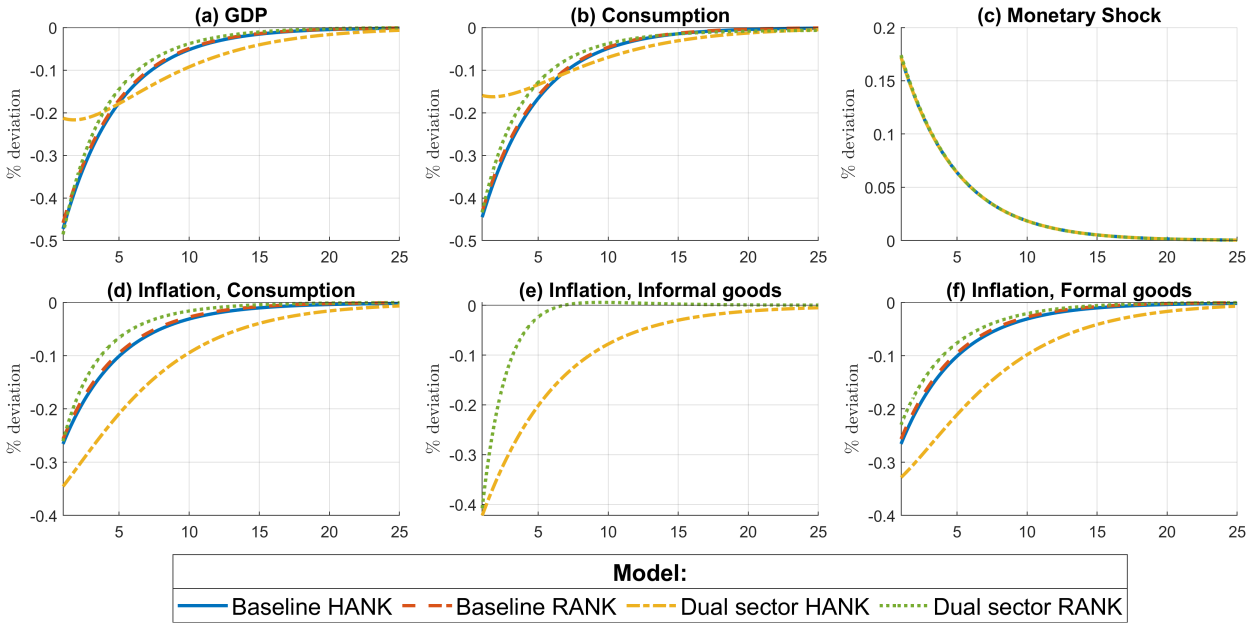


Figure 4: Transmission of monetary policy in different model Specifications

3.4. Model Comparison

With our objective in mind, we study, for each specification, the response of the economy to a positive (restrictive) monetary surprise represented by the Impulse Response Functions (IRFs) in Figure 4. The IRFs measure the percent deviation of key macroeconomic variables (GDP, Consumption, and Inflation) from their respective steady-state values following the realization of a similar size shock (panel (c)). We also report the cumulative percent deviations from steady state after the first year and the sacrifice ratio values in Table 5. We can identify two main takeaways from this figure. First, the presence of the informal sector weakens the transmission of the shock to GDP and Consumption. This is the outcome of the "buffer effect" of the informal sector as described in the literature. The effect is especially noticeable in our dual-sector HANK model where transmission is only half that of the standard HANK model, but the response becomes more persistent (panels (a)&(b)). The second finding is related to the monetary transmission to Consumption inflation, Formal inflation, and Informal inflation rates where, despite the lower losses in GDP and Consumption, the deviation of inflation is stronger and more persistent in our dual-sector HANK (panels (d),(e)&(f)). Working out the sacrifice ratio, described as the percent loss in output required to reduce consumption inflation by one percent from the cumulative deviation after the first year, reveals a significantly lower value from our model than the other specifications (0.697 vs 1.774, 1.786, 1.657). This result stands in contrast to other findings reported in

the literature, especially those of Alberola and Urrutia (2020) who also compares models with/without informality. At this stage, we should clarify that our takeaway is not claiming that the presence of informality improved policy transmission but that the inclusion of informality and heterogeneity in the model identified a more effective monetary transmission. For further clarification, we investigate the former claim in the next paragraph.

Cumulative Effect First Year (%)	$HANK_F$	$RANK_F$	$RANK_I$	$HANK_I$ 10%	$HANK_I$ 25%	$HANK_I$ 40%
GDP	-1.525	-1.478	-1.316	-0.835	-0.8257	-0.789
Consumption	-1.436	-1.392	-1.351	-0.716	-0.6193	-0.503
Consumption inflation	-0.860	-0.827	-1.228	-1.275	-1.184	-1.057
Informal inflation	-	-	-1.103	-1.510	-1.424	-1.299
Formal inflation	-0.860	-0.827	-0.730	-1.275	-1.184	-1.057
sacrifice ratio	1.774	1.786	1.657	0.6546	0.697	0.747

Table 5: Cumulative percent deviations from steady state after the first year (4 quarters)

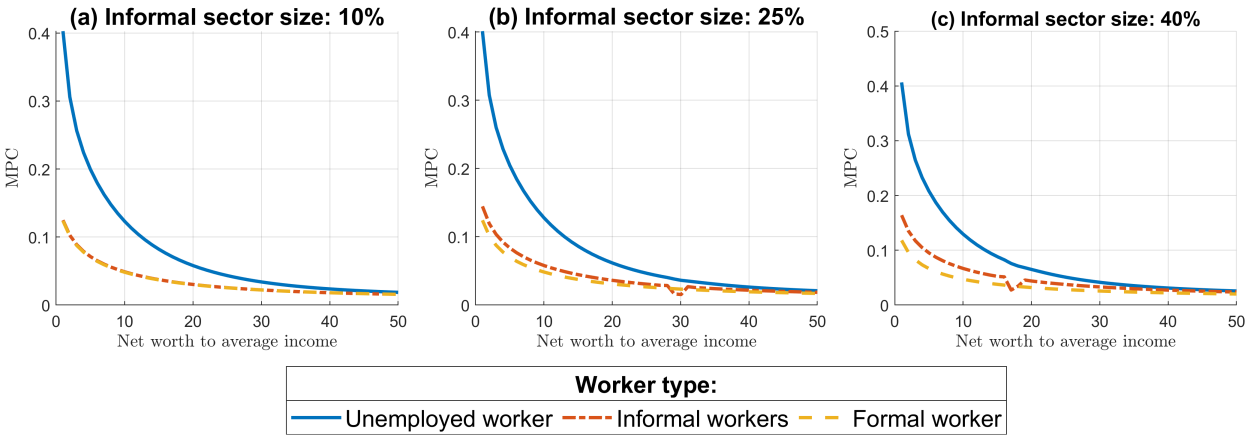


Figure 5: Marginal propensity to consume by informal sector size

We study the transmission of monetary policy at different informality levels, given household heterogeneity, using our dual-sector HANK model. We increase the size of the informal economy at steady state, $\frac{\bar{p}^i \bar{Y}^i}{G\bar{D}P}$, while retaining fixed values for the rest of our calibration parameters. In a first step, we compute the steady-state marginal propensity to consume (MPC), which measures the proportion of an increase in income that a person or household is likely to spend on consumption rather than saving, as a function of wealth and worker-type, and display the results in Figure 5. While at 10% informality size, formal and informal workers display close MPC values at any given wealth level, as the size increases, a wedge is formed between the two worker groups with the MPC curve for informal workers slowly shifting upward. We can rationalize this outcome by considering the increased probability of

uninsurable income shocks and equilibrium outcomes especially the expansion of the formal wage premium. By itself, this increase in MPC heterogeneity should amplify the trans-

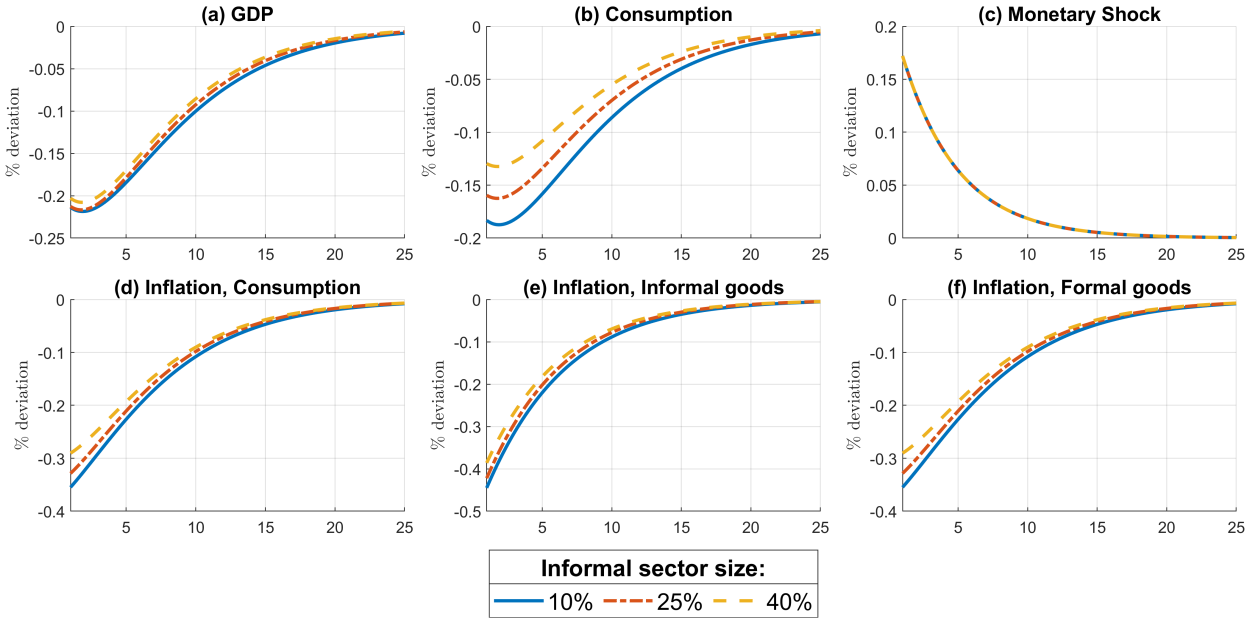


Figure 6: Transmission of monetary policy at different informal sector sizes

mission and redistributive effects of monetary policy, however, if we investigate the IRFs presented in Figure 6, we discover that the "buffer effect" offsets this outcome. In our model, we can see that the increase in informality leads to a weakening in transmission especially for Consumption (panel (b)) where downward divination is reduced by up to 30%. We also find a similar effect for inflation (panel (d)), especially for the formal goods' price inflation (panel (f)). As for the sacrifice ratio, we find a positive relationship with the informal sector size (Table 5).

Overall, we conclude that, from a modeling perspective, accounting for informality and heterogeneity is important for better policy transmission identification but once this is done, informal prevalence, by itself, increases households' MPC heterogeneity, dampens monetary transmission, and raises its cumulative output cost per unit of inflation stabilization.

4. An Estimated Dual-Sector HANK Model for Tunisia

In this section, we combine the insights from the previous two sections. We propose an extended version of the dual-sector HANK model to endogenously recreate the IEC. We

perform a Bayesian estimation on this continuous-time HANK model using discrete-time data using a novel workflow that we develop. Finally, we report the transmission results and give special focus to consumption across different household groups.

4.1. The Augmented Model

We augment our dual-sector HANK model with additional dynamics and shocks in preparation for estimation. The main features we include are; the heterogeneous informal budget share across households, to endogenously recreate the IEC, and investment in capital.

4.1.1. Households

To embed the heterogeneous informal budget share in our model, we separate the consumption utility of the two categories of goods. This is in contrast to the constant share across households implied by the existence of a single final consumption good. Our new assumption for household utility becomes:

$$u(c_t^f, c_t^i, l_t) = \frac{(c_t^f)^{1-\sigma_f}}{1-\sigma_f} + \Phi_i \frac{(c_t^i)^{1-\sigma_i}}{1-\sigma_i} - \Phi_l \frac{(l_t)^{1+\phi_l}}{1+\phi_l} \tag{25}$$

with σ_f and σ_i represent the elasticities of formal and informal goods respectively, ϕ_l is the inverse Frish elasticity of labor, and Φ_i and Φ_l are scaling parameters for informal consumption and labor respectively. The new household's asset holdings evolve following:

$$\dot{a}_t = w_t(z_t) z_t l_t + r_t a_t + T_t + \Pi_t - (1+\tau_c) p_t^f c_t^f - p_t^i c_t^i \tag{26}$$

Where τ_c is a value-added tax, applied only on the consumption of formal products. Also, we introduce a corporate profit tax, τ_f for formal firms.

4.1.2. Assets

We allow households to invest in capital alongside government bonds. Unlike in Kaplan et al. (2018), we do not separate wealth into illiquid and illiquid assets and retain a one-asset structure. Instead, we assume that wealth, a, is composed of bond holdings, b, and physical capital,k. Household wealth can be expressed as:

$$a_t = b_t + q_t k_t \tag{27}$$

Where q_t is the capital price. We assume that households can shift between the two types of assets without cost by imposing the non-arbitrage condition:

$$\frac{r_t^k - \delta q_t + \dot{q}_t}{q_t} = r_t \tag{28}$$

where r_t^k is the rental rate of capital to formal firms and δ is the depreciation rate of capital.

4.1.3. Goods Producers

We introduce capital in the production function of formal firms only:

$$y_{j_f,t}^f = Z_t^f k_{j_f,t}^{\alpha} (n_{j_f,t}^f)^{(1-\alpha)} \tag{29}$$

Where α is the share of capital in the production function. We also include two sector-specific cost-push shocks Ξ_t^f and Ξ_t^i from the formal sector and informal sector respectively. These shocks don't appear in the production functions but affect marginal cost and are similar in effect to markup shocks. We assume that these shocks follow the Ornstein–Uhlenbeck process:

$$d\Xi_t^s = -\theta_{\Xi_s} \Xi_t^s dt + \sigma_{\Xi_s} dW_t^{\Xi_s}; \quad s \in \{f, i\} \tag{30}$$

Since we separated the final consumption good into its components for household consumption, we no longer need to include a final consumption good producer. Aggregate formal consumption and aggregate informal consumption are expressed by:

$$C_t^f = \int c_t^f(a, z) d\mu_t \tag{31}$$

$$C_t^i = \int c_t^i(a, z) d\mu_t \tag{32}$$

4.1.4. Capital Producers

A competitive capital producer transforms formal goods into capital goods bought by households at the price q_t. We assume that the production process is subject to capital adjustment cost with a cost function $\Upsilon(\iota_t + \frac{\Omega_k}{2}(\iota - \delta)^2)K_t$. We note by ι_t the effective investment rate and by Υ_t the marginal efficiency of investment. The producer maximizes the expected profit stream, discounted at the stochastic discount factor of the household:

$$\begin{aligned} W_t = & \max_{\iota_t, K_t} \int_0^{\infty} \Lambda_{0,t} \left(q_t \iota_t - \Upsilon(\iota_t + \frac{\Omega_k}{2}(\iota_t - \delta)^2) \right) K_t dt \\ & s.t. \quad \dot{K}_t = (\iota_t - \delta) K_t \end{aligned} \tag{33}$$

We assume that Υ_t follows the Ornstein–Uhlenbeck process:

$$d\Upsilon_t = -\theta_\Upsilon \Upsilon_t dt + \sigma_\Upsilon dW_t^\Upsilon \tag{34}$$

4.1.5. Government

With the added revenue sources, the government budget constraint becomes:

$$\dot{B}_t^g + p_t^f G_t + T_t = \tau_w w_t^f L_t^f + \tau_c p^f C_t^f + \tau_f \tilde{\Pi}_t^f + r_t B_t^g \tag{35}$$

4.1.6. Equilibrium

For market clearing, we introduce the following modification to the previous equilibrium conditions. Asset market clearing:

$$A_t = \int a d\mu_t(a, z) \tag{36}$$

Assets are distributed into bonds and capital:

$$A_t = B_t^g + q_t K_t; \tag{37}$$

where $K_t = \int k_{j_f,t} dj_f$ is aggregate capital from formal firms.The formal goods market clearing condition becomes:

$$Y_t^f = C_t^f + I_t + G_t + \Theta_t^f \tag{38}$$

where I_t is gross investment.

4.2. Numerical Solution and Estimation Methodology

With the increasing prominence of HANK models in the literature, several computational tools were developed to facilitate their use. For continuous-time models, like the one we use in this section, Ahn et al. (2017) provides an easy-to-use toolbox for solving and simulating HANK models with aggregate shocks. Meanwhile, estimating the HANK model remains a more challenging task and an active area of research. Only a limited number of works like Bayer et al. (2024), Auclert et al. (2021), Fernández-Villaverde et al. (2023), and Acharya et al. (2023) attempted to estimate HANK models. Yet, the tools they develop are incompatible with our work since they treat discrete-time models, and, to our knowledge, we are not aware of any available tools appropriate for our case. Our solution is to develop a custom

workflow that would allow us to perform Bayesian inference on our continuous-time model using discrete-time data.

Our process can be divided into three steps. We first solve the continuous-time model using the toolbox Ahn et al. (2017) to obtain the model's rational expectation solution. The method they employ is based on the works of Achdou et al. (2021) to find the steady-state equilibrium using finite difference methods and of Reiter (2009), among others, to solve the linearized model with aggregate shocks. Second, we follow Christensen et al. (2024) to derive the exact discrete representation of the equilibrium dynamics. This method allows us to avoid discretization errors and construct the discrete-time ABCD state-space representation. Once this is done, we can, with some modifications, use the framework used in Liu and Plagborg-Møller (2023) for our Bayesian inference. We use a generic MCMC algorithm, in this case, the Metropolis-Hastings algorithm, to sample the posterior distribution, given a choice of prior densities. However, since this framework was developed to be used with the Dynare toolkit, we once again reference the work of Christensen et al. (2024) and utilize the Kalman filter they provide to evaluate the likelihood function. While we limit ourselves to a macrodata-only estimation, the proposed method in this article, like that in Liu and Plagborg-Møller (2023), should allow for the incorporation of microdata to perform a full-information estimation.

4.3. Data, Calibration, and Priors

For estimation, we use an observation sample of 7 macroeconomic variables, at a quarterly frequency, from the Tunisian economy from $2015Q2$ to $2022Q4$. We include in our sample the Policy rate, GDP inflation, CPI inflation, GDP, Consumption, Investment, and Government spending. We express quantities in real values and per capita terms. All variables are filtered using the one-way HP-filter. For Consumption, Investment, and Government spending, data is only available in annual frequencies so we opt for using a quadratic low-to-high-frequency transformation filter to obtain data at quarterly frequency [10].

We calibrate the model using standard values from the literature on New Keynesian models, and to replicate key features from the Tunisian economy. We present choices in Table 6. Since we will be evaluating the model at zero steady-state inflation, we set the discount factor to 1.5% (annualized) and the capital depreciation rate to 1.5%. We carefully

[10] We can instead perform a mixed frequency estimation facilitated by the algorithm provided in Christensen et al. (2024)

calibrate σ_f and σ_i to 1.45 and 1.85 since these two parameters are of great importance for fine-tuning the resulting slope of IEC. ξ_f and ξ_i are chosen to allow a mark-up rate of 20% and 10% in the formal and informal markets respectively. The average productivity $\bar{z}$ is chosen to match the average household consumption, in thousands, from the model with survey data. The relative productivity vector $[\tilde{z}_u, \tilde{z}_i, \tilde{z}_f]$ is set to $[0, 0.8, 1]$ implying that an informal workers, is on average 80% as productive as their formal counterparts. For the size of the informal sector, we choose a steady value of 25%. We choose the jump matrix for household employment status to obtain an employment rate of 15.75% and an informal employment rate of 44.71%. We report some of the steady-state results in Table 7.

Symbol	Parameter	Value
ρ	Discount factor, annualized	1.5%
α	Share of capital	0.3
δ	Capital depreciation rate	1.5%
σ_f	Risk aversion for formal consumption	1.45
σ_i	Risk aversion for informal consumption	1.85
ϕ_l	Inverse Frish elasticity	2
Φ_l	Labor disutility	25
ξ_f	Formal goods elast.	6
ξ_i	Informal goods elast.	11
Ω_f	Formal adj. cost	100
τ_w	Labor tax	25%
τ_f	Corporate profits tax	20%
τ_c	Consumption tax	18%
ϕ_y	Taylor output gap	0.3
$\bar{z}$	Average productivity	4.75
$[\tilde{z}_u, \tilde{z}_i, \tilde{z}_f]$	Normalized productivity vector	[0, 0.8, 1]
$\frac{\bar{B}}{G\bar{D}P}$	Government Debt to GDP, SS	60%
$\frac{\bar{T}}{G\bar{D}P}$	Government transfers to GDP, SS	6%
$\bar{\pi}$	Inflation, SS	0

Table 6: Calibrated values

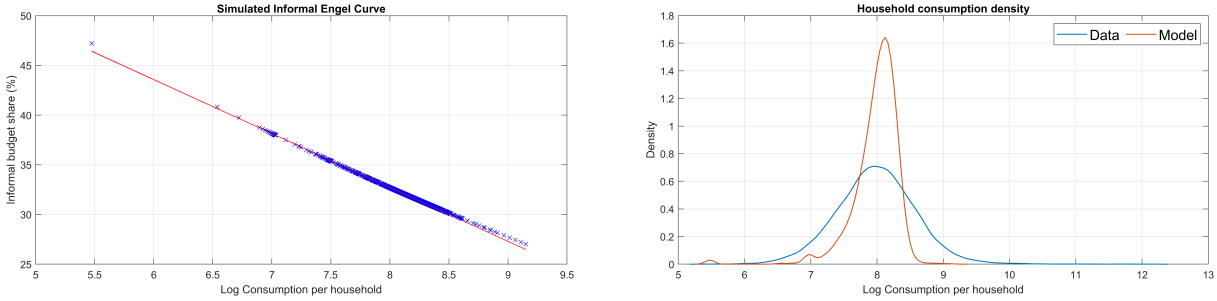
In Figure 7, we present the IEC generated from our model alongside a comparison of the survey and simulated kernel density estimates of the log household quarterly consumption distribution. We were able to recreate the IEC with a slope of -5.4262 against -5.423 that

Symbol	Name	Model	Data
Φ_i	Informal utility scaling parameter	0.2684	-
$\bar{r}$	Interest rate	0.358%	-
$\frac{\bar{p}^c\bar{C}}{\bar{GDP}}$	Consumption to GDP	69.39%	70.67%
$\frac{\bar{p}^I\bar{I}}{\bar{GDP}}$	Investment to GDP	15.39%	18.48%
$\frac{\bar{g}}{\bar{GDP}}$	Government spending to GDP	15.21%	20.26%
U	Unemployment	15.75%	15.78%
$\frac{\bar{p}^i\bar{Y}^i}{\bar{GDP}}$	Informal output share	25%	27.4% [a]
$\frac{\bar{L}^i}{\bar{L}}$	Informal employment share	44.71%	44.8% [b]
$\int \ln(c) d\mu_t$	Log quarterly expenditure per Household	7.988	7.984
$\int \frac{\bar{p}^i\bar{c}^i}{\bar{p}^c\bar{c}} d\mu_t$	Average share of informal consumption	32.74%	33.467%

[a] INS estimate for informal sector in official GDP, 2015.
[b] INS estimate for informal employment, 2019.

Table 7: Steady state results

we estimated earlier. Still, we were not quite able to replicate the consumption distribution. We can mainly attribute this to the inability of our model to create fat-tail distributions, which can be achieved using a two-asset model.



(a) Informality Engel Curve (b) Consumption distributions

Figure 7: Steady state IEC and consumption distributions

We ran a Metropolis-Hastings algorithm with 80,000 draws after a burn-in of 20000 draws. In total, we estimate 16 parameters for frictions and shock processes. We report the prior used and the resulting posterior in Table 8.

	Parameter description	Prior			Postorior	
		Distr	Mean	st.d	Mean	st.d
Ω_f	Formal adj. cost	Gamma	100	25	78.8495	0.0558
Ω_i	Informal adj. cost	Gamma	75	20	74.6879	0.0733

Ω_k	Capital adj. cost	Gamma	0.3	0.2	0.6698	0.1138
ϕ_π	Taylor rule, inflation	Normal	2	1	2.0772	0.0351
θ_{mp}	Reversion rate, mon. pol.	Beta	0.75	0.2	0.3711	0.0555
θ_{z_f}	Reversion rate, formal prod.	Beta	0.75	0.2	0.2893	0.0777
θ_{z_i}	Reversion rate, informal prod.	Beta	0.75	0.2	0.2427	0.1068
θ_g	Reversion rate, gov. exp.	Beta	0.75	0.2	0.3153	0.0858
θ_Υ	Reversion rate, MEI	Beta	0.75	0.2	0.3881	0.0651
σ_{mp}	Innovation size, mon. pol.	Inv-Gamma	0.25	Inf	0.3531	0.0691
σ_{z_f}	Innovation size, formal prod.	Inv-Gamma	0.25	Inf	0.4681	0.0719
σ_{z_i}	Innovation size, informal prod.	Inv-Gamma	0.25	Inf	0.7032	0.1387
σ_g	Innovation size, gov. exp.	Inv-Gamma	0.25	Inf	0.4661	0.0926
σ_{Ξ_f}	Innovation size, formal cost	Inv-Gamma	0.25	Inf	0.3922	0.0403
σ_{Ξ_i}	Innovation size, informal cost	Inv-Gamma	0.25	Inf	0.5749	0.1212
σ_Υ	Innovation size, MEI	Inv-Gamma	0.25	Inf	0.6139	0.1234

Table 8: Estimation priors and results

4.4. The Transmission of Monetary Policy in Tunisia

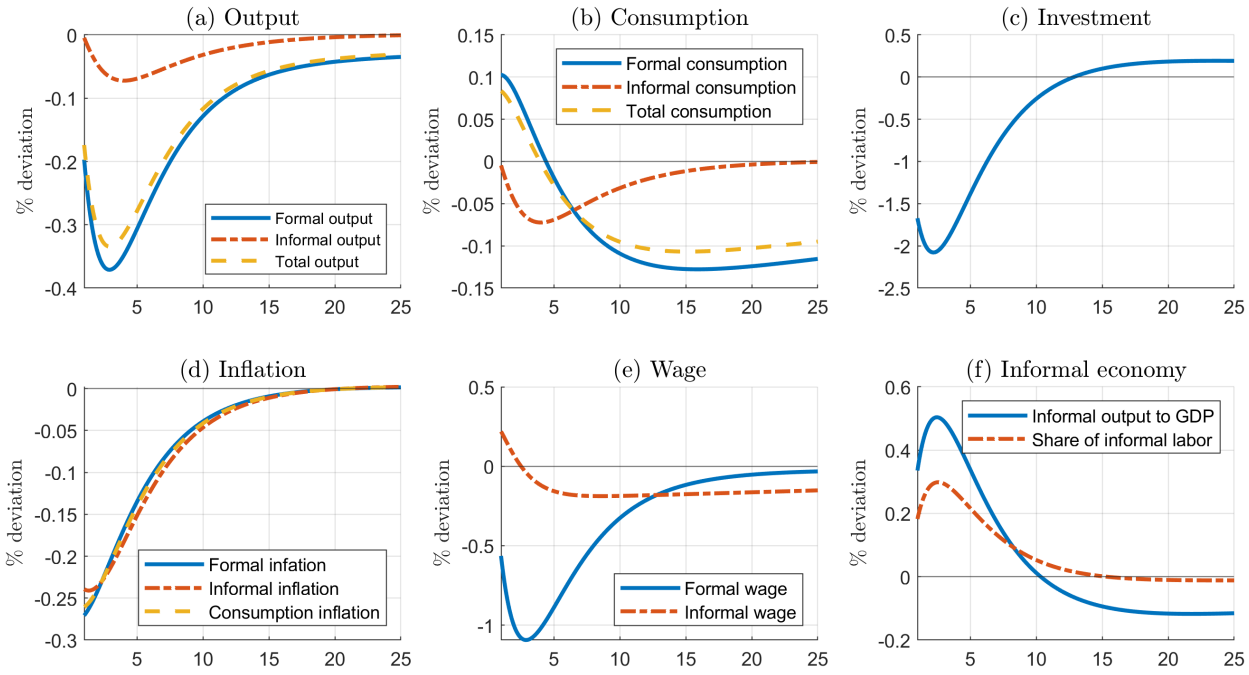


Figure 8: Impulse response function at the posterior mean

To evaluate monetary transmission in Tunisia, we examine the Impulse Response Func-

tions (IRFs) of main macroeconomic aggregates generated by the model at Posterior Mean[11] as a response to a positive 1 standard deviation shock at t_0. In general, results are in line with expectations based on existing literature. Formal output, informal output, GDP, and investment display a downward hump-shaped response, with the trough reaching its minimum by the third period. The buffer effect of informality is especially highlighted in panel (a) with a significantly less pronounced reaction and faster recovery from informal output (-0.06% vs -0.37% at maximum) yielding lower losses in aggregate output (-0.33%). Consumption and formal consumption (panel (b)), exhibit an atypical behavior where they increase on impact (by $+0.1\%$ and $+0.08\%$) before they undershoot their steady state by the fifth period where they persist for the long term. We try to rationalize this behavior by one or more of these reasons; First, an increase in government transfers as a result of surging demand for bonds exceeding revenue losses from taxation. Second, in this model specification, distributed profits increase with restrictive monetary policy as these are counter-cyclical[12]. Third, the presence of indirect effects due to a general equilibrium response from the two sectors or changes in household disposable income relative to the price of consumed goods. We further investigate the redistributive effects of policy on consumption in subsequent parts of our analysis. For inflation rates (panel (d)), we find closely similar responses with an initial drop of around -0.25%, before returning to the steady state by the 18^{th} period. Informal inflation deviates marginally lower and more persistently leading to slightly better transmission to consumption inflation. For the sacrifice ratio, we obtain a value of 0.995 in the first year after the shock which is quite low. We should note that, as our window widens, this sacrifice ratio should increase due to the persistence in the GDP's reaction. Comparing these results with those found in End et al. (2020) using a VAR model doesn't suggest a weakening of monetary transmission as a result of accounting the effects of informality to the model but, on the contrary, it supports our earlier claim that we can identify better transmission, in terms of sacrifice ratio, from our dual-sector model than from standard tools. However, the positive shock favors the expansion of the informal sector in the economy. We can observe in panel (e) that wages in the formal sector drop by as much as 1.1% while that of the informal sector get a slight boost at first before slightly undershooting below its steady-state level. As a response, the share of informal labor in total hours increases by up to 0.3% (panel (f)) while the size of the informal production to GDP goes up at first by up to 0.5%, before reversing its trajectory by the tenth period where it undershoots its steady state for the long term.

[11]Not to be confused with the mean IRFS generated which is often reported as Bayesian IRFs.

[12]In Kaplan et al. (2018) a similar problem arises and is dressed by controlling the share of profits distributed as liquid and illiquid assets, a specification which we can not achieve in our version.

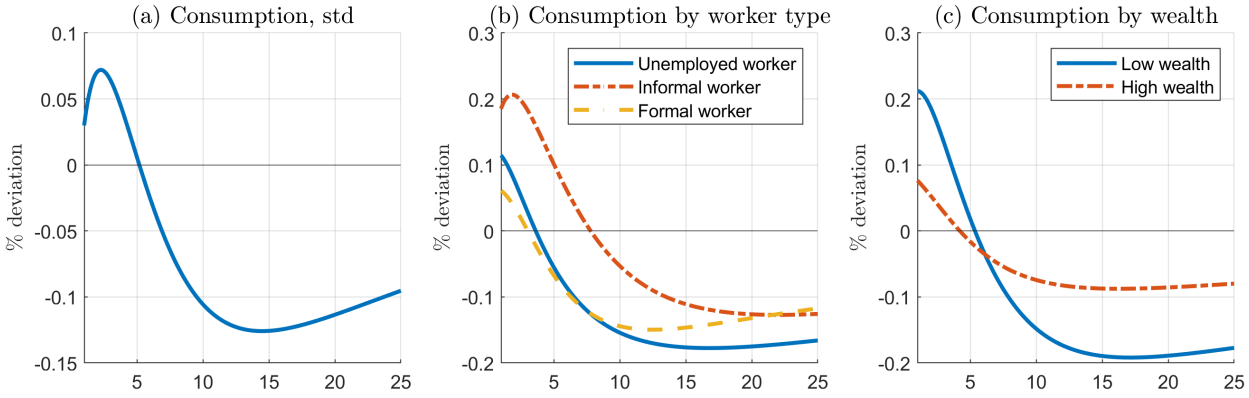


Figure 9: Impulse response function across household distribution

One advantage of HANK models is that they capture the dynamics of the household distribution. In Figure 9 , we take advantage of this feature to explore the reaction of household consumption spending. In panel (a), we report the response of the standard deviation. We find an initial increase in the first five periods followed by a decline in the long term with a maximum of -0.125%. There is still debate about the effects of monetary shocks on household consumption inequality but this result suggests that a restrictive policy reduces consumption inequality, but the effect is subtle. Panel (b) presents the consumption response across the different types of workers. We find that unemployed workers are the most affected by the shock, an expected result though informal workers are the least affected. This result can be, in part, explained by the reaction of informal wages and informal production. If we group households by their wealth level relative to median wealth (panel (c)), we find that low-wealth households are the ones most affected with high-wealth households' reaction being roughly around 65% lower. This is an expected result given the decreasing nature of MCP as a function of wealth.

Going through a more granular analysis, we generate the consumption response at different wealth levels and by worker type in Figure 10. As we saw earlier, the reaction of households with lower wealth is stronger. This is especially the case at the boundary a_{min} (panel (a)) where, despite the initial gains, consumption falls below its steady state level for the long term. The effect is most notable for unemployed workers. On the other side at a_{max} (panel(c)) we can see that consumption follows a similar trajectory across worker-groups, even if initially informal workers are better-off than the rest. These results suggest that wealth remains the deciding factor in shaping the reaction function of household consumption, but the worker type is especially important at lower levels of wealth.

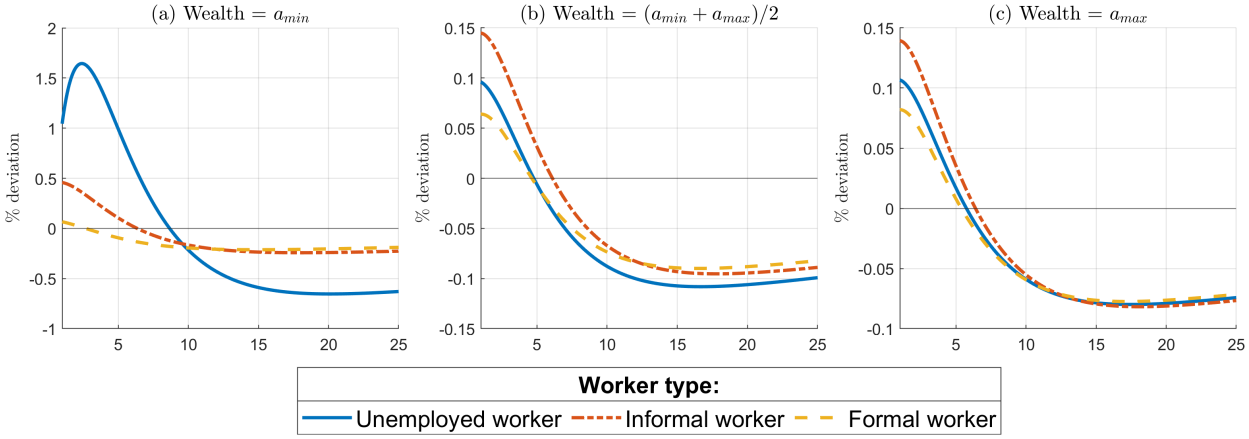


Figure 10: Impulse Response Function by Wealth and Worker Type

5. Conclusion

In this paper, we examine the transmission of monetary policy in an economy characterized by heterogeneous households and a sizable informal sector. A special focus in our analysis is given to household consumption and the informal budget share. We organize our work into three parts. The first part is dedicated to identifying the share of informal spending, using data from the 2021 National Survey on Household Budget, Consumption and Standard of Living (ENBCNV), and estimating the slope of the Informal Engel Curve (IEC). Across the three of our proposed identification strategies, we find statistically significant negative slope values. Second, we investigate the transmission of a monetary policy shock in the presence of informality and heterogeneity. We develop a one-asset dual-sector HANK model and compare transmission across different model specifications and at different informality levels. Results hint at the importance of the combined presence of informality and heterogeneity in identifying policy transmission with stronger and effective transmission, in terms of the sacrifice ratio, from our model. But, once we account for heterogeneity, the expansion of the informal sector dampens the transmission and increases its cost as measured by the sacrifice ratio. Finally, we extend our model to include, among other features, an endogenously reproduced IEC. We are able to estimate this continuous-time HANK model using discrete-time data by developing a novel workflow based on methods from three different toolboxes. The transmission results from the Bayesian estimation don't suggest any weakening in the effectiveness of monetary shocks as a tool to pursue price stability. However, we find that restrictive policy favors the expansion of the informal sector. By investigating household consumption by group, we find that the monetary shocks slightly reduce consumption inequality and that unemployed workers seem to be most affected while informal workers are

the least. We also find that employment status is only important at lower wealth levels and as it increases, households' reactions converge.

While we develop our tools for illustrative porpoises only, Monetary authorities in emerging economies, like Tunisia, should draw inspiration from our results to explicitly include the informal sector in their policy tools. Doing so will not only improve the efficacy of their decision-making but also improve public trust in their policy as a result of adopting a more appropriate representation of their economies. In particular, our result regarding monetary transmission in the presence of informality can provide policymakers with additional headroom to lean against the wind. Nevertheless, policy implication on the informal sector presents themselves as an additional externality for the central bank to consider. Also, coordination with fiscal authorities on, targeted direct transfers, can prove useful to mitigate harmful repercussions of monetary policy on vulnerable segments of the population without sacrificing policy objectives.

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