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Understanding Policy Transmission in Tunisia from a Heterogeneous Agents Model

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Monetary Policy and Informality: Understanding Policy Transmission in Tunisia From a Heterogeneous Agents Model*

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Abstract

We evaluate the transmission of monetary policy in an economy characterized by heterogeneous households and a sizable informal sector. We develop a Duel-sector Heterogeneous Agents new Keynesian Model to recreate the Informal Engel Curve observed in the survey data. Results suggest that, first, heterogeneity amplifies the buffering effect of informality. Second, estimating the model using the Bayesian method reveals that unemployed and informal workers are more vulnerable to the redistributive effect of monetary policy shocks than their formal counterparts, especially at lower wealth levels.

Keywords: Monetary policy, Heterogeneous agents, Informal sector, Informal Engel Curve, Bayesian estimation
JEL: E12, E52, E26, E31

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Contents

1. Introduction

In recent years, central banks' monetary policy quickly went from accommodating economic recovery following the COVID-19 pandemic to a restrictive stance in response to a worldwide surge in inflation. The effects of this transition were not felt equally among households. Indeed, the monetary policy presents itself as a systematic action that is often based on the evolution of macroeconomic aggregates but affects a wide selection of heterogeneous economic agents and in the process generates asymmetric outcomes, i.e. winners and losers. In the literature, this is often called the "redistributive effect" and recently, a growing number of papers have provided theoretical and empirical evidence for its importance for the transmission of monetary policy. However, these papers only focus on the case of developing economies and, as a result, some key features of emerging market and developing economies are omitted. Of these, a sizable informal sector might be one of the most prominent.

The informal economy, defined in this paper all economic activities that are hidden from official authorities and that would contribute to the officially calculated (or observed) Gross National Product if observed (Medina et al. (2018)), can have major implications on the transmission of monetary policy and its redistributive effects in more than one way. On the one hand, the prominence of informal labor is an additional source of employment insecurity and, by extension, increases the probability of uninsurable income shocks, especially in lower wealth quantities. These shocks are at the core of the redistributive effect of monetary policy (Kaplan et al. (2018)). On the other hand, the informal sector can dampen the effects of shocks especially when it comes to household consumption (Castillo and Montoro (2012), Alberola and Urrutia (2020)), which can lead to a lower marginal propensity to consumption (MPC), another important factor in the redistributive effect of monetary policy. In this paper, we aim to shed light on this topic by studying the transmission of monetary policy in an economy characterized by heterogeneous households and a sizable informal sector.

Since we are, to our knowledge, the first to examine this topic, we seek to justify, from a modeling perspective, the inclusion of informality and heterogeneity in monetary policy analysis. We first showcase the presence of distortionary effects introduced by the presence of informality. We compare the simulated Impulse Response Function generated by four illustrative models combining the aspects of "fully formal" and "with informality" with that of "representative agent" with "Heterogeneous agents". We find that combining informality with heterogeneity leads to a strengthening of the buffering effect of informality on output and consumption. We also find that this buffering effect increases with the size of the

informal sector. We then develop a medium size model specific to the Tunisian case. We set up the model to recreate the Informal Engel Curve (IEC). We follow the methodology used in Bachas et al. (2023) and estimate the IEC using data from the 2021 Tunisian National Survey on Budget, Consumption, and Living Standers (ENBCNV) and use the results to calibrate the distribution of consumption in our model. We then estimate a set of key parameters for friction and shocks using Bayesian inference. We use the use the toolbox developed in Liu and Plagborg-Møller (2023). Since we use continuous-time HANK models for their advantage (Ahn et al. (2017), Achdou et al. (2021)), we adopt the using use the method proposed in Christensen et al. (2024) based on an exact discrete-time representation for estimating continuous models using discrete data. From the generated results, we found typical reaction functions from output and inflation, showcasing the buffering effect, but an ambiguous reaction from consumption characterized by a long brief increase followed by a prolonged downward deviation. We also found that unemployed and informal workers seem to be most affected by the monetary shock, especially with lower wealth.

The paper will proceed as follows, In the first section, we showcase the distortionary effects of informality on the transmission of monetary policy using a set of illustrative models. In the second section, we examine key aspects of the informal sector in Tunisian before developing a medium-sized HANK model for monetary policy analysis that allows us to recreate these aspects. In the third section, we present our empirical strategy for calibrating and estimating the model using the Bayesian methods we then report the main results on the transmission for monetary policy and associated redistributive effect. Finally, we conclude and provide some recommendations.

2. Monetary Policy, Informality, and Heterogeneous Agents

We dedicate this section to highlighting the distortionary effect of heterogeneity and informality on the transmission of monetary policy. We proceed with this step to identify the existence of meaningful quantitative and qualitative effects of informal sector dynamics thus legitimizing, from a modeling perspective, their inclusion in the study on monetary policy analysis. To this end, we develop four illustrative models and simulate results under different conditions. At the center of these is the simple HANK model with informality. The rest three models represent the cases of representative agents without informality, representative agents with informality, and heterogeneous agents without informality. We also compare transmission for the cases of three economies with different levels of informality (low: 10%,

medium: 25%, and high: 40%). All models are developed in continuous time.

2.1. Small HANK Model with Informality

We develop an illustrative small-scale model with heterogynous agents. The model is a one-type-asset HANK model, in the spirit of Aiyagari (1994) and Krusell and Smith (1998), augmented with a dual-sector structure in an analog manner to works like Castillo and Montoro (2010), Castillo and Montoro (2012), or Alberola and Urrutia (2020). At this stage, we retain only the core parts necessary for our analysis which allows for the recreation of key stylized facts from the informal economy.

2.1.1. Households

The main feature of the HANK models is the explicit introduction of heterogeneous states between households. In this simple model, we assume that this heterogeneity can be summarized by two idiosyncratic states (a, z) where $a \in \mathbb{R}^+$ is the individual wealth of the household, $z \in \{0, z^i, z^f\}$ is the employment status and productivity of the household. Here, $z = 0$ indicates the unemployment state, while z^i and z^f indicate employment in the informal and formal sectors respectively. S Since on average, informal firms tend to be less productive than their formal counterpart. (Porta and Shleifer (2008) , La Porta and Shleifer (2014)), we impose that $z^f > z^i$. For simplicity, we assume that the labor, sector, and productivity are exogenous and follow a Poisson process with an arrival probability $\lambda^j; \quad j \in \{0, i, f\}$. We note by $\mu(da, dz)$ the joint distribution for the state of the economy. Households receive utility u from consumption c and disutility from labor l. Preferences are conditional on savings and the future discount rate $\rho \leq 0$. The household maximization problem differs between labor and sector states.

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t, l_t) dt. \tag{1}$$

We assume that household utility takes the form of a constant relative risk aversion (CRRA) function:

$$u(c_t, l_t) = \frac{(c_t)^{1-\sigma}}{1-\sigma} - \Phi \frac{(l_t)^{1+\phi}}{1+\phi} \tag{2}$$

with σ controlling the elasticity of consumption and ϕ is the inverse Frish elasticity of labor. Φ are scaling parameters.

Household's asset holdings evolve according to:

$$\dot{a} = w_t(z_t) z_t l_t + r_t a_t + T_t + \Pi - c_t \tag{3}$$

Where $w_t(z_t)$ is the wage level associated with the state z_t, andl_t is the labor supply. r_t is the real return on assets and T_t is a universal government transfer to all households. Profits are shared between households based on their productivity level in the form of a net transfer Π_t.

2.1.2. Goods Production

The production of goods in the economy follows a standard New Keynesian setup. Intermediate and final goods producers can be one of two types; Formal, indexed by f, and informal, indexed by i.

Intermediate Goods Producers:
There is a continuum of intermediate-good producers, indexed by j_s, operating in each sector $s; s \in \{f, i\}$ and using the production function:

$$y^s_{j_f,t} = n^s_{j_s,t} \tag{4}$$

Firms are subject to sector-specific quadratic price adjustment costs following Rotemberg (1982):

$$\Theta^s_t \left(\frac{\dot{p^s_t}}{p^s_t}\right) = \frac{\Omega^s}{2}\left(\frac{\dot{p^s_t}}{p^s_t}\right)^2 y^s_t \tag{5}$$

From the firm's optimization problem, we can derive the New Keynesian Phillips curves for each type of firm.

Final-Goods Producers:
Final goods produced by formal firms and final goods produced by informal firms are produced using intermediate inputs, y^f_t and y^i_t, from formal and informal intermediate goods producers respectively.

$$y^s_t = \left[\int_0^1 \left(y^s_{j_s,t}\right)^{\frac{1}{\xi_s}} dj_s\right]^{\xi_s} \quad s \in \{f, i\} \tag{6}$$

Consumption Goods Producer:
A competitive consumption goods producer combines formal and informal goods c^f_t and c^i_t respectively to produce the final consumption good c_t following the CES function:

$$C_t = \left[\alpha_c^{\frac{1}{\eta - c}}(C^f_t)^{\frac{\eta_c - 1}{\eta_c}} + (1-\alpha_c)^{\frac{1}{\eta - c}}(C^i_t)^{\frac{\eta_c - 1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c - 1}} \tag{7}$$

where α_c is the share of the formal good in final consumption goods and η_c is the elasticity of substitution between the two goods.

2.1.3. Monetary Policy

Monetary policy is set using the following Taylor rule:

$$i_t = \bar{r} + \phi_\pi \pi_t + \pi_y(y_t - \bar{y}) + \varepsilon_t^{MP} \tag{8}$$

ε_t^{MP} follows the Ornstein-Uhlenbeck process:

$$d\varepsilon_t^{mp} = -\theta_{mp}\varepsilon_t^{mp} + \sigma_{mp}.dW_t^{mp} \tag{9}$$

where dW_t is the innovation to a standard Brownian motion, θ_{MP} is the rate of mean reversion, and σ captures the size of innovations.

2.1.4. Government

The government budget satisfies the constraint given by:

$$G_t + T_t = \tau_t \int W(z, l^f)\mu_t(a, z, l^f)dadz + r_t B_t^g \tag{10}$$

The government can collect taxes only from formal sector workers and uses transfers T_t as its policy tool.

2.1.5. Equilibrium

There are five markets in our economy, the asset market, the formal and informal labor markets, and the formal and informal goods markets. In equilibrium, all markets are cleared.

2.2. Alternative Models

Along with the main model, we also develop three models that represent the cases of representative agents without informality, representative agents with informality, and heterogeneous agents without informality. These models are representatives from those used in several current literature strands, but in continuous time.

Standard HANK Model: In this model, we abstract from the inclusion of the informal sector and retain only the formal part of the economy. We are left with a standard baseline HANK model encompassing the core features used in the absolute majority of literature on heterogeneous agent models available.

RANK Model with Informality: In this representation, we abstract from the hypothesis of household heterogeneity and retain that of informality. We are left with a standard representative agent model with informality containing core features from the dual sector New Keynesian models used in the literature strands on monetary policy in the presence of informality, or the macroeconomics of informality in general.

Standard RANK Model: This model is representative of the core New Keynesian model and is akin to a textbook example. This model surveys as a baseline case for our comparison.

2.3. Calibration process

To evaluate the transmission of monetary policy generated by each case, simulate our models using calibrated values. We maintain key parameters constant across models. We attempt, to the best of our ability, to maintain the resulting steady-state equilibrium as closely as possible. We present our calibration choice in Table 1:

Symbol	Parameter	$HANK_I$	$HANK_F$	$RANK_I$	$RANK_F$
σ	CRRA parameter	1	1	1	1
ϕ	Inverse Frish elast.	2	2	2	2
Φ	Labor disutility	20.25	20.25	20.25	17.8
ρ	Discount rate	1.5%	1.5%	1.5%	1.5%
ξ_f	Elast. , Formal	6	6	6	6
ξ_i	Elast. , informal	11	-	11	-
Ω_f	Adj. cost, Formal	100	100	100	100
Ω_i	Adj. cost Informal	75	75	-	-
η_c	Elast,	1.2	-	1.2	-
α_c	Share of formal goods	0.8150	-	0.8153	-
π_{pi}	Taylor rule, inflation	1.25	1.25	1.25	1.25
τ_w	Labor tax	20%	20%	20%	20%
θ_{mp}	Mon. pol. reversion rate	0.25	0.25	0.25	0.25
σ_{mp}	Mon. pol. innovation size	0.2236	0.2236	0.2236	0.2236
$[z_u, z_i, z_f]$	Normalized prod, vector	[0,0.75,1]	[0,0.75,1]	-	-
$\bar{r}$	Steady state rate	1.38%	1.28%	1.38%	1.28%
π	Steady state inflation	0	0	0	0
$\frac{p^i Y^i}{Y^f}$	Informal sector size	25%	0	25%	0
$\frac{B}{Y^f}$	Asset size to GDP	10	-	10	-
$\frac{T}{Y_s s}$	Gov. transfers to GDP	7%	-	7%	-

Table 1: Calibration by model

2.4. Model Comparison

With our objective in mind, we simulate the response of the economy to a non-anticipated monetary policy innovation. We report the Impulse Response Functions (IRFs) generated by a positive (restrictive) monetary policy shock in Figure 1. The IRFs measure the percent deviation of key macroeconomic variables (GDP, Consumption, and Inflation) from their respective steady-state values following the realization of the shock. We mainly find that results from the HANK model with informality deviate substantially from the others. This effect can be seen in lower losses in consumption and GDP, in line with the 'buffering effect' characteristic of informality. This effect could be seen, to a lesser degree in the RANK model with informality. On the other hand, for inflation, we register a stronger reaction translated as a greater downward deviation of inflation. This comes in contrast to the RANK model where the response to inflation is lower than the models with only the formal part which is the case often reported in the literature (Castillo and Montoro (2010), Castillo and Montoro (2012), Alberola and Urrutia (2020)). We can also identify a stronger reaction from informal price inflation than the formal leading to a stronger transmission to consumption price inflation.

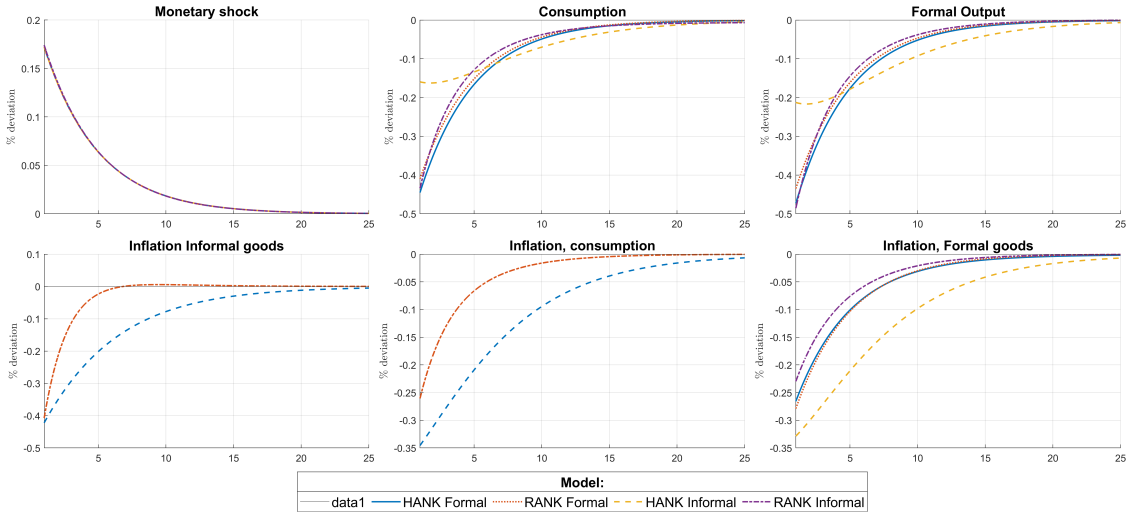


Figure 1: Transmission of monetary policy in different models

We now investigate the effect informal sector size can have on the transmission of monetary policy. In the first step, we test the effect of varying the size of the informal sector at a steady state, given the fixed calibration parameters. We simulate the response of the economy to a same-size policy shock and report the results in Figure 2. We find that, under these conditions, an increase in the informal sector size results in a strengthening of the "buffer

effect". This is especially seen in consumption where losses are reduced by up to 30%. On the other hand, the effect on inflation is less important as we find lower transmission as the informal sector grows. In the second step, we compute the marginal propensity to consume (MPC) for each type of worker at different informal sector sizes. The MPC which measures the proportion of an increase in income that a person or household is likely to spend on consumption rather than saving, plays an integral part in heterogeneous agents models (Kaplan et al. (2018), Auclert (2019)). Results are displayed in Figure 2. While at lower informality levels formal and informal workers display similar MPC at any given wealth level, as informality increases, there a diversion is created between the two types. These results indicate the existence of an important distortionary effect introduced by the prance on an informal sector. This effect seems to be amplified by the presence of household heterogeneity and the increased size of the informal sector.

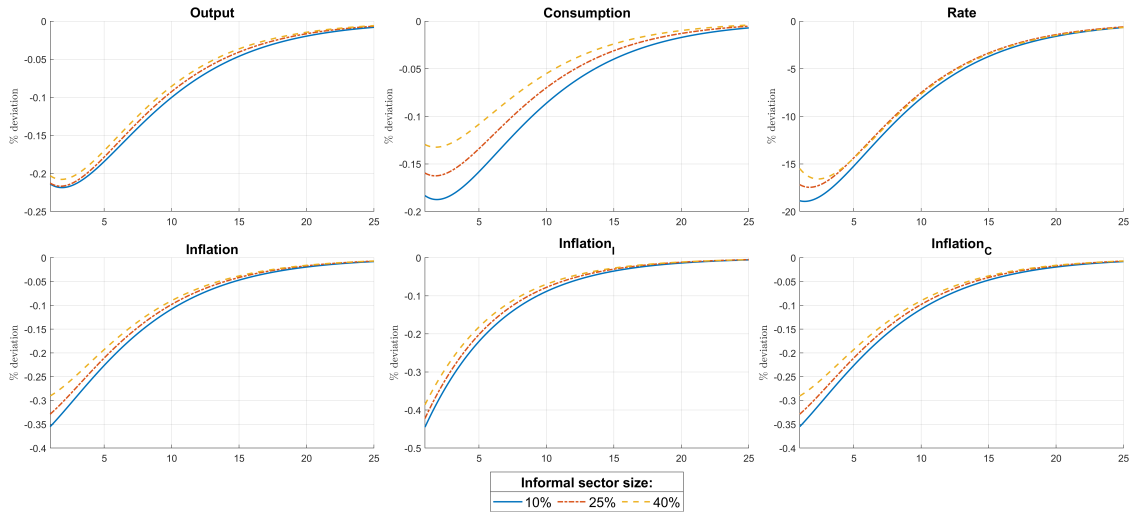


Figure 2: Transmission of monetary policy at different informal sector sizes

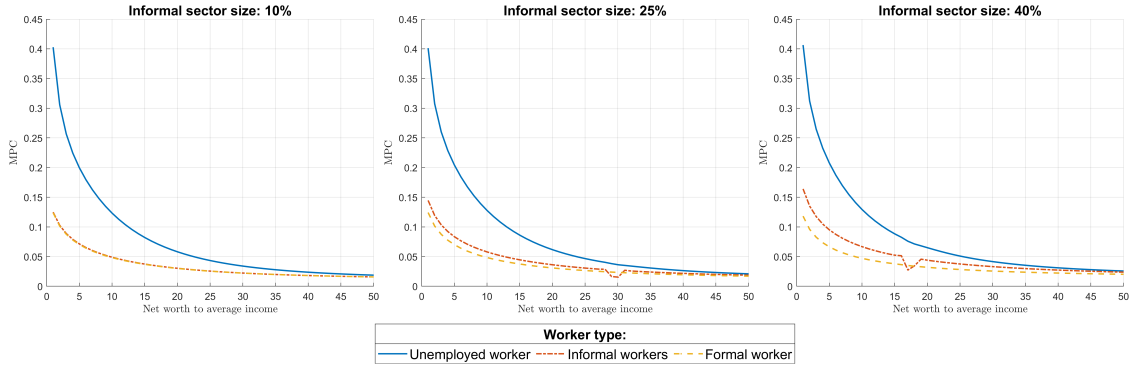


Figure 3: Marginal propensity to consume by informal sector size

3. A HANK model with Informality for Tunisia

3.1. Stylized Fact on Informality In Tunisia

Informality is a complex and multifaceted phenomenon present to a large extent in emerging market economies. Like most countries, there are several attempts to quantify various aspects of informality for the country. Most notably, Medina et al. (2018), Medina and Schneider (2019), and more recently Asllani et al. (2024) provide an estimate of the shadow economy using the Multiple-Indicators and Multiple-Causes (MIMIC) method. Elgin et al. (2021) also provided an estimation of the informal sector size using the MIMIC method and a general equilibrium estimate. Figure 4 presents the evolution of multiple informal size estimates for Tunisia across the years. With the exception of the national account estimate, the three other methods suggest the presence of a shadow economy representing more than 35% of GDP. The size however is following a downward trend followed by a small uptick in 2022. As for informal labor, estimates are harder to infer since they rely mostly on direct methods like surveys. ILO (2018) provide an estimate of 58.8 % based on the labor survey 2014 (citation). On the other hand, the Tunisian National Institute for Statistics estimates that in Q4 2019, the size of informal laborers represented 44.8% of the working force.

Informal budget share across households

Informality can affect households in a multitude of ways and the effects are far from equal. One of such effects comes down to the informal consumption behavior that can be defined as the expenditure on informal goods out of total expenditure. Bachas et al. (2023) document the existence and significance of this type of heterogeneity across households for a set of countries, including Tunisia, to assess the redistributive effect of taxes on consumption in developing countries. The authors showcase the existence of a downward-sloping informality Engel curve (IEC): the informal budget share steeply declines with household income, and by proxy consumption, in every country.

Data on Informal spending

To identify the share of informal spending in total consumption for households in Tunisia, we follow the methodology used in Bachas et al. (2023). We use the 2021 Tunisian National Survey on Budget, Consumption, and Living Standards (ENBCNV). The survey contains information on the purchase place of consumed items. These are then classified into traditional store type and modern store type. We use a simple assignment strategy based on the

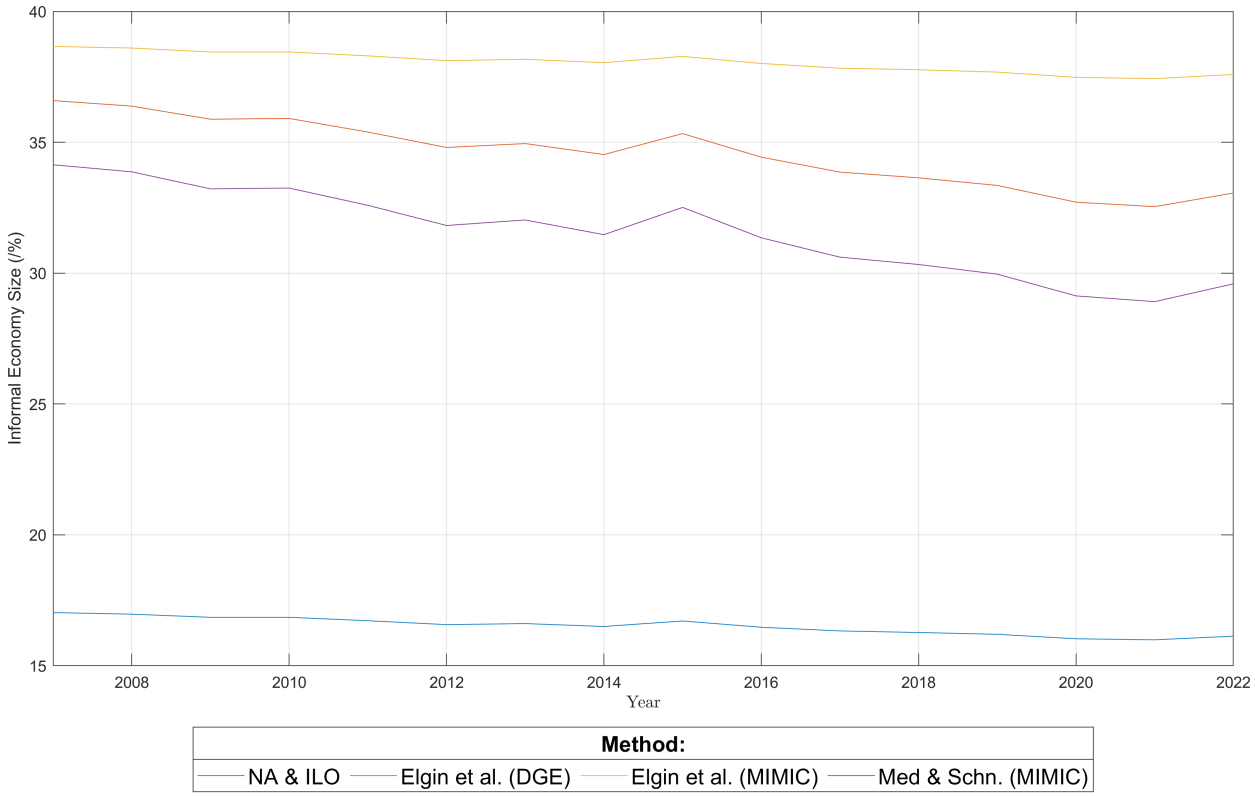


Figure 4: Asllani et al. (2024): The Size of Informal Economy In Tunisia

likelihood of the likelihood of taxes being levied on consumer prices in a given store type. As such, purchases made from traditional stores are accounted as informal expenditure and those made from modern stores are accounted as formal expenditure[1]. We can obtain the informal budget share for each household and estimate the slope of the Informal Engel Curve (IEC).

Variable	Obs	Mean	Std. Dev.	Min	Max
Consumption per capita	17,394	5468.35	6320.386	185.412	526271.2
Log consumption per capita	17,394	8.395	0.611	5.222	13.17357
Share of informal budget	17,394	35.60	12.74	0.349	100

Table 2: Summary Statistics

The Informal Engel Curve

Using the generated data, we estimate the slope of the IEC using the following regression:

$$Informalshare_i = \beta \ln(consumption_i) + \Gamma X_i + \epsilon_i \tag{11}$$

[1]We limit our analysis to the beeline case presented in this work, for simplicity.

	(1) Informal Budget Share	(2) Informal Budget Share
log consumption	-3.524***	-7.303***
	(0.156)	(0.185)
HH head size		-1.983***
		(0.0695)
HH head sex		1.271***
		(0.294)
HH head age		0.0234**
		(0.00801)
HH head edu		1.964***
		(0.126)
Urban		3.058***
		(0.213)
Constant	65.19***	96.14***
	(1.312)	(1.656)
N	17394	17118

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Slope of the IEC

Notes: The first model is a simple representation $Informalshare_i = \beta_0 + \beta_1 \ln(consumption_i) + \epsilon_i$. The second model is the full representation presented above.

X_i is a vector of control variables related to the household; size, head's sex, head's age, head's education level, and the urban state. We report the results in Table 3. Given household characteristics, we find a statistically significant slope coefficient β equal to -0.0644. Figure 5 showcases the downward trend of the IEC in Tunisia. The informal share goes down from upward of 50% to as low as 15% as higher levels of spending indicating a significant level of heterogeneity of informal consumption between households at different consumption levels. Inspired by this result, and motivated by the findings of the previous section, we investigate their combined effect on the transmission of monetary policy.

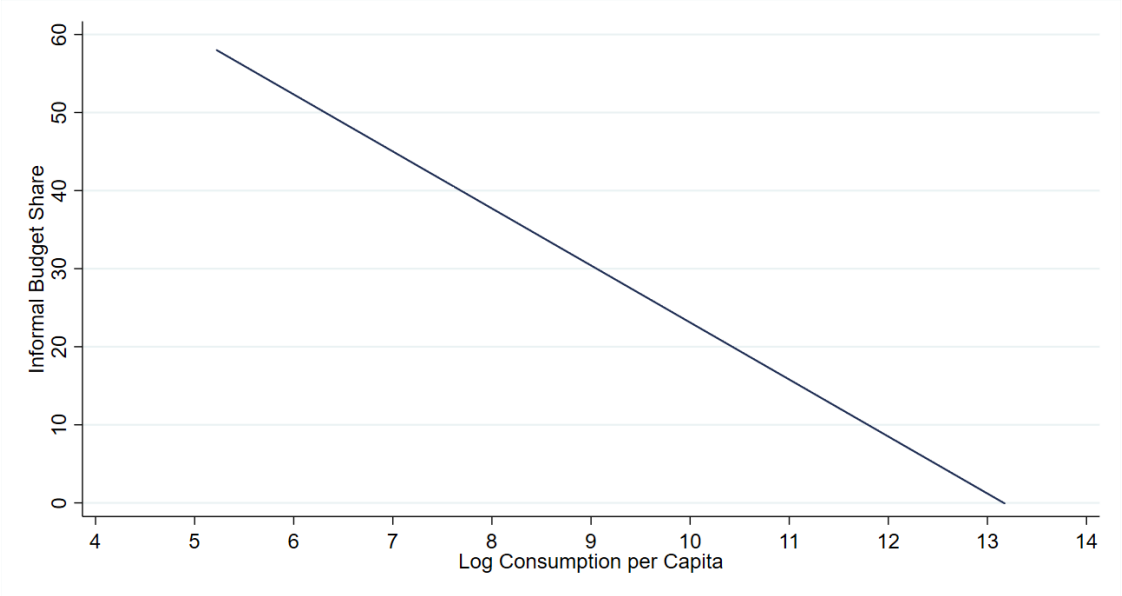


Figure 5: Informality Engel Curve (IEC) for Tunisia

3.2. The Extended Model

We develop an extended version of the HANK model with informality. The model is intended to recreate the stylized facts presented in the previous subsection.

3.2.1. Households

The economy is populated by a continuum of infinitely lived heterogeneous households. Heterogeneity between households can be summarised by two idiosyncratic states (a, z). Here, $a \in \mathbb{R}^+$ [2] is the individual wealth of the household, $z \in \{0, z^i, z^f\}$ is the employment status of the household and the sector of activity. Here, $z = 0$ indicates the unemployment state, while z^i and z^f indicate employment in the informal and formal sectors respectively. Since workers' productivity is generally lower in the informal sector, we impose that $z^f > z^i$. For simplicity, we assume that the labor, sector, and productivity are exogenous and follow a Poisson process with an arrival probability $\lambda^j; \quad j \in \{0, i, f\}$. We note by $\mu(da, dz)$ the joint distribution for the state of the economy. μ has a support $\mathcal{M} := \mathbb{R}^+ \times \times\{z^0, z^i, z^f\}$. Households receive utility u from consumption c and disutility from labor l. Preferences are conditional on savings and the future discount rate $\rho \leq 0$. The household maximization

[2] This means that there is no credit in the economy. Although this can be easily extended to the case where $a \in [\bar{a}, +\infty[$ as we will implement later.

problem differs between labor and sector states.

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t^f, c_t^i, l_t) dt. \tag{12}$$

To embed the heterogeneous informal budget share in our model, we propose to separate the consumption utility of the two categories of goods [3]. We assume that household utility follows:

$$u(c_t^f, c^i, l_t) = \frac{(c_t^f)^{1-\sigma_f}}{1-\sigma_f} + \Phi_i \frac{(c_t^i)^{1-\sigma_i}}{1-\sigma_i} + \Phi_l \frac{(l_t)^{1+\phi_l}}{1+\phi_l} \tag{13}$$

with σ_f and σ_i represent the elasticities of formal and informal goods respectively, ϕ_l is the inverse Frish elasticity of labor. Φ_i and Φ_l are scaling parameters for informal consumption and labor respectively.

Household's asset holdings evolve according to:

$$\dot{a} = w_t(z_t) z_t l_t + r_t a_t + T_t + \Pi - (1+\tau_c) c_t^f - p_t^i c_t^i \tag{14}$$

Where $w_t(z_t)$ is the wage level associated the state z_t i.e. formal sector wage $(1-\tau_w) w_t^f$ for z_f, w_t^i for z_i and 0 for unemployed workers and l_t is the labor supply. r_t is the real return on assets and T_t is a universal government transfer to all households. Combined profits from formal and informal firms are distributed to households as a dividend, Π_t, based on their productivity level [4].

Assets:
Household assets a can be split to two different types, government bonds b rewarded at the the real interest rate r, and capital stock k such that

$$a_t = b_t + q_t k_t \tag{15}$$

where q_t is the capital price. We assume that households can shift between the two types of assets without cost. As such we impose the non-arbitrage condition:

$$\frac{r_t^k - \delta q_t - \dot{q}_t}{q_t} = r_t \tag{16}$$

where r_t^k is the rental rate of capital to formal firms and *delta* is the depreciation rate of capital.

[3] This is in contrast to the constant share across households implied by a CES function where formal and informal goods are combined to produce a single final consumption good like in model presented in the previous section.

[4] As explained in Kaplan et al. (2018), the distribution process of assets plays a critical role in shaping the resulting distribution of wealth.

3.2.2. Goods Producers

In the economy, there are two types of firms; Formal, indexed by f, and informal, indexed by i. **Intermediate Goods Producers:**
There is a continuum of intermediate-good producers, indexed by j_s, operating in each sector $s; s \in \{f, i\}$ and using the sector specific production function:

$$\begin{aligned} y^f_{j_f,t} &= Z^f_t k^\alpha_t (n^f_{j_s,t})^{(1-\alpha)} \\ y^i_{j_i,t} &= Z^i_t n^s_{j_s,t} \end{aligned} \tag{17}$$

Here, Z^f_t and Z^i_t are two aggregate productivity shocks associated with the formal and informal sectors respectively. Both shocks follow the Ornstein-Uhlenbeck process:

$$dZ^s_t = -\theta_{z_s} Z^s_t dt + \sigma_{z_s} dW^{z_s}_t; \quad s \in \{f, i\} \tag{18}$$

We assume that the marginal cost of firms in both sectors is subject to a cost-push shock. We have:

$$\begin{aligned} mc^f_t &= \frac{\Xi^f_t}{w^f_t} \left(\frac{w^f_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r^k_t}{\alpha} \right)^{\alpha} \\ mc^i_t &= \frac{\Xi^i_t}{Z^i_t} w^i_t \end{aligned} \tag{19}$$

The two cost-push shocks, Ξ^f_t and Ξ^f_t, follow the Ornstein-Uhlenbeck process:

$$d\Xi^s_t = -\theta_{\Xi_s} \Xi^s_t dt + \sigma_{\Xi_s} dW^{\Xi_s}_t; \quad s \in \{f, i\} \tag{20}$$

Firms are subject to sector-specific quadratic price adjustment costs following Rotemberg (1982):

$$\Theta^s_t \left(\frac{\dot{p^s_t}}{p^s_t} \right) = \frac{\Omega^s}{2} \left(\frac{\dot{p^s_t}}{p^s_t} \right)^2 y^s_t \tag{21}$$

From the firm's optimization problem, we can determine the following New Keynesian Phillips curves:

$$\begin{aligned} \left(r_t - \frac{\dot{y^f_t}}{y^f_t} \right) \pi_t &= \frac{\xi_f}{\Omega^f} \left(m^f_t - \frac{\xi_f - 1}{\xi_f} \right) + \dot{\pi}_t \\ \left(r_t - \frac{\dot{y^i_t}}{y^i_t} \right) \pi^i_t &= \frac{\xi_i}{\Omega^i} \left(m^i_t - p^i_t \frac{\xi_i - 1}{\xi_i} \right) + \dot{\pi^i_t} \end{aligned} \tag{22}$$

where π_t is the inflation rate of formal goods and π^i_t is the inflation rate of informal goods.

Final-Goods Producers:
Final goods produced by formal firms and final goods produced by informal firms are produced using intermediate inputs, y_t^f and y_t^i, from formal and informal intermediate goods producers respectively.

$$y_t^s = \left[\int_0^1 \left(y_{j_s,t}^s\right)^{\frac{1}{\xi_s}} dj_s\right]^{\xi_s} \quad s \in \{f, i\} \tag{23}$$

3.2.3. Capital Producer

A competitive capital producer transforms formal goods into capital goods bought by households at the price q_t. We assume that the production process is subject to capital adjustment cost and that the cost function of the capital producer is $\Upsilon(\iota_t + \frac{\Omega}{2}(\iota - \delta)^2)K_t$ where ι_t is the investment rate and Υ_t is a marginal efficiency of investment shock. The producer maximizes the expected profit stream, discounted at the stochastic discount factor of the household:

$$\begin{aligned} W_t = &\max_{\iota_t, K_t} \int_0^\infty \Lambda_{0,t} \left(q_t \iota - \Upsilon(\iota_t + \frac{\Omega}{2}(\iota - \delta)^2)\right) K_t dt \\ &s.t. \quad \dot{K}_t = (\iota_t - \delta) K_t \end{aligned} \tag{24}$$

3.2.4. Monetary Policy

Monetary policy is set using the following Taylor rule:

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y (y_t - \bar{y}) + \varepsilon_t^{MP} \tag{25}$$

ε_t^{MP} follows the Ornstein-Uhlenbeck process:

$$d\varepsilon_t^{mp} = -\theta_{mp} \varepsilon_t^{mp} + \sigma_{mp}.dW_t^{mp} \tag{26}$$

where dW_t is the innovation to a standard Brownian motion, θ_{MP} is the rate of mean reversion, and σ captures the size of innovations.

3.2.5. Government

The government budget satisfies the constraint given by:

$$\dot{B}_t^g + G_t + T_t = \tau_w \int w_t^f z^f l^f d\mu_t(a, z) + \tau_c \int c_t^f d\mu_t(a, z) + \tau_f \tilde{\Pi}_t^f + r_t B_t^g \tag{27}$$

Where the government adjusts lump transfers T_t to satisfy the budget constraints. We assume that government expenditure follows the Ornstein-Uhlenbeck process:

$$dG_t = -\theta_g G_t dt + \sigma_G dW_t^G \tag{28}$$

Finally, to maintain fiscal the budget balance, we assume that the government uses transfers, T_t, as its policy instrument[5].

3.2.6. Equilibrium

The model has 6 markets, the bonds market, the capital market, the formal and informal labor markets, and the formal and informal goods markets. The bond market clearing:

$$B_t^g = \int b_t d\mu_t(a, z) \tag{29}$$

The capital market clearing:

$$K_t = \int k_t d\mu_t(a, z) \tag{30}$$

Clearing labor markets for the formal and informal sectors:

$$\begin{aligned} L^f &= \int z^f l_t \mu_t(a, z^f) da \\ L^i &= \int z^i l_t \mu_t(a, z^i) da \end{aligned} \tag{31}$$

The formal goods market clearing condition:

$$Y_t^f = C_t^f + I_t + G_t + \Theta_t^f \tag{32}$$

And finally clearing the informal goods market

$$Y_t^i = C_t^i + \Theta_t^i \tag{33}$$

4. Estimation and Results: The Transmission of Monetary Policy In Tunisia

In this section, we lay down our approach to empirically quantify the Transmission of monetary policy in the case of Tunisia.

[5]The choice of fiscal instrument has an important role in the transmission of shocks as showcased in Kaplan et al. (2018). Other specifications could utilize taxes or government expenditures. Exploring these alternative cases is left out of our analysis

4.1. Numerical Solution and Estimation Algorithm

Nowadays, solving the HANK model can be considered a feasible task thanks to toolboxes kindly provided online for the scientific community. For continuous-time models, Ahn et al. (2017) provides an easy-to-use toolbox, for solving and simulating HANK models with aggregate shocks. The method is based on the works of Achdou et al. (2021) to find the steady state equilibrium using finite difference methods and of Reiter (2009), among others, to solve the linearized model. On the other hand, estimating the HANK model is still a new area of research, and unlike model solutions, it is a more challenging task. Most of the available literature uses calibrated models made to replicate key empirical moments for the heterogeneous states (from income or consumption surveys). Only a minority of works attempt to estimate HANK models including Bayer et al. (2024),Auclert et al. (2021), Fernández-Villaverde et al. (2023), and Acharya et al. (2023) to name a few. For our case, we use the estimation method proposed in Liu and Plagborg-Møller (2023). While we limit ourselves to a macro data-only estimation, the proposed method in this article allows for the incorporation of microdata for a full-information estimation, a task we are willing to tackle in future works. However, the inference method presented in Liu and Plagborg-Møller (2023) is developed for discrete-time models. Applying it to our continuous time model requires additional tinkering. To solve this problem, we use the method proposed in Christensen et al. (2024) for estimating continuous models using discrete data. The approach is based on an exact discrete-time representation.

4.2. Data

For estimation, we use an observations sample of 7 macroeconomic variables, at a quarterly frequency, from the Tunisian economy from $2015Q2$ to $2022Q4$. We include in our sample the policy rate, GDP inflation, CPI inflation, GDP, household final consumption, investment, and government spending. All variables are filtered using the one-way hp-filter. Real quantities are expressed in real values and per capita terms. For household final consumption, investment, and government spending, data is only available in annual frequencies. We opt to apply a quadratic low to high-frequency transformation filter to obtain data at quarterly frequency[6].

[6]We instead of using mixed frequency estimation facilitated by the algorithm provided in Christensen et al. (2024)

4.3. Calibration and Priors

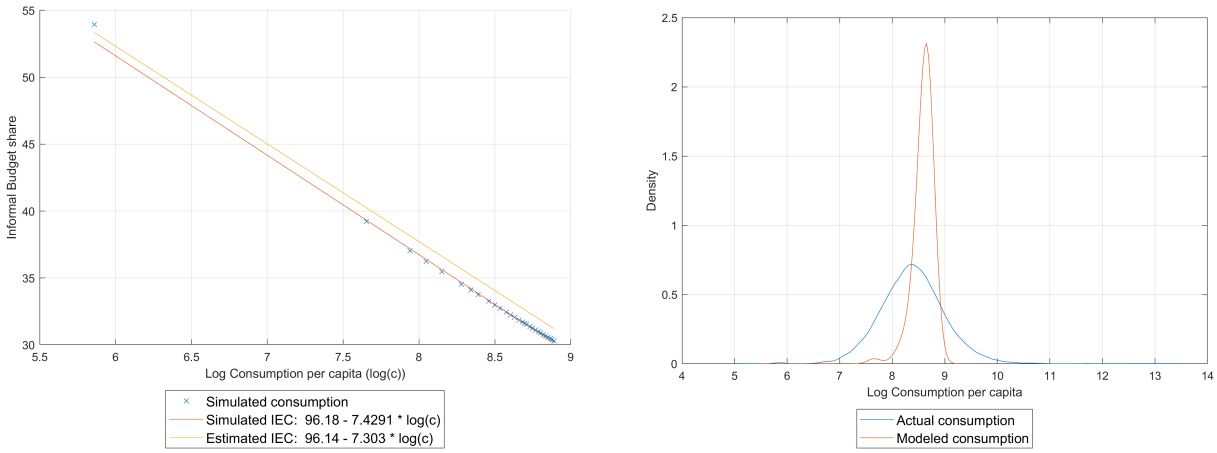
We calibrate the model using standard values from the literature on New Keynesian models, and to replicate key features from the Tunisian economy. Since we will be evaluating the model at zero steady-state inflation, we set the discount factor to 0.01 and the capital depreciation rate to 0.0125. σ_f and σ_i are calibrated to 1.3 and 1.825 to control the slope of the implied IEC generated by the model. ξ_f and ξ_i are chosen to allow a mark-up rate of 20% and 10% in the formal and informal markets respectively. $\bar{z}$ is chosen to match the average household consumption from the model with the one observed in the survey data. The normalized productivity vector $[z_u, z_i, z_f]$ is set to [0, 0.8, 1] implying that an informal worker, is on average 80% as productive as its formal counterpart. For the size of the informal sector, we choose a steady value of 35%. While this value is at the higher end of the estimates presented by Asllani et al. (2024), we found it necessary to obtain a 44% share of informal labor as reported by the Tunisian National Institute of Statistics. We present our calibration choice in Table 4 along with some resulting values and steady-state great ratios.

Symbol	Parameter	value
ρ	Discount factor	1%
α	Share of capital	0.35
δ	Capital depreciation rate	1.25%
σ_f	Risk aversion for formal consumption	1.3
σ_i	Risk aversion for informal consumption	1.825
ϕ	Inverse Frish elasticity	4
Φ	Labor disutility	4
ξ_f	Formal goods elast.	6
ξ_i	Informal goods elast.	11
τ_w	Labor tax	25%
τ_f	Corporate profits tax	20%
τ_c	Consumption tax	18%
ϕ_y	Taylor output gap	0
$\bar{z}$	Average productivity	2.5
$[z_u, z_i, z_f]$	Normalized productivity vector	[0, 0.8, 1]%
π	Steady state inflation	0
$\frac{B}{y^f}$	Government Debt to GDP	0.7%
$\frac{T}{y^f}$	Government transfers to GDP	0.07%

$\frac{p^i y^i}{y^f}$	Informal sector size	35%
Φ_i	Informal utility scaling parameter	0.2469
r	Interest rate	0.718 %
$\frac{c^f}{y^f}$	Consumption to GDP	63.46%
$\frac{i}{y^f}$	Investment to GDP	18.52%
$\frac{g}{y^f}$	Government spending to GDP	18.01%
U	Unemployment	15.56%
$\frac{L^i}{L}$	Informal labor share	44.28%
$\ln(\bar{c})$	Log average consumption per capita	8.6038 %

Table 4: Calibrated Values

In Figure 6, we present results from our measurement of the implied IEC generated by the model and the kernel density estimates of the log per capita consumption distribution. We compare these with the impractical findings obtained from the household consumption survey. While, with some more fine-tuning we could get the simulated and estimated IECs to match, approximating consumption distribution posed a bigger challenge. We suspect that this is a result of our limited heterogeneous state space.



(a) Informal Engel Curve (b) Consumption distributions

Figure 6: Steady State Informal Engel Curve and Consumption Distribution

For our estimation, we use a random walk Metropolis-Hasting chain with 100000 draws after a burn-in of 20000 draws. We estimate in total 16 parameters, focusing on frictions and shock processes. We report the prior used and the resulting posterior in Table 5.

	Parameter description	Prior			Postorior	
		Distr	Mean	st.d	Mean	st.d
Ω_f	Formal adj. cost	Gamma	100	25	75.233	0.0530
Ω_i	Informal adj. cost	Gamma	75	20	75.106	0.0740
Ω_k	Capital adj. cost	Gamma	0.3	0.2	0.644	0.1138
ϕ_π	Taylor rule, inflation	Normal	2	1	2.082	0.0361
θ_{mp}	Reversion rate, mon. pol.	Beta	0.75	0.2	0.3626	0.0523
θ_{z_f}	Reversion rate, formal prod.	Beta	0.75	0.2	0.2758	0.0740
θ_{z_i}	Reversion rate, informal prod.	Beta	0.75	0.2	0.2408	0.1003
θ_g	Reversion rate, gov. exp.	Beta	0.75	0.2	0.3007	0.0863
θ_Υ	Reversion rate, MEI	Beta	0.75	0.2	0.4052	0.0628
σ_{mp}	Innovation size, mon. pol.	Inv-Gamma	0.25	Inf	0.3578	0.0655
σ_{z_f}	Innovation size, formal prod.	Inv-Gamma	0.25	Inf	0.4780	0.0676
σ_{z_i}	Innovation size, informal prod.	Inv-Gamma	0.25	Inf	0.7036	0.1436
σ_g	Innovation size, gov. exp.	Inv-Gamma	0.25	Inf	0.4361	0.0912
σ_{Ξ_f}	Innovation size, formal cost	Inv-Gamma	0.25	Inf	0.4033	0.0407
σ_{Ξ_i}	Innovation size, informal cost	Inv-Gamma	0.25	Inf	0.5585	0.1179
σ_Υ	Innovation size, MEI	Inv-Gamma	0.25	Inf	0.6328	0.1215

Table 5: Estimation priors and results

4.4. The Transmission of Monetary Policy In Tunisia

To evaluate the transmission of monetary policy in Tunisia, we study the generated Impulse Response Functions (IRFs) generated by a positive 1% standard deviation monetary policy shock at t0. We report the IRFs generated at the mean posterior distribution[7]. In general, results are qualitatively in line with expectations based on existing literature. For output, we find that formal output is more affected than informal output but recovers slightly faster. However, we find that consumption showcases an opposite pattern. Formal consumption deviates upward at first before diving below its steady-state level. We should note that consumption movement is less important exhibiting only one-fifth that of output deviation. For Inflation, we find that formal and informal price inflation exhibit similar path trajectories although informal inflation deviates slightly lower leading to a slightly lower consumption price inflation. These results are in line with those estimated in End et al. (2020)

[7]Not to be confused with the mean IRFS generated which is often reported as Bayesian IRFs.

using a var model and other estimates for the Tunisian economy using DSGE models. As a result, the size of the informal sector to GDP goes up at first by up to 0.3% before reversing its trajectory by the fourth period and maintaining a downward deviation.

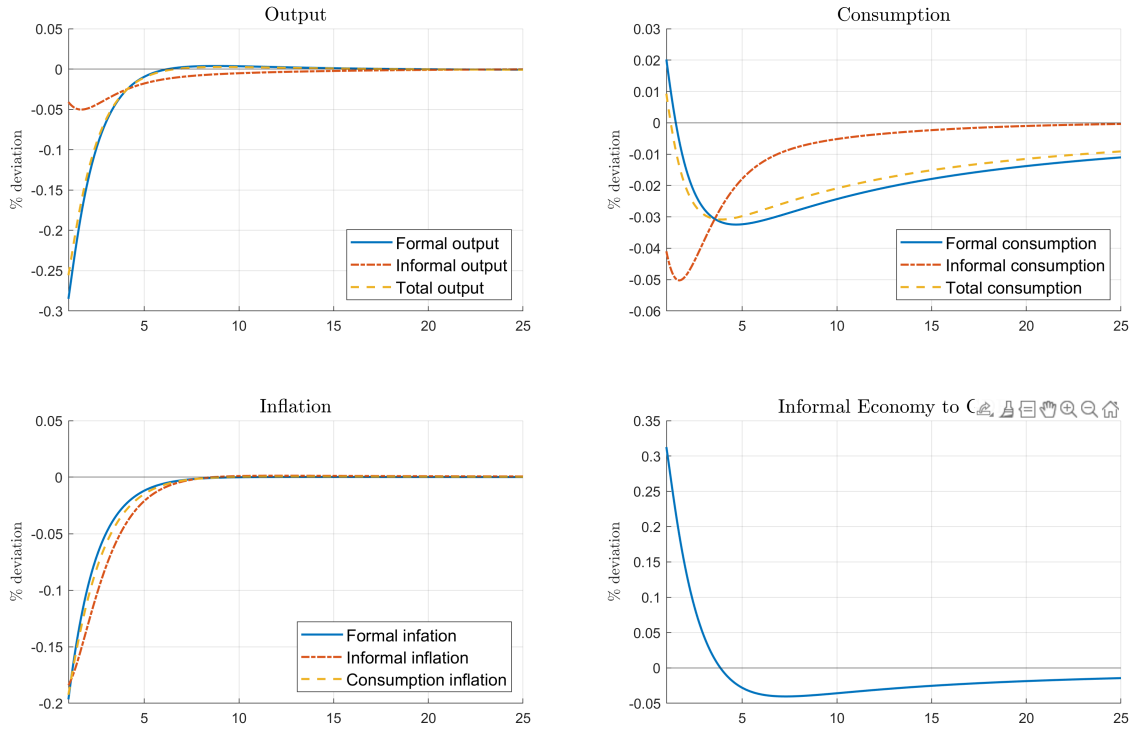


Figure 7: Impulse Response Function at Posterior Mean

One advantage of heterogeneous agent models is that they can be used to generate specific IRFs for each type of household. If we take consumption, we generate the formal consumption reaction of each worker type at the extremities of our grid and a midpoint. Results are presented in Figure 8. Household formal consumption is not homogeneous across the two states. First, size and diversion between reactions seem to decrease with wealth. Second, all results seem to exhibit qualitatively similar trajectories to aggregate consumption i.e. positive deviation at first followed by a prolonged downward deviation. We attribute this reaction to three factors. First, the indirect effects due to general equilibrium response to changes in household disposable income. Second, the increase of government transfers as a result of increased demand for bonds exceeding revenue losses from taxation. Finally, in this model specification distributed profits increase with restrictive monetary policy as these are counter-cyclical, in the presence of only price rigidities[8].

[8]In Kaplan et al. (2018) a similar problem arises and is dressed by controlling the share of profits distributed as liquid and illiquid assets.

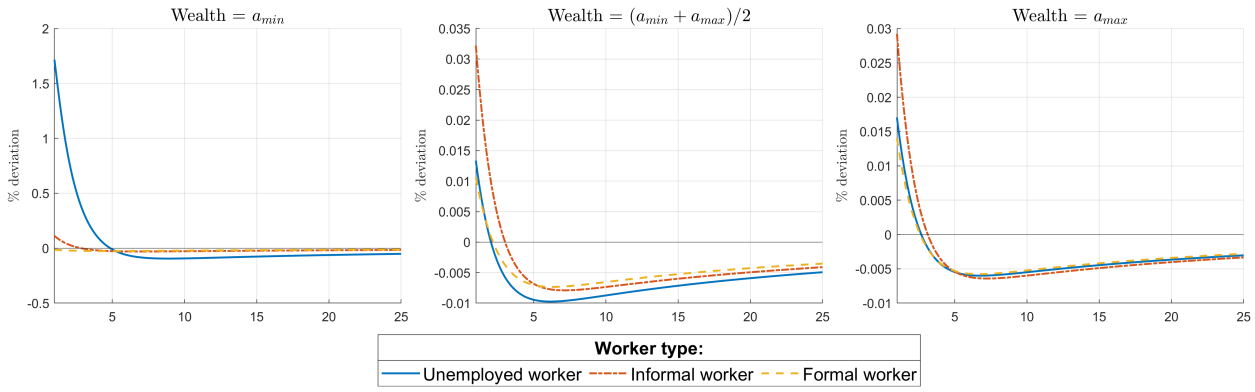


Figure 8: Impulse Response Function by Wealth and Worker Type

Another advantage of the HANK model is that it provides the ability to map the reaction of household distributions to aggregate shocks. In our case, we report the reaction of wealth distribution to a monetary shock in Figure 9. For the clarity of the illustration, we increased the size of the initial shock. We can note that the wealth distribution shifts to the left. This effect is especially noticeable for unemployed and informal workers and at lower wealth levels. This result suggests that unemployed and informal workers are more vulnerable to the redistributive effect of monetary policy in comparison with their formal counterparts. By t = 4 (one year after the shock), the distribution returns to its steady state.

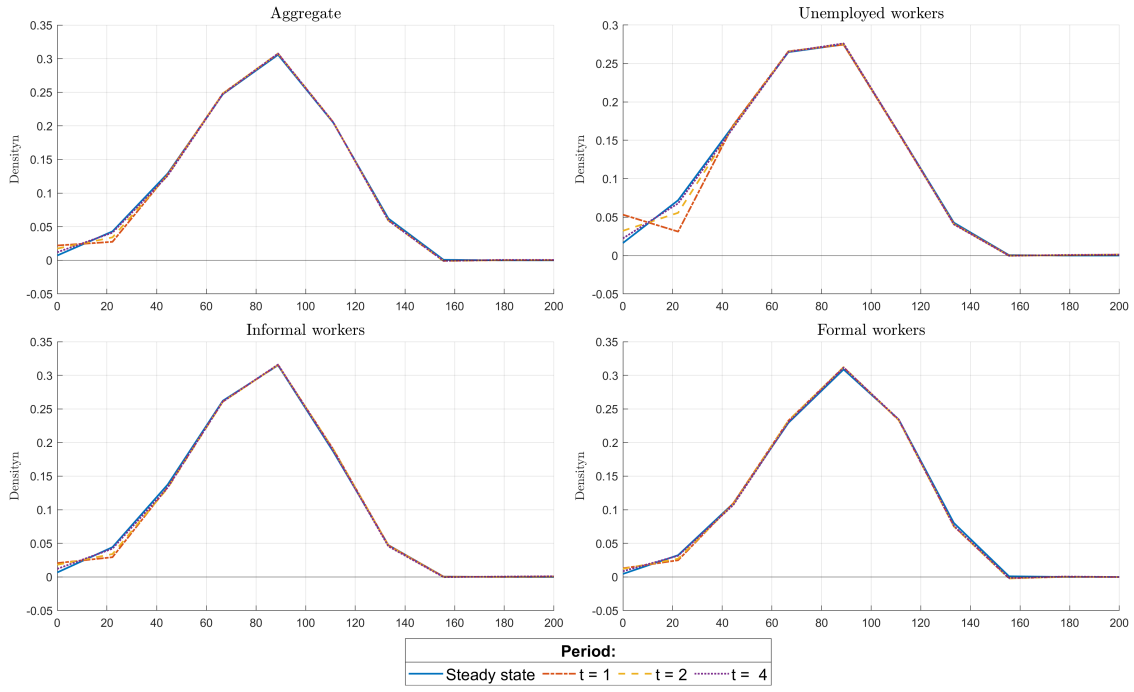


Figure 9: Impulse response function

5. Conclusion

In this paper, we examine the transmission of monetary policy in an economy characterized by heterogeneous households and a sizable informal sector. In the first step, we showcase the presence of distortionary effects introduced by the presence of informality. We compare the simulated Impulse Response Function generated by four illustrative models combining the characteristics of "fully formal" and "with informality" with that of "representative agent" with "Heterogeneous agents". We find that combining informality with heterogeneity leads to a strengthening of the buffering effect of informality on output and consumption. We also find that this buffering effect increases with the size of the informal sector. In the second step, we develop a medium size model specific to the Tunisian case. To enrich the model, we embed heterogeneous informal consumption behavior as a function of health that gives rise to an Informality Engel Curve (IEC). We estimate the IEC using data from the 2021 Tunisian National Survey on Budget, Consumption, and Living Standers (ENBCNV) and attempt to recreate it using our model. We then estimate a set of key parameters for friction and shocks using Bayesian inference. From the results generated by estimating our model, we found that unemployed and informal workers seem to be most affected by the monetary shock, especially those with lower wealth. These results showcase the importance of informality and heterogeneity in monetary policy analysis. Monetary authorities in emerging economies like Tunisia can use these insights to improve the efficacy of their policy at lower social costs. This can be especially important for achieving mandates related to price stability, fostering economic growth, improving public trust, and the accountability of these institutions. Also, coordination with fiscal authorities can prove useful to mitigate harmful repercussions of monetary tightening on vulnerable segments of the population without sacrificing policy objectives.

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