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Abstract

This paper assesses levels and trends in vulnerability to multidimensional poverty for two different years in Algeria (2012/13 and 2019) and Tunisia (2012 and 2018). Using as benchmark the M-gamma multidimensional poverty measures as developed by Alkire and Foster (2019), it follows the approach suggested by Gallardo (2022). To preserve the multidimensional nature of poverty, the joint probability of being poor and deprived in each dimension is modelled using multidimensional Bayesian networks classifiers and the vulnerability by mean risk approach (VMR) to vulnerability measurement. Despite similar levels of multidimensional poverty, vulnerability measures are higher in Tunisia than in Algeria. In addition, the achievements in poverty reduction are more fragile in Tunisia than in Algeria. The results show that moderate vulnerability prevails over severe vulnerability both in Algeria and Tunisia. Trends over time indicate that in Algeria, vulnerability seems to be shifting more towards moderate vulnerability while the opposite is observed in Tunisia. The indicators that differentiate severe from moderate vulnerability are mainly related to health and education dimensions both in Algeria and Tunisia. We show that chronic poverty among the vulnerable is larger in Tunisia than in Algeria. Our results reveal also different trajectories in the evolution of the vulnerability components in these two countries.

Keywords: vulnerability, multidimensional poverty, Bayesian networks, downside risk, Algeria, Tunisia

JEL Classifications: I31, I32, D63, D81

ملخص

تقيم هذه الورقة مستويات واتجاهات التعرض للفقر متعدد الأبعاد لمدة عامين مختلفين في الجزائر (2012/13 و 2019) وتونس (2012 و 2018). باستخدام معايير M-gamma متعددة الأبعاد للفقر كما طورها Alkire and Foster (2019)، وتتبع النهج الذي اقترحه Gallardo (2022). وللحفاظ على الطابع المتعدد الأبعاد للفقر، فإن الاحتمال المشترك للفقر والحرمان في كل بُعد قد صيغ باستخدام مصنفات شبكات بايزية متعددة الأبعاد وقابلية التأثر بنهج متوسط المخاطر بقياس الضعف. وعلى الرغم من وجود مستويات مماثلة من الفقر المتعدد الأبعاد، فإن تدابير الضعف أعلى في تونس منها في الجزائر. وبالإضافة إلى ذلك، فإن الإنجازات في مجال الحد من الفقر أكثر هشاشة في تونس منها في الجزائر. تظهر النتائج أن الضعف المعتدل يسود على الضعف الشديد في كل من الجزائر وتونس. وتشير الاتجاهات على مر الزمن إلى أن الضعف في الجزائر يبدو أنه يتحول أكثر نحو ضعف معتدل بينما يلاحظ العكس في تونس. وتتصل المؤشرات التي تختلف اختلافا شديدا عن الضعف المعتدل أساسا بالأبعاد الصحية والتعليمية في كل من الجزائر وتونس. نبين أن الفقر المزمن بين الضعفاء أكبر في تونس منه في الجزائر. تكشف نتائجنا أيضًا عن مسارات مختلفة في تطور مكونات الضعف في هذين البلدين.

Introduction

As other countries in the world, Mena countries adopted the vision of the UN agenda 2030 and the Sustainable Development Goals (SDGs). At the core of the SDGs is a pledge to ensure that "no one is left behind". The first SDG 1 of ending poverty in all its forms everywhere remains one of the most challenging issues in the MENA region due to the fragile context. Despite significant progress in poverty reduction over the last years, the current crises facing countries around the world will severely impact the well-being of the population in the years to come threatening progress achieved in poverty reduction. The resulting negative economic shocks illuminate the need to pay more attention not only to the current poor but also to those who are vulnerable to poverty (or those at the risk of future poverty). Therefore, a better understanding of vulnerability can support the development of more effective and efficient policies to combat poverty in a sustainable way.

Inclusion of vulnerability in poverty analysis dates back to the 2000's following the pioneering study by the World Bank on social risk and management (2001). A number of approaches have been proposed to assess and estimate vulnerability to poverty but they are not yet been widely adopted. Indeed, since vulnerability is by definition forward looking, most measures require long panel data. However, for many countries, only cross-sectional data are available. This reduces the range of concepts and measures that allow the use of such kind of data. In addition, although poverty is now-well recognized as a multidimensional phenomenon, empirical studies on vulnerability assessment are dominated by the monetary approach to poverty. Yet, vulnerability should also reflect the fact that it can occur in different dimensions of well-being. The analysis of vulnerability to multidimensional poverty is still poorly developed. There are very few studies that take a multidimensional approach to vulnerability. To the best of our knowledge, the only existing studies are those by Calvo (2008), by Abraham and Kavi (2008), by Feeny and McDonald (2016), the extended cross dimensional poverty line introduced by OPHI (2018) using the MPI (*Multidimensional Poverty Index from UNDP*) as the reference indicator and by Gallardo (2020, 2022) within the framework of the MPI for Latin American countries and Chile. Except the study of Lyons et al. (2021) on Syrian refugees in Lebanon which draws on Feeny and McDonald‘s approach, there is also a lack of studies assessing vulnerability to poverty in the Mena region.

The objective of the present study is to fill this gap. Drawing on the study by Bérenger (2023) which assesses levels and trends in multidimensional poverty in Algeria, Iraq and Tunisia the
present paper proposes to examine vulnerability to multidimensional poverty following the approach developed by Gallardo (2022) and to investigate the complex relationship between multidimensional poverty and vulnerability in Algeria and Tunisia. According to the study by Bérenger (2023), although these two countries have very similar levels of multidimensional poverty, it is interesting to examine whether their population face the same risk of poverty in the future. Vulnerability to multidimensional poverty is estimated using the downside mean semideviation approach proposed by Gallardo (2013). To estimate the risk of being multidimensional poor in the future we draw on Gallardo (2022) that implements multidimensional Bayesian network classifiers. This study is currently one of the two rare applications of Bayesian networks to the analysis of welfare and poverty.[1]

[1] Ceriani, L., Gigliarano, C., (2020) used Bayesian networks to model the dependence structure among the different dimensions of well-being for a selection of Western and Eastern European countries.

Our study will be organized as follows. Section 1 presents a review of the literature on vulnerability to poverty concepts. Section 2 describes our methodological strategy which includes three steps: the multidimensional poverty measures based on the M-*gamma* family measures suggested by Alkire and Foster (2019), the Bayesian network strategy to estimate conditional probabilities and the mean-risk approach developed by Gallardo (2013) to estimate vulnerability to poverty. Section 3 shows the results obtained using data from the UNICEF- MICS for Algeria and Tunisia. Section 4 will conclude the study by highlighting some of the policy issues for reducing poverty and vulnerability.

1. Conceptualizations and assessments of vulnerability to poverty

This section presents a brief review of the main approaches to conceptualize and measure vulnerability to poverty.

Poverty and vulnerability to poverty are different but closely linked concepts as both of them are measures of well-being. The main difference is that poverty is an ex-post and vulnerability an ex-ante measure of well-being. A measure of poverty is typically done ex-post and from the observed level of household well-being below the poverty line at some point in time and hence it is a static measure of well-being. Yet, poverty is not a permanent characteristic of the households but a stochastic phenomenon as poor people today may exit poverty while others may remain or fall into poverty in the near future because of their exposure to shocks. Poverty measures are not able to capture these transitions in and out households' poverty in a given period of time. Consequently, they may lead to inclusion and exclusion errors in poverty alleviation programs.

In contrast, vulnerability is explicitly dynamics as it does not focus on the current status but it is forward looking. More generally, vulnerability refers to the threat of experiencing poverty in the future. As argued by Calvo and Dercon (2013), vulnerability is always more than mere exposure to risks. It is also about deprivations and shortfalls. Therefore, vulnerability is a combination of two elements: poverty and risk (Chaudhuri et al., 2002). Vulnerability refers to a future situation, using present information which describes the exposure to poverty rather than the result of poverty per se (Hernandez and Zuluaga, 2021). Vulnerability to poverty today is in fact the risk of being poor tomorrow.

So the main distinction between the two concepts is the uncertainty about the future as a consequence of risks that households or individuals face. Although the definition of vulnerability as the risk of being poor in the future seems easy and intuitive to understand, the stochastic nature of the future adds some complexity to the ex-ante estimation of vulnerability. The literature produced many definitions and corresponding approaches but no consensus has yet been reached on a single definition. Authors such as Hoddinott and Quisumbing (2003, 2008), Ligon and Schechter (2003), Calvo and Dercon (2013), Klasen and Povel (2013) and Gallardo (2018) surveyed all the existing literature. They reviewed strengths and weaknesses of the most influential approaches on vulnerability to poverty.

They can be grouped into three main categories : *vulnerability as expected utility* (VEU) proposed by Ligon and Schechter (2003), *vulnerability as uninsured exposure to risk* (VER) proposed by Tesliuc and Lindert (2002), *vulnerability as expected poverty* (VEP) by Chaudhuri et al. (2002). Recently Gallardo (2018) added to this list a new category: *Vulnerability by mean risk* (VMR). As mentioned by the authors, each category includes several approaches. We limit ourselves to mention the main categories. VEU measures and compares the difference between utility associated with a certainty equivalent level of well-being (as benchmark) and household's own expected utility given its uncertain prospect. Its main limitation is its dependence on a functional form of utility and its symmetric approach to the risk (Klasen and Povel, 2013 and Gallardo, 2018). VER is based on assessment of the extent to which a given shock imposes a welfare loss due to the absence of effective and efficient risk management tools.[2]

VEP focuses on the probability that a given shock moves a household's well-being below the poverty line in the near future. This approach has been widely used in the empirical literature. The main reason is that estimations of vulnerability can be obtained using cross-sectional surveys which are more frequent than panel data in developing countries. Some limitations have been raised against this approach. Its implementation assumes that past distribution of well- being reflects future distribution and that all households are exposed to the same distribution of changes in well-being. It also requires the assumption of a specific probability distribution function. As argued by Gallardo (2018), VEP considers neither risk sensitivity, nor the depth of expected poverty as it only defines the probability of falling below the poverty line.

VMR includes the mean deviation approach developed by Chiwaula et al. (2011) and the downside mean semi-deviation proposed by Gallardo (2013). These two approaches identify vulnerable people based on a preference ordering between welfare outcomes determined according to the expected mean and a risk parameter. The risk parameter is the variance in the first approach and the standard downside semi-variance in the second one. Rather than considering the risk as symmetric, the downside mean semi-deviation approach is based on the premise that the risk of falling into poverty is asymmetric in nature. Individuals do not fear random variations in well-being per se but losses below expected values of well-being. This definition encapsulates in a single measure two kinds of situations, both expected poverty and the downside risk of falling into poverty. In addition, measures of individual vulnerability can be aggregated using standard FGT indexes. While this approach has been defined in the framework of monetary poverty (Gallardo, 2013), it has been recently applied to measure vulnerability to multidimensional poverty (Gallardo, 2020 and 2022).

In summary, all of these approaches incorporate the idea that people face diverse risks. All of them build models that predict a measure of well-being and hence of the risk of poverty. However, they differ in their definition of well-being and in their modeling of risk. Most of them are based on expected mean and variance of household's consumption and are defined relative to a benchmark. While VEP and VMR can be evaluated using cross-sectional data, VER and VEU require lengthy panel data. The non-availability of panel data has limited research efforts to measure vulnerability. Recent developments in microeconometric modeling have made it possible to estimate vulnerability using cross-sectional data. For instance, Chaudhuri et al. (2002) and Chaudhuri (2003) developed a methodology which estimates the expected mean and variance of (log) consumption conditional on a bundle of covariates using the three-step feasible generalized least square (FGLS) procedure with single cross section data.

Despite the recognition of the multidimensional nature of poverty, most of these definitions and their implementations in the empirical literature used income or consumption

[2] This category now includes new versions that incorporate asymmetric conception of risk, either in terms of lack of insurance to cover the risk of falling under the poverty line (Cafiero and Vakis, 2006) or on the basis of downside risk (Dutta et al. 2011, Povel, 2010, 2015).

expenditures as proxy for poverty measurement. It is only recently that a few studies explored vulnerability to multidimensional poverty. The majority of these studies employed the VEP approach applied it to the households' deprivation score following the Alkire and Foster (2011) approach and using the methodology developed by Chaudhuri (2003). Vulnerability is estimated as the conditional probability of the deprivation score to fall above a predetermined poverty line. Using cross-sectional data, this methodological strategy has been adopted by Feeny and McDonald (2016) for Solomon Islands and Vanuatu in Melanesia, by Azeem et al. (2018) for Pakistan, by Tigre (2019) for Ethiopia, by Gebrekidan et al. (2020) for Dugu'a Tembien District in Ethiopia, by Liu et al. (2021) in rural China, Lyons et al. (2021) on Syrian refugees in Lebanon and by Hernandez and Zuluaga (2022) for Colombia. However, this approach raises several issues. One of the main limitations lies in the loss of the multidimensionality that characterizes households' deprivation scores since in a way it reintroduces unidimensionality. This method does not model the joint probability distribution over the whole dimensions of the household or individual's deprivation score. Consequently, it is not possible to investigate the vulnerability profiles by dimension of the vulnerable people to multidimensional poverty. In addition, the VEP measure only provides estimates of the incidence of vulnerability since it is not sensitive to variability.[3] It says nothing about how vulnerable the vulnerable people are to multidimensional poverty.

To overcome some of these limitations, Pham et al. (2021) used the measure of Chiwaula et al. (2011) to investigate vulnerability to poverty across multiple dimensions in Vietman. Instead of using the framework of Alkire and Foster, this study applied the fuzzy approach to income and each of six non-monetary dimensions using the three waves of a panel survey. Following a similar approach, Gallardo (2020) measured vulnerability to multidimensional poverty in Chile using the mean-risk behavior approach (Gallardo, 2013). The methodology calculates an estimate of the probability that the household is not poor for each indicator of the multidimensional poverty index (MPI) using a multilevel Probit model. Then, the approach follows the Alkire and Foster approach to derive aggregate multidimensional measures, using dimensional vulnerability thresholds and a multidimensional poverty threshold. However, the study does not fully resolve the issue of the multidimensionality.

From our point of view, the study by Gallardo (2022) appears to provide the best answers to the weaknesses previously mentioned. Indeed, in order to preserve the multidimensionality of poverty in the estimation of vulnerability, Gallardo used a multidimensional Bayesian network classifier to estimate the conditional probabilities of being multidimensional poor and the VMR approach using standard downside semi-deviation as the risk parameter. The method provides estimates of vulnerability at the individual level that can be then summarized to provide some Foster-Greer-Thorbecke (FGT) vulnerability measures. In addition, this approach enabled a breakdown of vulnerability to multidimensional poverty by dimensions. This is the reason why this paper employs this methodological strategy.

2. Methodological strategy

In this section we present the methods we used to provide measures of vulnerability to multidimensional poverty. We follow the approach developed by Gallardo (2022). This approach involves three steps : the first step refers to the assessment of multidimensional poverty, the second one concerns the modeling of uncertainty present in the conditional probabilities of being multidimensional poor and deprived in each well-being dimension,

[3] The VEP approach can lead to cases in which increases in variance or risk can reduce the probability of being vulnerable.

using Bayesian network classifiers and the last step is the measurement of vulnerability based on mean- downside semi-deviation as developed by Gallardo (2013, 2022).

3.1 Assessment of multidimensional poverty

This section describes M-*gamma* family multidimensional poverty measures suggested by Alkire and Foster (2019) that we used as benchmark to assess vulnerability to multidimensional poverty. This class of measures is analogous to FGT-*alpha* measures in the case of ordinal variables.

The central features of the counting-based approach to poverty of Alkire and Foster (2011) are the use of binary variables and of a dual cut-off method to identify the multi-poor.

Given a population of n individuals ($i = 1, . . , n$), m indicators ($j = 1, . . , m$) of well-being and weights (w_j) assigned to each indicator, two cut-offs are used to identify individuals who are multi-dimensionally poor: the dimension specific poverty lines (z_j) and the cross dimensional cut-off (k). Individual deprivations in every dimension are first assessed by comparing achievements in a given dimension j with a dimension-specific poverty line (z_j). An overall deprivation count (c_i) is then computed for each individual by summing up weighted deprivations suffered by each individual. In a second step, a cross dimensional cut-off value (k) which indicates the minimum deprivation count an individual should experience to be considered as multi-dimensionally is used to distinguish the individuals who are multi-dimensionally poor from those who are not poor. In the case of the UNDP's global MPI, the value of k is set at 1⁄3 of the weighted dimensions.

Finally, individual poverty levels are aggregated to derive a measure of poverty in multiple attributes. In the case of ordinal variables, Alkire and Foster (2019) define the M-*gamma* class of poverty measures as*:*

$$M_0^\gamma = \frac{1}{n}\sum_{i=1}^{n} c_i^\gamma(k) \quad for\ \gamma \geq 0$$

When γ=0, we obtain H the multidimensional headcount ratio or the incidence of multidimensional poverty. When γ=1, M_0^1 corresponds to the famous MPI which is similar to the poverty gap in unidimensional case as M_0^1 can be expressed as the product of the incidence (H) and the intensity of poverty (A) or the average deprivation counts among the poor:

$M_0^1 = H \times A$ with $A = \frac{1}{q}\sum_{i=1}^{q} c_i\,(k)$ and q being the number of poor.

The main advantage of M_0^1 is that it is decomposable by sub-group of population and by dimension.[4] Such a break down allows us to stress the contribution of each indicator to overall poverty and the deprivation profile of the poor.

When γ=2, the measure M_0^2 is an extension of the squared poverty gap ($FGT_{\alpha=2}$) in the

[4] In particular, M_0^1 can be expressed as an average of the censored headcount ratios of indicators weighted by their relative weights. the intensity of poverty A can also be expressed as a weighted average of deprivations in each indicator among the poor.

multidimensional case. M_0^2 is sensitive to inequality among the poor.[5]

2.2 Modeling uncertainty using Bayesian network classifiers

Ex-post identification of the multidimensional poor individuals can then be used to derive an ex-ante measure of poverty that is not simultaneously observable. Put differently, we must now consider the multidimensional poverty status and its various indicators as random variables. Due to the binary nature of variables, the two possible outcomes are being poor/not poor deprived or not deprived/ deprived. These possible events are represented by a Bernoulli distribution. In addition, given that deprivations in each dimension may depend on households' characteristics that are proxies of the determinants of poverty, the objective is to estimate a joint probability distribution that allows to predict both the risk of experiencing multidimensional poverty and to be deprived in each dimension in the near future.

In order to preserve the multidimensionality of poverty, the approach to be adopted requires to take into account the complex relationships among the several latent variables of deprivations and the characteristics of the households.[6]

Consider our n individuals $i = 1, \dots, n$. Each individual i is now characterized by a m-random vector $Y_i = (Y_{i1}, \dots, Y_{im})$ where each random variable for given attribute j takes the value of one in the event that individual is not deprived and zero otherwise. In addition, let a n-random vector $Y^{MP} = (Y_1^{M}, \dots Y_i^{MP} \dots, Y_n^{MP})$ with only two possible realizations for each Y_i^{MP}: one if the individual i is classified as not being multidimensional poor and 0 otherwise. The realizations of Y_i^{MP} depend on the values taken by Y_i. In addition, the random variables in Y_i depend on values taken by a q-random vector $X_i = (X_{i1}, \dots, X_{iq})$ of q categorical variables corresponding to household and community characteristics to which a person belongs.

The uncertainty regarding multidimensional poverty can be modeled by the following joint probability distribution function:

$$P(Y_i^{MP}, Y_i, X_i) = P\left(y_i^{MP}, y_{i1}, \dots, y_{im};\ x_{i1}, \dots, x_{iq}\right), \qquad i = 1, \dots, n$$

The objective is to estimate simultaneously both the probability of being poor/ non-poor conditional on the set of deprivations y_i ie

$$P(y_i^{MP} | y_i)$$

and the probability of being deprived/non-deprived in each attribute of well-being conditional on the households' characteristics x_i:

$$P\left(y_{ij} | x_i\right).$$

A multidimensional Bayesien network classifier (MBC) seems particularly appropriate for this task. To ease the presentation in what follows, we ignore the indices relating to individuals.

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[5] M_0^2 can be easily decomposed into the three 'I's of poverty (Jenkins and Lambert, 1997) i.e. the incidence, intensity and also inequality of multidimensional poverty as follows:

$$M_0^2 = H \times A^2 \times \left[1 + 2GE_2(c_p)\right] = M_0^1 \times A \times \left[1 + 2GE_2(c_p)\right]$$

with $GE_2(c_p)$ a Generalised Entropy measure of inequality among the poor applied to the distribution of deprivation counts among the poor (c_p). When γ=2, $GE_2(c_p)$ is half of the square of the coefficient of variation.

[6] The use of Logit and Probit models would not be satisfactory in this case, except to restrict ourselves to estimating the probability of being poor by not taking into account the risk of deprivation in each dimension.

MBC is a Bayesian network classifier characterized by a restricted topology to solve classification problems which include multiple class variables in which instances described by a number of features have to be assigned to a combination of classes (see Zaragoza et al., 2011). A Bayesian network is a probabilistic graph model that represents a set of random variables with their conditional dependencies through the use of a directed acyclic graph (*DAG*). In the (*DAG*), random variables are modelled as nodes, probabilistic relationships are captured by directed arcs between the nodes and conditional probability distributions associated with the nodes. For instance, an arc from Y to X_j indicates that a value taken by X_j depends on the value taken by Y. Nodes$Y$ is referred to the parent of X_j and Y is the child of X_j. This naming can be extended to the descendants of a node X_j from the nodes reachable from by repeatedly following the arcs. In addition, the structure of the network encodes that each node is conditionally independent of its non-descendants given its parents. This condition is important for the factorization of the joint probability distribution over the entire set of random variables.[7]

More formally, a BN is a pair $B = \{G, \Theta\}$ where G is a directed acyclic graph (*DAG*) whose nodes are the random variables and Θ a set of parameters that quantifies the dependencies between the variables within G, Θ contains the conditional probability distributions. It is formed by a parameter $\theta_{xj|pa} = P\ (x_j|pa_{(x}\ ))$ for each possible values x_j of X_j, given each (x_j) combination of the direct parent variables of X_j denoted by $(pa_{(x_j)})$. Then, the network represents the following joint probability distribution:

$$P(X_1, \dots, X_q) = \prod_{i=1}^{q} P\left(x_j|pa_{(x_j)}\right)$$

In turn, a Bayesian network classifier is a Bayesian network where variables are partitioned into class variables Y and feature variables $X = (X_1, \dots, X_q)$ of binary or categorical variables. A class variable Y has no parent and each attribute X_j has the class variable(s) as parents. BN computes the joint probability distribution as:

$$P(Y, X_1, \dots, X_q) = P(Y) \prod_{i=1}^{q} P(X_j|Y)$$

The classification problem can be stated as learning the posterior conditional distribution of the class variable Y conditioned on the attribute levels X_j. For an instance of the feature variables X, the goal is to find the most probable assignment of the class variable Y, known as the maximum posterior (MAP) estimation:

$$argmax_y P(Y = y|x_1, \dots, x_q)$$

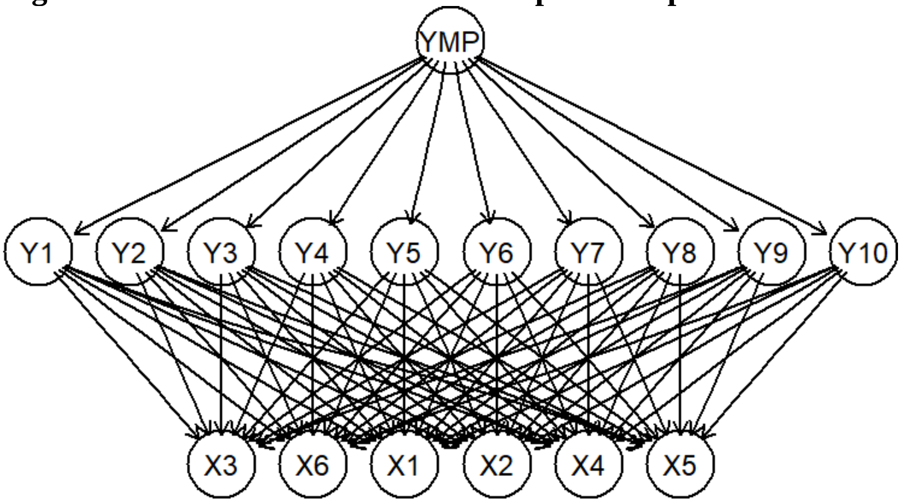
where the corresponding posterior conditional probabilities $P(Y|X)$ can be computed using the Bayes rule as $P(Y|X) = P(Y, X_1, \dots, X_q)/ P(X)$.

In our case, the structure of the bayesian network classifier is more complex since it includes two levels: in the first level the q feature variables X_j are used to predict m class variables Y_j while in the second level, the m class variables Y_j act as a vector of feature variables to predict the super-class variable Y^{MP} (Figure 1).

[7] This property is used to reduce the number of parameters required to characterize the joint probability distribution (JPD).

Our aim is to obtain for each individual i the posterior conditional probabilities which will be denoted by p_i and for y_i^{MP} and by p_{ij} for y_{ij} in each dimension j=1,…,m from the implementation of MBC to estimate vulnerability to multidimensional poverty.

Figure 1: MBC to estimate conditional posterior probabilities



Note: Our implementation of MBC includes the ten indicators used to measure multidimensional poverty and six households features as covariates of deprivation in each indicator.

2.3 Vulnerability to multidimensional poverty's measurement indicators

We employ the *Vulnerability by Mean Risk* (VMR) approach developed by Gallardo (2013) that uses the mean risk criterion to calculate risk (see Gallardo, 2013). It encapsulates in a single measure two kinds of situations both expected poverty and the downside risk of falling into poverty.

Consider y_i a random variable representing the well-being of individual i, μ_i the expected value of well-being and r_i the risk to fall below μ_i. Given the asymmetric nature of the risk of falling into poverty, Gallardo suggested the downside mean-semi deviation as the risk parameter:[8]

$\tilde{\sigma}_i = E\{min[(y_i - \mu_i), 0]^2\}^{1/2}$

This measure considers only random deviations of well-being below its expected value given that individuals seek to maximize μ_i and minimize $\tilde{\sigma}$.

From μ_i and $\tilde{\sigma}_i$, the risk-adjusted mean parameter for individual i is defined as follows:

$$\tilde{\mu}_i = \mu_i - \lambda\tilde{\sigma}$$

where λ is a risk aversion parameter defined in the interval [0,1]. It reflects the social planer concern about the trade-off between mean and risk of losses in well-being.

In fact, the λ parameter acts as a weight in the expression of $\tilde{\mu}_i$ as λ = 0 corresponds to the case of risk neutrality while when values of λ are large and tend toward 1, the gains in the expected value of well-being are at least as preferred as avoiding losses due to risk.

[8] Following the criticism of the use of the variance as a measure of risk in finance literature, Markowitz (1959) suggested the Semivariance which takes into consideration the asymmetry and the risk perception of investors.

Given z the poverty line under certainty, the identification criterion is then given by: An individual i is vulnerable to poverty if only if $\tilde{\mu}_i = \mu_i - \lambda\tilde{\alpha} \leq z$

Moreover, since $\tilde{\mu}_i$ includes both expected poverty and risk of falling into poverty, it enables to distinguish individuals experiencing severe vulnerability if $\mu_i \leq z$ from those facing moderate vulnerability when $\mu_i > z \wedge \mu_i - \lambda\tilde{\alpha} \leq z$.

Once the identification criterion has been applied, standard FGT poverty measures can be computed as follows:

$$V_\alpha = \frac{1}{n}\sum\left[\frac{z-\tilde{\mu}_i}{z}\right]^\alpha I_{\tilde{\mu}_i \leq z}$$

with $\alpha \geq 0$ and $I\tilde{\mu}i\leq z$ the identification function that takes a value of 1 if $\tilde{\mu}i\leq z$ and 0 otherwise.

In addition, $V\alpha$ can be decomposed into two indicators: vulnerability induced by poverty $V\alpha P$ and vulnerability induced by risk V_α^R :

$$V_\alpha = V_\alpha^P + V_\alpha^R$$

with $V_\alpha^P = \frac{1}{n}\sum\left[\frac{z-\tilde{\mu}_i}{z}\right]^\alpha I_{\mu_i \leq z}$ and $V_\alpha^R = \frac{1}{n}\sum\left[\frac{z-\tilde{\mu}_i}{z}\right]^\alpha I_{\mu_i > z \wedge \tilde{\mu}_i \leq z}$ with $\alpha \geq 0$

As shown by Gallardo (2013), these indexes can easily be extended to measure vulnerability to multidimensional poverty as defined in 2.1., using the counting-based approach of Alkire and Foster.

Given that for each individual i, y_i^{MP} is a random Bernoulli variable, the identification criterion requires the choice of a vulnerability threshold value which corresponds to a probability threshold. We apply the most common vulnerability benchmark of 50%:[9]

In this case, the downside semi-deviation of y_{iMP} takes the following expression:

$$\tilde{\sigma}_i^{rp} = [p_i^2(1-p_i)]^{1/2}$$

where pi is the probability of being multidimensionally non-poor for individual i. Note that we used k=1/3 of the weighted dimensions as the threshold value to identify the multidimensional poor. It follows that $\tilde{\mu}_i$ is then defined by

$$\tilde{\mu}_i^{rp} = p_i - \lambda\,\tilde{\sigma}_i^{rp}.$$

Therefore, individual i is deemed vulnerable to multidimensional poverty if $\tilde{\mu}_i^{rp} \leq 0.5$.

As the realization of y_i^{MP} $yiMP$depend on the values taken by the m-random vector yi=(yi1,…,yim) of Bernoulli variables with p_{ij} the probability that individual i is not deprived in dimension j, the identification criterion applies to each dimension using z^p=0.5 as the probability threshold. The person i is vulnerable in dimension j if $\tilde{\mu}_i^{rp}\leq 0.5$.

[9] For a discussion regarding the choice of this probability threshold, see Azam and Imai (2009) and Chaudhuri, S. (2003).

Since the estimation of the MBC provides estimates of these probabilities for each individual i over the whole dimensions (p_i) and in each dimension pij with j=1,…,m, it is then easy to obtain aggregate measures of vulnerability to multidimensional poverty V_α^{MP} and of vulnerability in each dimension V_α^{JP}.

For the FGT measure, vulnerability to multidimensional poverty V_α^{MP} can be expressed as follows:

$$V_\alpha^{MP} = \frac{1}{n}\sum_{i=1}^{n} g_i^\alpha \, I_{\tilde{\mu}_{ij}^{rp} \leq z^p} \text{ with } \alpha \geq 0$$

and $g_i = \left(\frac{z^p - \tilde{\mu}_{ij}^{rp}}{z^p}\right)$ the vulnerability gap for person i relative to the probability threshold z^p=0.5.

Analogous to FGT measures, V_α^{MP} becomes the vulnerability headcount ratio V_0^{MP} for α=0, the vulnerability gap ratio V_1^{MP} for α=1, and the square V_2^{MP} vulnerable gap ratio for α=2. It is also possible to decompose V_α^{MP} and V_α^{JP} for j=1,…,m into poverty induced and risk induced vulnerability denoted by sub-indexes P and R respectively :

$$V_\alpha^{MP} = V_{\alpha,P}^{MP} + V_{\alpha,R}^{MP}$$

$$V_\alpha^{JP} = V_{\alpha,P}^{JP} + V_{\alpha,R}^{JP}$$

In particular, in the empirical part of this paper, we investigate the extent of overlap between ex-post multidimensional poverty and ex-ante vulnerability to multidimensional poverty. We also make use of the vulnerability headcount ratio in each dimension to examine the profile by dimension of the different categories of vulnerable people.

3. Results and discussion

We now apply the empirical strategy described in section 2 to Algeria and Tunisia. These countries are all classified as middle-income countries but have adopted different economic models. While Tunisia based its development on an export-oriented labor-intensive model and tourism, Algeria belongs to oil producing countries. Between 2012 and 2018, with an annual average GDP per capita growth rate at 1.4 %, economic growth has been sluggish in Tunisia due to instability and terrorist attacks. In Algeria with an annual average GDP per capita growth rate at 0.6%, economic performance has been highly dependent on oil price volatility. In addition, according to the Human Development Index, Algeria and Tunisia rank in high HDI category with values above that in most Arab countries. Despite their commitment to SDG 1, monetary measures based on the international and national poverty lines remain predominant to monitor progress. However, the last estimate of monetary poverty dates back to 2011 for Algeria, to 2015 for Tunisia. There are very few studies that take a multidimensional approach to poverty measurement in those countries. To the best of our knowledge, the most recent studies on multidimensional poverty in these countries are a series of papers edited by Bérenger and Bresson (2013). Except the global MPI by UNDP and the revised Arab MPI (2021), very few studies rely on the counting-based approach of Alkire and Foster. The sole studies are those by Abu-Ismail et al. (2015) on Jordan, Iraq and Morocco, by Bérenger (2017) on Egypt and Jordan, by Bérenger (2021) on Algeria, Iraq and Tunisia, by Nasri and Belhadj (2017), by Ben Hassine and Sghairi (2021) using 2010 Tunisia household budget surveys and by Oznur and Eleftherios (2021) in selected MENA countries. While studies that adopt a multidimensional approach to poverty are scarce, they are almost non-existent with respect to the measurement of vulnerability to poverty in this region. To our knowledge, the only study is Lyons et al. (2021) on Syrian Refugees in Lebanon.

3.1 Data description

We use data from UNICEF's Multiple Indicators Cluster Survey (MICS) for two different years in Algeria (2012/13, 2018/19) and in Tunisia (2012, 2018). Table 1 presents the list of the indicators with the same dimensions as the HDI, namely education, health, and standard of living. However, drawing on proposals from ESCWA for an Arab MPI (2017 and 2021), secondary school level is used as deprivation cut-off for years of education as well as the duration of compulsory school for deprivation in school attendance. We also include three additional indicators:

- overcrowding as a form of deprivation in the context of rising prices of real estate and housing in Arab countries;
- prevalence of obesity among children alongside undernutrition which is an increasing concern in Arab countries
- and early pregnancy or early marriage for women under 28 years old as a major factor behind women's deaths.

The MPI is composed of ten indicators which corresponds to $(Y_1, \dots, Y_m)$ grouped in three dimensions using the same weighting structure as the MPI by UNDP.

Table 1: List of dimensions and indicators

Dimension	**Indicators**	**Deprivation Cut-off**	**Relative weight**
Education	School attendance	Any school-aged child (6-16) is not attending school or is two years or more behind the right school grade	1/6
	Years of education	No household member aged 17 years or older has completed secondary school	1/6
Health	Nutrition	Any child (0-59 months) is stunted or overweight (weight for height > +2SD)	1/9
	Mortality	Any child from a household who has died	1/9
	Early pregnancy or marriage	A woman less than 28 years old got first pregnancy or marriage before being 18 years old	1/9
Standard of Living	Water	No access to safe drinking water source within 30 minutes one-way distance from the residence	1/15
	Sanitation	Household sanitation facility is not improved or improved but shared.	1/15
	Overcrowding	Household has 2.5 people per sleeping room	1/15
	Floor	Household has rudimentary or cement floor	1/15
	Assets	Household has less than two assets for accessing to information (radio, TV, phone) or less than two livelihood assets (refrigerator, washing machine, air conditioner, water heater, stove) and household has less than two mobility assets (car, bike, motorcycle)	1/15

Source: Author's calculation based on UNICEF-MICS data.

The households' characteristics that we used as variables to implement the MBC are reported in Table 1.A in Annex. Due to the constraint availability of data, six variables were selected; they corresponds to $(X_1, \dots, X_q)$. As the implementation of the MBC requires that features variables be categorical, Table 1.A shows the categorization implemented.

The multidimensional poverty measures have been computed using the poverty threshold k=1/3. In addition, as mentioned in section 2, vulnerability measures based on downside semi- deviation depends arbitrarily on the choice of the value of the risk aversion parameter λ. The analysis of levels and trends in vulnerability is performed using λ = 1 and a vulnerability threshold value of 0.5.

Since the vulnerability measures taken as a reference the poverty measures constructed on the basis of the Alkire and Foster approach, we begin by presenting the results of these measures. We then present the results from the implementation of the MBC and the vulnerability measures that it enables to construct. Finally, we examine the overlap between vulnerability and multidimensional poverty in order to identify different categories of vulnerable people.

3.2 Multidimensional poverty measures

Table 2 reports multidimensional poverty estimates of the MPI (M_0^1) and of its two components- the incidence (H) and the intensity (A) and of M_0^2 for Algeria and Tunisia and for two years. Comparisons across countries show that Algeria in 2019 and Tunisia in 2018 exhibit very similar levels of poverty. Let us now take a look at the trends of the poverty indices for each country and by areas of residence (Table 2.A in Annex). At the national level, all countries experienced a reduction in their multidimensional poverty. However, there are striking differences between and within the two countries. Algeria registered the fastest reduction in its MPI (from 0.120 in 2013 to 0.049 in 2019) by 13.80% per year thus allowing Algeria to catch up with Tunisia where M_0^1 was even significantly lower in the first period (0.079). In both countries, progress was due to the joint impact of decreases in H and in A which were significantly faster in Algeria (13% and 1 % per year reps.) than in Tunisia (7.20% and 0.70% per year resp.). In Tunisia, rural poverty decreased at a faster rate than in urban areas mitigating the urban-rural divide. By contrast, in Algeria poverty measures (M_0^1, H, M_0^2) decreased at a slower rate in rural areas than in urban areas deepening the gap with urban areas. In Table 2.A (Appendix), the estimates of M_0^2 that is sensitive to inequality among the poor indicate that poverty decrease has been accompanied in both countries by a decline in inequality among the poor. While in Tunisia the decline has been faster in rural than in urban areas, this has not been the case in Algeria. In Algeria, the urban poorest seem to benefit more from the poverty decline than the rural poorest.

Table 2: Observed multidimensional poverty using the M-gamma family measures

	$\boldsymbol{H}$	M_0^1	$\boldsymbol{A}$	M_0^2
Algeria				
2013	0.259	0.120	0.463	0.058
	(0.007)	*(0.003)*	*(0.002)*	*(0.002)*
2019	0.113	0.049	0.437	0.022
	(0.004)	*(0.002)*	*(0.003)*	*(0.001)*
$\boldsymbol{ARC}$	-0.130	-0.138	-0.010	-0.148
Tunisia				
2012	0.176	0.079	0.451	0.038
	(0.008)	*(0.004)*	*(0.004)*	*(0.002)*
2018	0.112	0.049	0.432	0.022
	(0.006)	*(0.003)*	*(0.004)*	*(0.001)*
$\boldsymbol{ARC}$	-0.072	-0.078	-0.007	-0.086

Note: $\boldsymbol{ARC}$ is the average annualized change. Standard errors are reported between brackets. $\boldsymbol{ARC}$ are statistically significant at α=0.01. Source: Author's calculation based on UNICEF-MICS data.

Now we need to take an ex-ante approach to poverty in order to examine the patterns of vulnerability in these two countries.

3.3 Results from the MBC implementation

We apply the MBC described in section 2.2. It enables us to obtain for each individual i the posterior conditional probabilities which will be denoted by pi for $\boldsymbol{y_i^{MP}}$ and by p_{ij} for y_{ij} in each dimension j=1,…,m . These probabilities are then used to construct measures of vulnerability to multidimensional poverty. In line with Gallardo (2022) and with similar assessments implemented in the literature (Gil-Begue et al. 2020 and Zaragoza et al. 2011), the predictive accuracy of Bayesian Classifiers has been estimated using two measures. The first is the overall accuracy which corresponds to the accuracy of predicting correctly the values of y^{MP} of the multidimensionally poor and non-poor. The second measure is the average accuracy over the class variables y_j which is the mean of the prediction accuracies obtained from each class variables separately. The results of these two measures are presented in Table 3. The measures of overall accuracy range from 0.83 to 0.90 which is quite good. Regarding the measures of accuracy by dimension, we observe that the performance of the MBC are the best for the outcomes of early pregnancy, mortality and nutrition whatever the period and the country considered. By contrast, the predictions of the model are less accurate (accuracy less than 0.8) in indicators of floor material and assets over the whole period in the two countries. Not surprisingly, the average accuracy is lower than overall accuracy.

Table 3: Predictive accuracy of the Bayesian network classifiers with five-fold cross validation

	Algeria 13	Algeria 19	Tunisia 12	Tunisia 18
Accuracy by Dimension				
Sanitation	0.88	0.88	0.88	0.97
Water	0.82	0.93	0.93	0.86
Floor mat.	0.75	0.72	0.72	0.83
Overcrowding	0.75	0.80	0.80	0.89
Assets	0.73	0.73	0.73	0.78
Nutrition	0.91	0.92	0.92	0.95
Early Pregnancy	0.98	0.97	0.97	0.98
Mortality	0.94	0.96	0.96	0.97
School attendance	0.77	0.88	0.88	0.92
Years of Education	0.82	0.83	0.83	0.77
Average accuracy dimensions	0.84	0.86	0.86	0.89
Overall Accuracy	0.83	0.90	0.90	0.91

Note: In order to assess the predictive accuracy of the Bayesian network classifier, we applied a 5-fold cross- validation procedure. The idea behind this procedure is to randomly split the original data set into k-folds (or subsets). For each fold, a model is trained on the k-1 folds of the dataset and the remaining set is used as a validation test. The procedure is repeated until the k-folds have served as test sets. At each step, the accuracy of the model is recorded and the cross-validation accuracy is simply the average of the k recorded accuracy.

As explained in section 2.2., we used the probabilities provided by the Bayesian network classifier to compute vulnerability measures presented in section 2.3.

3.4 Measures of vulnerability to multidimensional poverty

The probabilities obtained from the implementation of the Bayesian network classifiers are used to compute the risk adjusted probabilities for each individual of being non-poor $\tilde{\mu}_i^{rp}$ or of being non-deprived in each indicator $\tilde{\mu}_{ij}^{rp}$ and the vulnerability measures V_α^{MP} for α=0,1,2.

However, these measures are obtained from the individual vulnerability gaps, using only the information contained in $\tilde{\mu}_i^{rp}$. Per se, they do not enable one to obtain information on the components of the vulnerability suffered by the vulnerable individuals. In addition, the results of such measures are not easily understandable since their interpretation is expressed in terms of probabilities. For instance, V_1^{MP} is the amount of the adjusted probability that would be required as proportion of the vulnerability threshold, to overcome vulnerability. Therefore, we propose to combine the information provided by the identification of the vulnerable people to multidimensional poverty provided by the headcount ratio and the information regarding the vulnerability in each dimension to follow an approach similar to that of Alkire and Foster in constructing the MPI. These measures will be denoted by V_{01}^{MP} and V_{02}^{MP} which are the analogous to M_0^1 and M_0^2 of the multidimensional poverty measures respectively.[10] All these measures were computed using λ=1 for the risk parameter. However, Figure 1.A. in the Appendix presents values of the vulnerability headcount ratio obtained for alternative values of λ. Table 4 reports the results of these several measures at the national level for each country. The results obtained by area of residence are also given in Table 2.A. in Appendix.

Table 4: Measures of Vulnerability to Multidimensional Poverty using $\boldsymbol{\lambda}$ = 1

Multidimensional Vulnerability based on risk-adjusted mean					
	V_0^{MP}	V_1^{MP}	V_{A1}^{MP}	V_2^{MP}	V_0^{MP} / H
Algeria					
2013	0.451	0.266	0.590	0.201	1.741
	0.007	*0.006*	*0.006*	*0.005*	
2019	0.176	0.091	0.518	0.072	1.568
	0.005	*0.004*	*0.009*	*0.003*	
$\boldsymbol{ARC}$	-0.145	-0.163	-0.021	-0.157	
Tunisia					
2012	0.317	0.170	0.537	0.129	1.805
	0.007	*0.005*	*0.009*	*0.004*	
2018	0.210	0.112	0.535	0.087	1.867
	0.008	*0.005*	*0.010*	*0.005*	
$\boldsymbol{ARC}$	-0.067	-0.067	-0.001	-0.064	
Multidimensional vulnerability based on dimensional vulnerability					
	V_0^{MP}	V_{01}^{MP}	V_{A01}^{MP}	V_{02}^{MP}	V_{02ineg}^{MP}
Algeria					
2013	0.451	0.226	0.501	0.122	0.037
	0.007	*0.004*	*0.002*	*0.003*	
2019	0.176	0.075	0.425	0.035	0.047
	0.005	*0.002*	*0.002*	*0.001*	
$\boldsymbol{ARC}$	-0.145	-0.168	-0.027	-0.188	
Tunisia					
2012	0.317	0.138	0.436	0.067	0.054
	0.007	*0.003*	*0.003*	*0.002*	
2018	0.210	0.085	0.404	0.038	0.053
	0.008	*0.004*	*0.004*	*0.002*	
$\boldsymbol{ARC}$	-0.067	-0.078	-0.013	-0.091	

Note: $\boldsymbol{ARC}$ is the average annualized relative change. Standard errors are reported between brackets. $\boldsymbol{ARC}$ are statistically significant at α=0.01. Source: Author's calculation based on UNICEF-MICS data.

In all cases vulnerability headcount ratios are significantly higher than poverty headcount ratios, suggesting that current poverty estimates tell us only part of the story. First of all, while Algeria and Tunisia registered similar levels of multidimensional poverty, the estimates in Table 4 show that vulnerability measures are higher in Tunisia than in Algeria. In addition, the vulnerability to poverty ratios (V_0^{MP}/H) in Table 4 indicate that for each poor people in the population there are 1.5 and 1.8 vulnerable persons in Algeria in 2019 and in Tunisia in

[10] A complete use of the information conveyed by vulnerability in each dimension would require taking into account the vulnerability gap in each dimension. This could be achieved by using the multidimensional measures proposed in the literature.

2018 respectively. This ratio decreased over time in Algeria while it slightly increased in Tunisia. As well as poverty, vulnerability has decreased over time in these two countries. However, the decrease in vulnerability has been faster in Algeria than in Tunisia (at 14.5% per year and 6.7% per year resp. in V_0^{MP}) for all the vulnerability measures. Comparing the evolution of vulnerability to that of multidimensional poverty provide interesting insights regarding the evolution paths of poverty in these two countries. Whatever the approach adopted to measure vulnerability, vulnerability decreased at a faster rate than poverty, both at the national level and in urban areas (Table 2.A.) while the opposite can be observed in Tunisia where the decrease in vulnerability was slower than the decrease in poverty both at the national level and in rural areas (Table 2.A.). This finding suggests that the achievements in poverty reduction are more fragile in Tunisia than in Algeria. Similar trends can be also observed with the measures that account for the intensity and inequality among the poor. Moreover, as mentioned previously, the decomposition of the vulnerability measures V_{01}^{MP} which is the analogous of M_0^1 or the MPI in V_{A01}^{MP} to deprivations among the vulnerable people. In particular, even if Algeria registers less vulnerability in 2019 than Tunisia in 2018, the intensity in vulnerability is higher than in Tunisia since the vulnerable are at risk of being deprived in 42.5 % of the attributes of well-being compared to 40.4% in Tunisia. While Algeria and Tunisia register similar levels of multidimensional poverty in the last period, vulnerability assessments mitigate this conclusion.

Let us now take a look at the composition of vulnerability into its risk induced $V_{\alpha,R}^{MP}$ and poverty induced $V_{\alpha,R}^{MP}$ components. In what follows, we limit ourselves to presenting the decomposition of the vulnerability headcount ratio V_0^{MP} and the intensity of vulnerability within each vulnerable group as our aim is to concentrate our attention on the trends in these two components. Thus Table 5 reports $V_{0,P}^{MP}$ which corresponds to the percentage of individuals whose vulnerability is due to low expected level of well-being, also called by Gallardo (2013) severe vulnerability, and $V_{0,R}^{MP}$which gives the percentage of individuals who suffer vulnerability due to the volatility of their well-being and called moderate vulnerable. Table 5 also presents the measures of intensity of vulnerability in terms of the risk adjusted probability gap (V_{A1P}^{MP}; V_{A1R}^{MP}) and also in terms of the proportion of dimensions in which vulnerable individuals face a risk of being deprived (V_{A01P}^{MP}; V_{A01R}^{MP}). Table 3.A. in Appendix provides also the results by areas of residence. Table 5 shows that moderate vulnerability prevails over severe vulnerability both in Algeria and Tunisia. Algeria experienced the greatest achievements in the reduction of both vulnerability components, compared to Tunisia. In Algeria, the decrease in the headcount ratio of severe vulnerability was faster than that of moderate vulnerability (15.8% and 13.6% resp.), although improvements in the intensity of vulnerability benefitted slightly more the moderate vulnerable than the severe vulnerable people (0.177 and 0.067 of weighted dimensions according to V_{A01R}^{MP} and V_{A01P}^{MP} resp.). These trends are particularly evident in rural areas (Table 3.A). On the other hand, trends are more ambiguous in urban areas despite the most significant decrease recorded in the vulnerability headcount ratio. The intensity of vulnerability even seems to have increased for the urban severe vulnerable individuals (Table 3.A). As a result of these trends in Algeria, vulnerability seems to be shifting more towards moderate vulnerability as the contribution of severe vulnerability to overall vulnerability (V_{0P}^{MP}/V_0^{MP}) decreased from 40.8% in 2013 to 37.1% in 2019.

Table 5: Decomposition of vulnerability into severe and moderate vulnerability

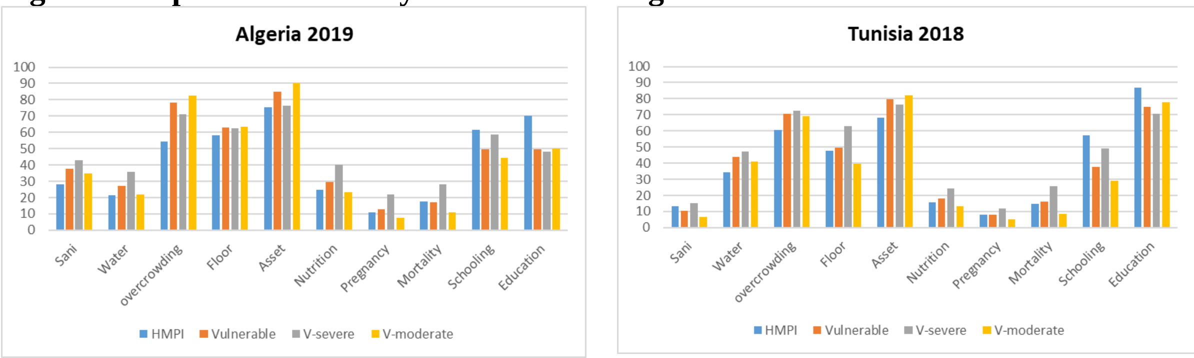
	V_0^{MP}	V_{0P}^{MP}	V_{0R}^{MP}	V_{A01P}^{MP}	V_{A01R}^{MP}	V_{A1P}^{MP}	V_{A1R}^{MP}	V_{0P}^{MP}/V_0^{MP}
Algeria								
2013	0.451	0.184	0.267	0.537	0.476	0.367	0.222	0.408
	0.007	*0.006*	*0.004*	*0.003*	*0.003*	*0.007*	*0.004*	
2019	0.176	0.065	0.111	0.470	0.399	0.349	0.169	0.371
	0.005	*0.003*	*0.003*	*0.005*	*0.003*	*0.012*	*0.004*	
ARC	-0.145	-0.158	-0.136	-0.022	-0.029	-0.009	-0.045	
Tunisia								
2012	0.317	0.103	0.214	0.528	0.392	0.298	0.239	0.324
	0.007	*0.005*	*0.006*	*0.007*	*0.003*	*0.012*	*0.006*	
2019	0.210	0.091	0.119	0.452	0.367	0.398	0.137	0.434
	0.008	*0.005*	*0.004*	*0.005*	*0.005*	*0.012*	*0.004*	
ARC	-0.067	-0.020	-0.094	-0.026	-0.011	0.049	-0.089	

Note: ***ARC*** is the average annualized relative change. Standard errors are reported between brackets. ***ARC*** are statistically significant at α=0.001. Values of ARC for V_{A01P}^{MP} and V_{A01R}^{MP} are easier to interpret by considering the absolute variation which gives outcomes in terms of share of weighted dimensions. Source: Author's calculation based on UNICEF-MICS data.

On the other hand, opposite trends can be observed in Tunisia. The most significant decreases in vulnerability concerned the moderate vulnerability group both in terms of the headcount ratio (V_{0R}^{MP}) and the intensity of the risk of multiple deprivations (whatever the approach adopted to measure intensity in vulnerability). These trends are particularly noticeable in rural areas (Table 3.A) for the moderate vulnerability people. However, regarding the severe vulnerability group, the results are less obvious since the approaches used to measure intensity in vulnerability provide opposite results both at the national level and by area of residence (Table 3.A). However, it is interesting to emphasize that the decline in severe vulnerability registered at the national level conceals an increase in the percentage of the severe vulnerability in urban areas which may suggest that some moderately vulnerable people have slipped into severe vulnerability. As a result, in Tunisia, vulnerability seems to be shifting more towards severe vulnerability as the contribution of severe vulnerability to overall vulnerability (V_{0P}^{MP}/V_0^{MP}) increased from 32.4% in 2012 to 43.4% in 2018.

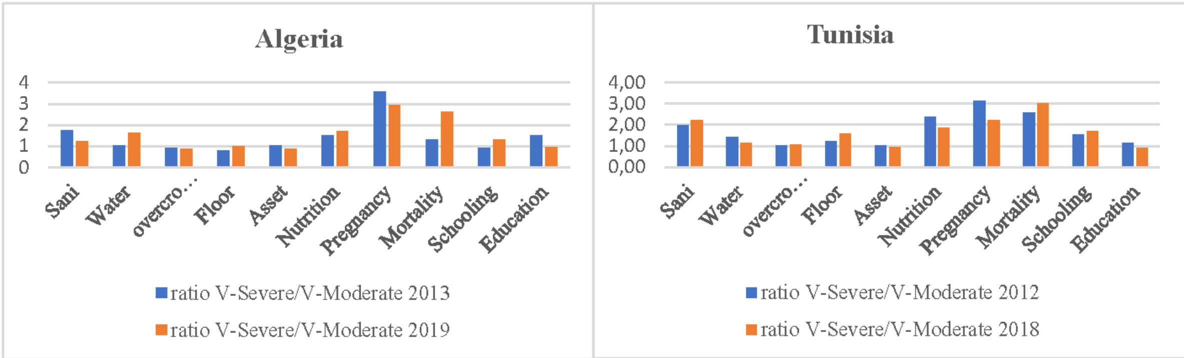
Let us have a further look at the dimensional composition of vulnerability and its two main components. Our aim is to examine whether severe vulnerability differs from moderate vulnerability in terms of its dimensional composition. For that purpose, we computed deprivation rates in each indicator among all the vulnerable, the severe vulnerable and the moderate vulnerable. In order to ease the presentation, Figure 3 presents deprivation rates among the different groups of vulnerable for the last year of the survey for Algeria and Tunisia. For comparison purpose, we also report the deprivation rates among the multidimensional poor.

Figure 3: Deprivation rates by dimension among the vulnerable



Note: V-Severe and V-moderate refer to the severe vulnerable and to the moderate vulnerable groups of people respectively. HMPI is the headcount ratio of multidimensional poverty.

Figure 4: Deprivation ratio by dimension among the severe and moderate vulnerable



Source: Author's calculation based on UNICEF-MICS data.

Figure 4 complements Figure 3 by showing the evolution over time of the ratios of deprivation rates among the severe vulnerable to the deprivation rate among the moderate vulnerable in each indicator and for each country.

Figure 3 shows that in Algeria the risk of deprivation among the vulnerable is the highest in three indicators of living standard dimension (assets, overcrowding and floor materials) followed by school attendance and years of education. The lowest risks are found in sanitation, access to water and in the three indicators of health (nutrition, mortality and early pregnancy). However, we can identify the deprivations that differentiate severe vulnerability from moderate vulnerability. By looking at Figure 3 and particularly Figure 4, the indicators where the differences between severe and moderate vulnerability are the largest are early pregnancy, mortality, nutrition, access to water and school attendance. They correspond to dimensions of structural poverty. In contrast, deprivations in the remaining indicators are much more similar between the two groups of vulnerable. In addition, Figure 4 shows that the ratios in mortality, nutrition and schooling increased between 2013 and 2019. The results obtained by areas of residence (Figure 3.A.) allow us to emphasize that early pregnancy, mortality, access to water and nutrition differentiate the two types of vulnerable people particularly in urban areas since the ratios of deprivation between severe and moderate vulnerability are significantly higher than in rural areas (Figure 4.A).

For Tunisia, the risk of deprivation among the vulnerable are the highest in assets, years of education, overcrowding and floor materials. As for Algeria, the indicators that differentiate the severe and moderate vulnerable are mortality, early pregnancy, sanitation, nutrition followed by school attendance. In Figure 4, it is interesting to note that differences between the two types of vulnerability increase regarding the risk of deprivation in mortality, sanitation and school attendance between 2012 and 2018. In addition, although they are not reported here, deprivations in sanitation and nutrition increased among the severe vulnerable over time. Finally, as shown in Figure 4.A, the ratios of indicators that differentiate the severe from the moderate vulnerable are significantly higher in urban than in rural areas and increased over time.

This analysis provides interesting information that is particularly suitable to policy targeting, the design and implementation social policies. It makes it possible to differentiate the indicators or dimensions that would require specific attention for the design and implementation of social policies.

3.5 Overlap between vulnerability and multidimensional poverty

As shown in Table 4, the percentage of individuals that are vulnerable to multidimensional poverty are 1.6 and around 1.9 times more numerous than the observed multidimensional poor in Algeria in 2019 and in Tunisia in 2018 respectively. Thus, it may be interesting to examine the overlap between different forms of vulnerability and poverty. This should make it possible to identify among the poor and the non-poor those who are at risk of remaining poor or falling into poverty and those who are likely to escape from poverty. Given that severe vulnerability and moderate vulnerability used two different vulnerability thresholds, following Chaudhuri et al. (2002) and Feeny and McDonald (2016), the vulnerable population can be divided into four distinct groups which enables us to differentiate the chronic poor from the transient (frequently poor) poor, and the highly (severely) vulnerable non-poor (vulnerability to chronic poverty) and relative (moderately) vulnerable non-poor. Table 6 presents the cross tabulation of vulnerability by distinguishing severe and moderate vulnerability and observed multidimensional poverty in Algeria and Tunisia for the last survey year.

Table 6: Vulnerability to poverty and observed multidimensional poverty

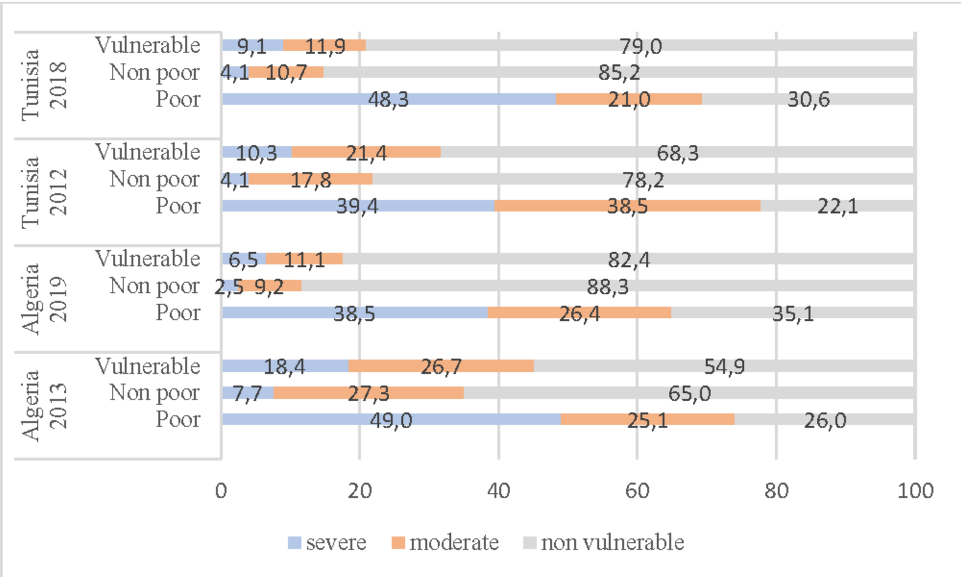
	Algeria 2019			
Observed multidimensional poverty				
Current poor 11.25%	Current non-poor 88.75%			
Estimated vulnerability	Total vulnerability 17.64%			
		Chronic poor 4.33%	Vulnerability to chronic poverty 2.21%	Severe vulnerability 6.54%
		Frequently poor 2.97%	Vulnerability to frequent poverty 8.13%	Moderate vulnerability 11.10%
	Not Vulnerable 82.36%	Infrequently poor 3.95%	Not vulnerable and not poor 78.41%	
	Tunisia 2018			
Observed multidimensional poverty				
Current poor 11.23%	Current non-poor 88.77%			
Estimated vulnerability	Total vulnerability 20.96%			
		Chronic poor 5.43%	Vulnerability to chronic poverty 3.66%	Severe vulnerability 9.09%
		Frequently poor 2.36%	Vulnerability to frequent poverty 9.52%	Moderate vulnerability 11.88%
	Not Vulnerable 79.04%	Infrequently poor 3.44%	Not vulnerable and not poor 75.60%	

Source: Author's calculation adapted from Chaudhuri et al. (2002) and Tesliuc and Lindert (2004). Shaded area is vulnerability.

Figure 5 complements information in Table 6 by providing vulnerability incidence by poverty status for each country and the two years, results by areas of residence are reported in Figure 5.A. in Appendix.

As shown in Table 6, of the 17% and 20% of the vulnerable population in Algeria and Tunisia in the last year of the survey respectively, 62.9% and 56.6% are vulnerable due to transitory factors. It can be seen that chronic poverty is more important among the vulnerable in Tunisia than in Algeria (43.4% and 37.1% resp.). Among the currently poor in Algeria and Tunisia, 38.5% and 48.3% (resp.) remain chronically poor with a high probability of experiencing multidimensional poverty in the future while 26.4% in Algeria and 21% in Tunisia face frequent poverty due to the volatility of their expected level of well-being (moderate vulnerability). Finally, among the currently poor, 26.4% in Algeria and 21% in Tunisia are infrequently poor suggesting that they are likely to escape from poverty (Figure 5). Regarding the incidence of vulnerability among the similar percentage of non-poor individuals in both countries (around 88%), only 2.5% in Algeria and 4.1% in Tunisia are found to qualify as vulnerable to chronic poverty. In addition, Figure 5 provides interesting insights on the evolution of vulnerability by poverty status over time in the two countries. It reveals clearly different trajectories in the evolution of the vulnerability components.

Figure 5: Incidence of vulnerability by poverty status in Algeria and Tunisia



Source: Author's calculation based on UNICEF-MICS data.

For the case of Algeria, the proportion of individuals that are severely vulnerable among the poor decreased significantly from 49% in 2013 to 38.5% in 2019 while the proportion of the moderately vulnerable slightly increased (from 25.1% to 26.4%). This trend is particularly evident in rural areas as shown in Figure 5A. while both severe and moderate vulnerability decreased in urban areas. Among the non-poor, severe and moderate vulnerability decreased almost at the same rate although slightly different trajectories are observed between urban and rural areas. Indeed, the decrease in vulnerability among the non-poor stems from higher relative decline in severe than moderate vulnerability in rural areas while the opposite is true in urban areas (Figure 5A.). For Tunisia, we note that the decrease in moderate vulnerability among the poor (from 38.5% in 2012 to 21% in 2018) came at the expense of an increase in severe vulnerability (from 39.4% in 2012 to 48.3% in 2018). This is clearly evident both in urban and rural areas (Figure 5.A). Among the non-poor. the decrease in vulnerability is essentially due to moderate vulnerability (from 17.8% to 10.7%). whereas severe vulnerability has not changed significantly despite a slight increase in urban areas (Figure 5.A).

4. Conclusion

Our objective was to assess levels and trends in vulnerability to multidimensional poverty in Algeria and Tunisia. Unlike the few existing studies that explored vulnerability to multidimensional poverty and employed the popular Chaudhuri et al. (2002) approach, we followed the approach suggested by Gallardo (2022). We modeled the joint probability of being poor and deprived in each dimension using multidimensional Bayesian networks classifiers. We relied on the VMR (vulnerability by mean risk) approach using the standard downside semi- deviation as the risk parameter for measuring vulnerability. To our knowledge, this study is currently the only application that adopted such an approach following Gallardo's study (2022) for the case of Chile. Analogous to the famous FGT-*alpha* indices in unidimensional case, our study provided measures of vulnerability to multidimensional poverty when γ = 0, 1 and 2 and decomposition of vulnerability by distinguishing severe and moderate vulnerability. Four key findings are noticeable.

First, both in Algeria and Tunisia, vulnerability headcount ratios are significantly higher than poverty headcount ratios suggesting that current poverty estimates tell us only part of the story. Despite similar levels of multidimensional poverty, vulnerability measures are higher in Tunisia than in Algeria. In addition, the achievements in poverty reduction are more fragile in Tunisia than in Algeria. Similar trends were also observed with the measures that account for the intensity and inequality among the poor.

The second finding is that moderate vulnerability prevails over severe vulnerability in both countries. Trends over time indicate that in Algeria, vulnerability seems to be shifting more towards moderate vulnerability as the contribution of severe vulnerability to overall vulnerability decreased between 2013 and 2019. Opposite trends were observed in Tunisia where the most significant decreases in vulnerability concerned moderate vulnerability particularly in rural areas. However, severe vulnerability increased in urban areas suggesting that some moderately vulnerable people have slipped into severe vulnerability. As a result, in Tunisia, vulnerability seems to be shifting more towards severe vulnerability between 2012 to and 2018.

Third, the dimensional decomposition of the two main components of vulnerability enabled us to identify the indicators where the differences between severe and moderate vulnerability were the largest are early pregnancy, mortality, nutrition, access to water and school attendance in Algeria; mortality, early pregnancy, sanitation, nutrition followed by school attendance in Tunisia. They correspond to dimensions of structural poverty. In contrast, deprivations in the remaining indicators are much more similar between the two groups of vulnerable. This makes it possible to differentiate the indicators or dimensions that would require specific attention for the design and implementation of social policies. Especially, it is worrisome to note that the risks of vulnerability seem to have increased. particularly in the nutrition indicator in both countries, in early pregnancy in Algeria and in sanitation in Tunisia.

Fourth, the analysis of the overlap between different forms of vulnerability and poverty showed that chronic poverty among the vulnerable is more important in Tunisia than in

Algeria. Among the currently poor in Algeria and Tunisia, 38.5% and 48.3% (resp.) remain chronically poor with a high probability of experiencing multidimensional poverty in the future while 26.4% in Algeria and 21% in Tunisia are infrequently poor suggesting that they are likely to escape from poverty. Our results revealed different trajectories in the evolution of the vulnerability components in these two countries. Severe vulnerability among the poor decreased in Algeria between 2013 and 2019 while moderate vulnerability slightly increased particularly in rural areas. On the other hand, in Tunisia, the decrease in moderate vulnerability among the poor came at the expense of an increase in severe vulnerability between 2012 and 2019.

These results highlight interesting differences in the nature of the vulnerability faced by the population in these two countries. These differences can be explained by the social policies at work in these two countries following the Arab Spring.

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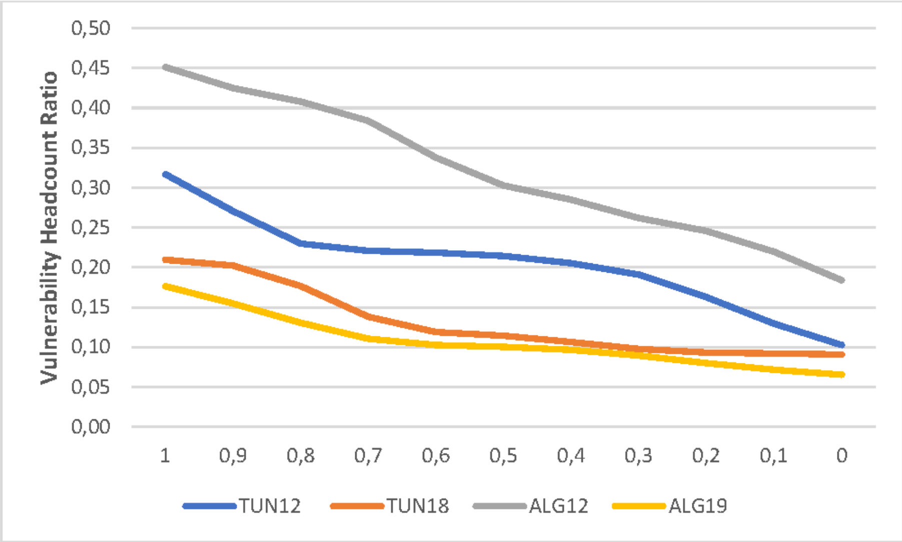
Appendix

Table 1.A: Discretization of feature household variables

Households characteristics		Algeria		Tunisia	
		2013	2019	2012	2018
Household head gender	Woman	0.11	0.11	0.15	0.16
	Man	0.89	0.89	0.85	0.84
Household head age	less than 35	0.98	0.08	0.09	0.08
	36-45	0.34	0.34	0.33	0.33
	46-55	0.35	0.35	0.35	0.36
	56-65	0.24	0.23	0.23	0.23
Household head education	no	0.32	0.28	0.27	0.21
	primary	0.46	0.47	0.37	0.34
	secondary and higher	0.22	0.25	0.36	0.45
Household size	1 person	0.08	0.11	0.21	0.23
	2 persons	0.12	0.13	0.16	0.18
	3 persons	0.18	0.19	0.22	0.25
	4 persons	0.19	0.23	0.21	0.20
	5 persons	0.16	0.16	0.11	0.09
	6 persons	0.27	0.09	0.05	0.04
	7 persons or more		0.09	0.04	0.02
Area of residence		0.65	0.64	0.68	0.71
		0.35	0.36	0.32	0.29
Region	1	0.36	0.33	0.25	0.26
	2	0.15	0.15	0.15	0.14
	3	0.15	0.17	0.11	0.12
	4	0.06	0.07	0.23	0.23
	5	0.14	0.14	0.05	0.12
	6	0.05	0.05	0.04	0.08
	7	0.09	0.09	0.03	0.05
	8			0.09	
	9			0.05	

Notes Regions in Algeria are : Nord Centre. Nord Est. Nord Ouest. Hauts Plateaux Centre. Hauts Plateaux Est. Hauts Plateaux Ouest. Sud. Regions in Tunisia 2012 : District Tunis. Nord Est. Nord Ouest. Centre Est . Kasserine. Kairouan. Sidi Bouzid. Sud Est. Sud Ouest.Tunisia 2018 : District Tunis. Nord Est. Nord Ouest. Centre Est . Centre Ouest. Sud Est. Sud Ouest. Values computed using the household as the unit of analysis. Source: Author’s calculation based on UNICEF-MICS data.

Figure 1: A Headcount ratio of vulnerability to multidimensional poverty for different values of $\boldsymbol{\lambda}$



Note: V_0^{MP} increases for higher values of λ. It could be possible to identify the value of λ for which vulnerability headcount ratio is closed to the poverty headcount ratio. These values would be around 0.3 for Algeria in 2013. 0.2 for Tunisia in 2012. 0.7 for Algeria in 2019 and 0.6 for Tunisia in 2018.
Source: Author's calculation based on UNICEF-MICS data.

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Table 2.A: Observed multidimensional poverty using the M-*gamma* family measures by areas of residence

		Observed multidimensional poverty				Multidimensional Vulnerability based on dimensional vulnerability			
		H	M_0^1	A	M_0^2	VH	VM_0^1	VMA	VM_0^2
Algeria									
2013	**Urban**	0.189	0.083	0.438	0.038	0.352	0.163	0.462	0.081
		0.006	*0.003*	*0.003*	*0.001*	*0.007*	*0.003*	*0.003*	*0.002*
	Rural	0.381	0.185	0.485	0.094	0.623	0.336	0.539	0.192
		0.013	*0.007*	*0.003*	*0.004*	*0.010*	*0.006*	*0.003*	*0.004*
2019	**Urban**	0.067	0.028	0.424	0.012	0.080	0.031	0.386	0.013
		0.003	*0.001*	*0.003*	*0.001*	*0.003*	*0.001*	*0.004*	*0.001*
	Rural	0.191	0.085	0.445	0.039	0.343	0.151	0.441	0.073
		0.010	*0.005*	*0.004*	*0.002*	*0.010*	*0.005*	*0.003*	*0.002*
ARC Urban		-0.159	-0.163	-0.005	-0.169	-0.220	-0.243	-0.030	-0.262
ARC Rural		-0.109	-0.122	-0.014	-0.135	-0.095	-0.124	-0.033	-0.150
Tunisia									
2012	**Urban**	0.076	0.032	0.428	0.014	0.123	0.049	0.404	0.022
		0.006	*0.003*	*0.005*	*0.001*	*0.006*	*0.003*	*0.006*	*0.001*
	Rural	0.367	0.169	0.461	0.082	0.690	0.309	0.447	0.153
		0.018	*0.009*	*0.005*	*0.005*	*0.012*	*0.006*	*0.004*	*0.004*
2018	**Urban**	0.057	0.023	0.409	0.010	0.095	0.033	0.348	0.013
		0.004	*0.002*	*0.004*	*0.001*	*0.005*	*0.002*	*0.005*	*0.001*
	Rural	0.232	0.103	0.445	0.048	0.457	0.196	0.429	0.092
		0.014	*0.006*	*0.005*	*0.003*	*0.014*	*0.007*	*0.004*	*0.004*
ARC Urban		-0.047	-0.054	-0.008	-0.063	-0.041	-0.064	-0.025	-0.088
ARC Rural		-0.073	-0.079	-0.006	-0.085	-0.067	-0.073	-0.007	-0.081

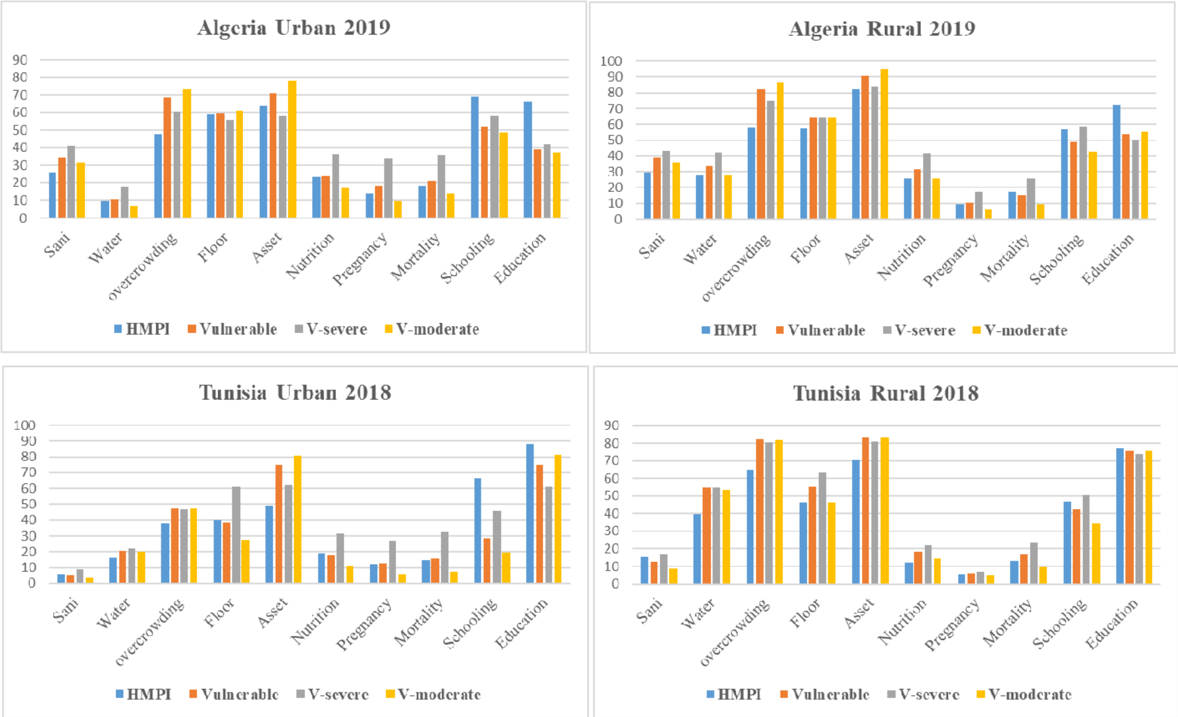
Note: $\boldsymbol{ARC}$ is the average annualized relative change. Standard errors are reported between brackets. $\boldsymbol{ARC}$ were statistically significant at α=0.01. Source: Author's calculation based on UNICEF-MICS data.

Table 3.A: Decomposition of vulnerability into severe and moderate vulnerability by areas of residence

		H	V_0^{MP}	V_{0P}^{MP}	V_{0R}^{MP}	V_{A01P}^{MP}	V_{A01R}^{MP}	V_{A1P}^{MP}	V_{A1R}^{MP}	V_{0P}^{MP}/V_0^{MP}
Algeria										
2013	**Urban**	0.189	0.352	0.103	0.250	0.493	0.449	0.261	0.212	0.291
		0.006	*0.007*	*0.004*	*0.005*	*0.004*	*0.003*	*0.007*	*0.003*	
	Rural	0.381	0.623	0.325	0.298	0.560	0.515	0.471	0.233	0.522
		0.013	*0.010*	*0.009*	*0.007*	*0.004*	*0.004*	*0.010*	*0.006*	
2019	**Urban**	0.067	0.080	0.028	0.052	0.441	0.356	0.330	0.175	0.351
		0.003	*0.003*	*0.002*	*0.002*	*0.007*	*0.005*	*0.014*	*0.007*	
	Rural	0.191	0.343	0.130	0.213	0.481	0.417	0.356	0.166	0.379
		0.010	*0.010*	*0.008*	*0.002*	*0.006*	*0.004*	*0.015*	*0.005*	
ARC	**Urban**	-0.159	-0.220	-0.195	-0.231	-0.019	-0.038	0.040	-0.032	
ARC	**Rural**	-0.109	-0.095	-0.142	-0.054	-0.025	-0.035	-0.046	-0.054	
Tunisia										
2012	**Urban**	0.076	0.123	0.028	0.095	0.495	0.377	0.206	0.237	0.227
		0.006	0.006	0.003	0.005	0.011	0.007	0.019	0.013	
	Rural	0.367	0.690	0.247	0.443	0.535	0.399	0.330	0.239	0.357
		0.018	*0.012*	*0.012*	*0.013*	*0.007*	*0.004*	*0.015*	*0.007*	
2018	**Urban**	0.057	0.095	0.031	0.064	0.415	0.315	0.303	0.151	0.327
		0.004	*0.005*	*0.003*	*0.003*	*0.008*	*0.005*	*0.021*	*0.007*	
	Rural	0.232	0.457	0.220	0.237	0.463	0.398	0.441	0.130	0.482
		0.014	*0.014*	*0.011*	*0.008*	*0.006*	*0.007*	*0.014*	*0.005*	
ARC	**Urban**	-0.047	-0.041	0.019	-0.063	-0.029	-0.029	0.067	-0.073	
ARC	**Rural**	-0.073	-0.067	-0.019	-0.099	-0.024	0.000	0.050	-0.097	

Note: $\boldsymbol{ARC}$ is the average annualized relative change. Standard errors are reported between brackets. $\boldsymbol{ARC}$ are statistically significant at α=0.01. Values of ARC for V_{A01P}^{MP} and V_{A01R}^{MP} are easier to interpret by considering the absolute variation which gives outcomes in terms of share of weighted dimensions. Source: Author's calculation based on UNICEF-MICS data.

Figure 3.A. Deprivation rates by dimension among the vulnerable by areas of residence



Note: V-Severe and V-moderate refer to the severe vulnerable and to the moderate vulnerable groups of people respectively. HMPI is the headcount ratio of multidimensional poverty. **Source:** Author's calculation based on UNICEF-MICS data.

Figure 4. A. Deprivation ratio by dimension among the severe and moderate vulnerable by areas of residence

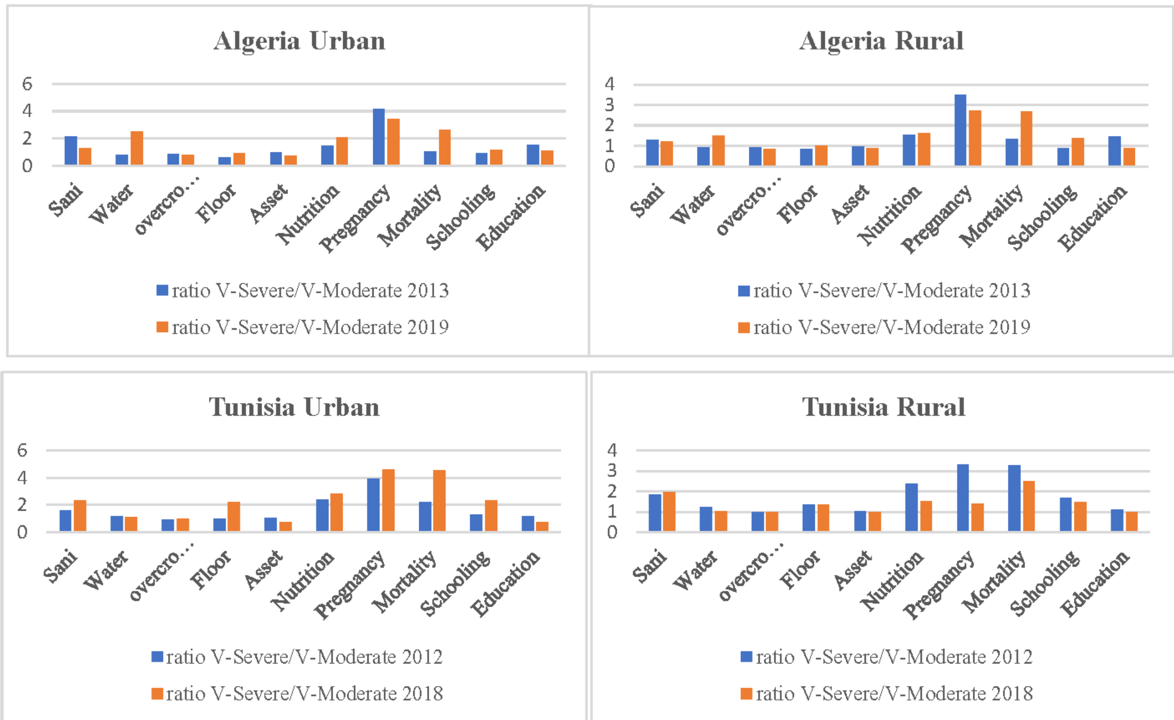
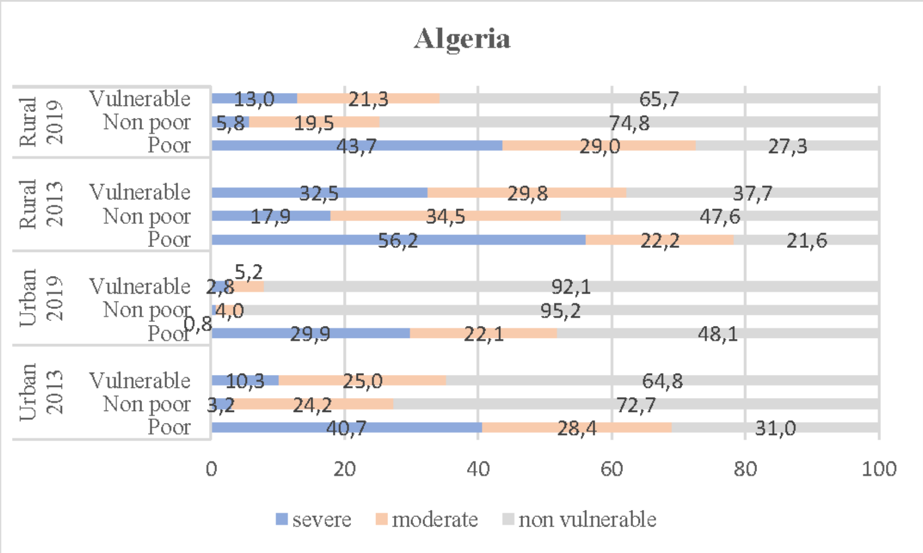
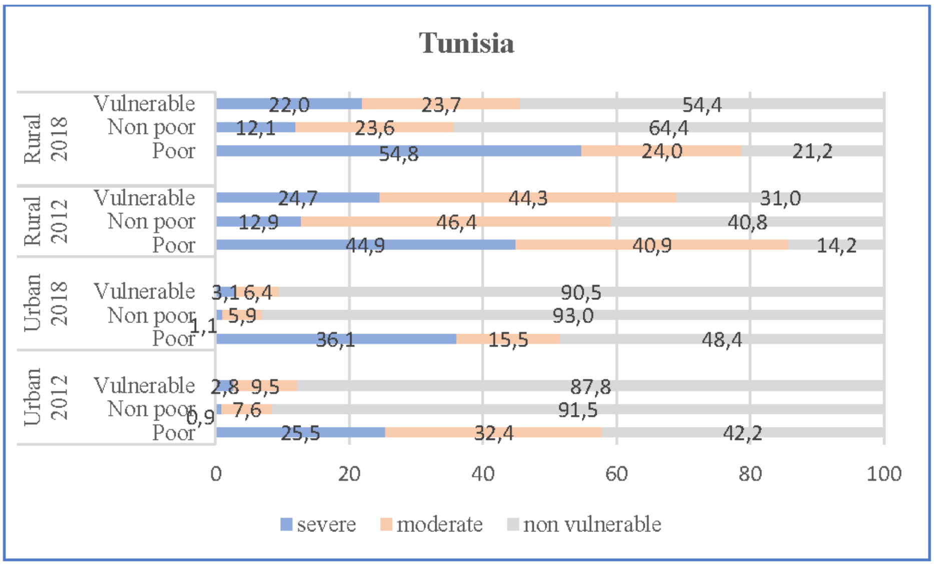


Figure 5.A. Incidence of vulnerability by poverty status in Algeria and Tunisia by areas of residence





Source: Author's calculation based on UNICEF-MICS data.