More Competition to Alleviate Poverty? A General Equilibrium Model and An Empirical Study

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December 22, 2021

Abstract

In this paper, we theoretically and empirically analyze the impact of competition on poverty. We consider a general equilibrium framework with vertical preferences and compare poverty in a Monopoly setting versus a Duopoly setting considering explicitly the ownership structure. We consider both price competition and quantity competition to find the equilibrium under Duopoly. Poverty is measured by the size of the population living below an absolute poverty line. Theoretical results show that the impact of competition on poverty is contingent to the ownership structure, the poverty line and the relative dispersion of the individuals with respect to their intensity of preference for quality and sensitivity to effort: competition can improve or worsen poverty depending on the model's parameters. Empirical findings for the three poverty lines are consistent to some extent with our theoretical results.

Keywords: Poverty, Price Competition, Quantity Competition, Vertical Differentiation, General Equilibrium, Ownership Structure.

JEL Classification: I32, L13.

Proposal Areas: Development Economics, Industrial Organization.

1 Introduction

"Our mission at the World Bank Group is defined by two goals: to end extreme poverty by 2030 and to boost prosperity among the poorest 40 percent in low- and middle-income countries...We are the first generation in human history that can end extreme poverty. This is our great challenge and our great opportunity" (Jim Yong Kim, President of the World Bank, 2015). While the previous words from the World Bank President seem reassuring, poverty statistics are still alarming. According to the World Bank 2017 estimates[1], 9.2 percent of the world's population lives on less than \$ 1.9 a day. The same estimates show that both developed and developing

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[1]https://www.worldbank.org/en/topic/measuringpoverty

countries are concerned with poverty[2]. Despite a constant decrease in poverty rates over the last 20 years, the goal to end poverty has suffered a setback with the COVID-19 pandemic bringing about 120 million additional people under the poverty line and the expectation to reach 150 million by the end of 2021. Moreover climate change is expected to exacerbate poverty: 68 million to 132 million people will be driven into poverty by 2030 due to climate change[3] through entangling access to drinking water, deteriorating health conditions, and threatening food security in many countries in Africa, Asia, and Latin America (Abeygunawardena et al. (2009).)[4].

Bhagwati (1998) distinguishes between two approaches that could be used to reduce poverty: "the indirect route" and "direct route" [5] . Competition falls under the indirect routes aiming to reduce poverty: Namely, more competition leads to lower prices, higher wages and more employment opportunities which improves the living standards of the poor. In this paper we aim at checking this statement in theoretical and empirical terms, while we highlight the role of ownership and quality.

A World Bank study (2016) shows that removing four cartels (in maize, wheat, poultry, and pharmaceuticals) in South Africa led to a decrease in the overall national poverty rate by 0.4 percentage points. Argent and Begazo (2015) find that poverty in Kenya would be reduced by 1.5 percent if trade barriers in the sugar market are relaxed, and by 1.8 percent if the maize price falls by 20 percent in both rural and urban areas.

A series of OECD reports explores channels through which competition helps poor consumers and workers. In particular, the 2013 *Competition and Poverty Reduction* report from the OECD[6] portrays some of these channels. First, the poor as consumers of essential goods and services benefit from competition through lower prices. In necessary goods market, where the elasticity of demand is high, even a small price reduction will significantly improve the poor's access to the goods. Second, competition can make a positive difference by helping the poor earn more as workers through access to better job opportunities. Most interestingly, the report casts a shadow of doubt over the positive impact of competition on the poor. In fact, “in some circumstances, competition could take away poor people's opportunities to succeed with small businesses", by forcing some businesses to exit the market resulting in job losses. In particular with technological innovation, older technologies become obsolete threatening poor entrepreneurs and workers. In addition, the effect of competition on poor consumers depends on their spending decisions. For instance, price reduction leads to an extra saving that many poor use in alcohol and tobacco consumption. According to the above mentioned report, the impact of competition on poverty is therefore inconclusive.

Furthermore, the Chinese experience between 1978 and 1983 shows that the shift from collective property rights to individual property rights had an important impact on productivity growth and poverty reduction. The ownership structure pops out then as a natural candidate factor impacting poverty.

This paper contributes to the understanding of the relationship between competition and poverty by conducting an extensive theoretical and empirical study of this relationship and by explicitly taking into account the ownership structure. To establish our theoretical results, we use a general equilibrium model with vertical preferences for one good first introduced by Kahloul et al. (2017) (also used by Lahmandi-Ayed and Laussel (2018), Kahloul et al. (2019) and Ghazzai et al. (2021)) considering a population of individuals who are potentially consumers, workers, and shareholders. Each individual is doubly characterized by his/her intensity of preference for quality and his/her sensitivity to effort. The whole population is divided into two groups: the shareholders and the non-shareholders. The proportion of shareholders may take any value between 0 and 1.

Two market structures are considered: Monopoly and Duopoly. In the case of Duopoly, we consider two types of competition: price competition and quantity competition. Our objective is to examine the effect of moving from

[2] https://unctad.org/meetings/en/Contribution/IGE2013_RT2_OECD1_en.pdf
https://www.weforum.org/agenda/2015/07/how-increasing-competition-can-reduce-poverty/
https://www.oecd.org/investment/globalforum/40315399.pdf
http://documentos.bancomundial.org/curated/es/662481468180536669/pdf/104736-REPF-Competition-and-Poverty.pdf

[3] https://www.worldbank.org/en/topic/poverty/overview

[4] https://documents1.worldbank.org/curated/en/534871468155709473/pdf/521760WP0pover1e0Box35554B01PUBLIC1.pdf

[5] ”With the amelioration of poverty as the target, the policy instruments designed to achieve that target can be divided into two main classes: (i) the indirect route, i.e., the use of resources to accelerate growth and thereby impact on the incomes and hence the living standards of the poor; and (ii) the direct route, i.e., the public provision of minimum-needs-oriented education, housing, nutritional supplements and health, and transfers to finance private expenditures on these and other components of the living standards of the poor.”

[6] https://www.oecd.org/daf/competition/competition-and-poverty-reduction2013.pdf

Monopoly to Duopoly (thus increasing competition) on the size of the population living under some given poverty level. We prove that the impact of competition on poverty depends in a complex way on the distribution of the individuals with respect to their taste for the quality and their sensitivity to effort, the ownership structure (proportion of shareholders), the competition type and the level of the poverty line.

Indeed, the effect of competition on poverty is not obvious. For non-shareholders, competition alleviates poverty as it offers individuals higher wages and more employment opportunities, thus reducing unemployment. However the effect of competition on poverty for shareholders is more ambiguous. Indeed, they may be negatively impacted by more competition as it decreases their profit share. Hence, the size of shareholders in the population becomes a critical factor in determining the effect of competition on poverty.

For the theoretical analysis, under each type of competition, three cases are possible depending on the level of relative dispersion of individuals w.r.t. taste and sensitivity to effort. For each case, we identify in the space (ownership, poverty line), the effect of competition on poverty. We then empirically test our theoretical findings using the headcount ratios at $1.90 a day, $3.2 a day, and $5.5 a day as three alternative proxies for poverty. Among the six cases identified theoretically, we eliminate four cases which are not consistent with the empirical findings obtained with the three headcount ratios, and we keep two cases, one under quantity competition and one under price competition.

Related Literature

Several empirical papers prove that more competition decreases poverty or inequality. Rodriguez-Castelan and Rodriguez-Chamussy (2018) study the relationship between poverty and market concentration in the retail sector in Mexico. Using a comprehensive municipality-level panel data and market concentration measures produced with economic censuses, they show that higher market concentration in the retailing sector raises the poverty headcount in the municipality. In a more recent work, Rodriguez-Castelan et al. (2021) carry a simulation to analyze the effects of competition on poverty and inequality using data from the Mexican Household Income and Expenditure survey. The paper illustrates the likely distributional effects of reducing concentration in two markets in Mexico: mobile telecommunication and corn products. In the microsimulation tool they propose (WELCOM), the number of firms is gradually augmented and the effects on the specific good price are calculated. Poverty and inequality indicators are then deduced. Their findings confirm that combining increased competition in the mobile telecommunication and the corn industry leads to a reduction in poverty and inequality. The actual effects are likely bigger than estimated as their work is restricted to the effect through the price channel. Alternative channels including wages, employment, profit shares, and quality improvement are not taken into consideration. In the general equilibrium model we develop, these channels come to jointly affect poverty.

Another stream of research focuses on the effects of trade liberalization on poverty and income inequalities. The general result is that more trade liberalization alleviates poverty (Liyanaarachchi et al. (2016), Dollar and Kraay (2002, 2004), Acharya et al. (2012))[7] . In the same line, Mahadevan et al. (2017) and Warr (2005) show that trade protectionism increases poverty. Howevever, the results do not converge when it comes to income inequalities. Indeed, while Liyanaarachchi et al. (2016) and Acharya et al. (2012) prove that trade liberalization increases income inequalities, Mahadevan et al. (2017) show that the effect of trade protectionism on income inequalities depends on the trade policy implemented. They find that export tariffs and limited import tariffs have no impact on income inequalities and that substantial import tariffs increase it.

Other empirical works show that more competition positively affects welfare. Atkin et al. (2018) prove a positive effect of foreign supermarket entry on household welfare in Mexico. Urzua (2013) presents evidence of welfare losses due to the exercise of monopoly power in Mexico. These losses are not only significant, but also larger, in relative terms, for the poor. Finally, Ghani and Reed (2015) highlight the importance of market entry in Sierra Leone in promoting growth and welfare [8]. However, all the above mentioned literature on competition and poverty completely ignores the ownership structure. The only exceptions in this respect are Rodriguez-Castelan (2015) and Kahloul et al.(2019). [9]

[7] Kis-Katos and Sparrow (2015) find mitigated results: input tariff liberalization alleviates poverty for Indonesia whereas output tariff liberalization worsens it.

[8] Dixit and Stern (1982) is one of the first theoretical papers to focus on the relationship between competition and welfare.

[9] An important literature on ownership structure exists. However, the questions addressed are not related to the poverty issue. To cite few of them, Phung and Mishra (2016) analyze the relationship between ownership structure and firm performance. Dhillon and Rossetto (2015) examine the determinants of a firm's ownership structure when risk-averse shareholders make decisions over risk through a majority vote.

We use the same model as in Kahloul et al. (2019). They consider only two extreme ownership structures: concentrated where the owners of the firms are negligible; and egalitarian where the firms are equally owned by the population. They show that, competition reduces poverty when the ownership structure is concentrated, but the effect is ambiguous in the egalitarian case. We assume as in Ghazzai et al. (2021) that the proportion of shareholders varies between 0 and 1, which allows richer theoretical results and thus a richer empirical study.

Rodriguez-Castelan (2015) [10] addresses the same question as in our paper but considers a completely different model where two consumption homogeneous goods are produced in the economy: one in a competitive market and one in an oligopolistic market. Consumers are modeled using two representative consumers differing by their level of productivity and own different shares of the total profits of the oligopolistic firms. He proves that if the share of the oligopolistic profits of low-income consumers is sufficiently large and there is a significant productivity gap between less and more highly skilled workers, then an increase in market concentration may decrease poverty. We use a completely different model with a vertically differentiated good and a continuum of individuals who are heterogeneous with respect to their preference for the quality and their sensitivity to effort (productivity). In our paper, only a proportion of the individuals (shareholders) own equal shares in firms' profits.

The remainder of the paper is organized as follows. In section 2, we describe the model. Sections 3 and 4 give respectively the results for price competition and quantity competition. Section 5 provides the empirical analysis. We conclude in section 6. All proofs are given in the appendix.

2 The model and Preliminary Results

We consider the model first introduced by Kahloul et al. (2017) and used by Kahloul et al. (2019) and Ghazzai et al. (2021). There is an economy with an indivisible differentiated good produced using labor as the unique input. Producing one unit of the differentiated good requires one unit of labor (constant returns to scale). The population is made of individuals who are potentially consumers, workers, and shareholders. Each individual is endowed with one unit of labor and a quantity of numeraire e. Heterogeneity among individuals is threefold. Each individual is characterized by:

1. his/her sensitivity to effort $\alpha \in [0, \overline{\alpha}]$;
2. his/her intensity of preference for the product quality $\theta \in [0, \overline{\theta}]$;
3. his/her ownership status i.e. whether he/she owns shares in the firm(s) or not.

We assume that individuals are uniformly distributed over $[0, \overline{\alpha}] \times [0, \overline{\theta}]$ with a density normalized to 1. Concerning the ownership structure, only a fraction $\mu \in [0, 1]$ of the population are shareholders. The model is thus an extension of Kahloul et al. (2017) and (2019) who considered only two extreme ownership structures ($\mu = 0$ and $\mu = 1$).

We denote by $\lambda \geq 0$ the individual's share in the firms' profit. For non-shareholders, $\lambda = 0$. Each shareholder holds the share $\lambda = \frac{1}{\mu\overline{\alpha}\overline{\theta}}$ in firms' profit. Firms' profits are thus equally distributed among all shareholders and the lower the proportion of shareholders μ, the higher their share in the firms' profits.

We model two market structures: Monopoly and Duopoly. Under each market structure, each individual makes first the choice between remaining idle ($\overline{W}$) and working (and if so, s/he chooses between Firms 1 and 2 under Duopoly.) S/he second chooses between not consuming ($\overline{C}$) and consuming one unit of the differentiated product (and if so, s/he chooses between the products of Firms 1 and 2 under Duopoly.)

The individual utility is derived from the consumption of the differentiated product and the numeraire as follows:

$$V(x, t) = \theta v x + t, \tag{1}$$

where x is the quantity of the differentiated product of quality v and t is the quantity of the numeraire good.The consumption bundle (x, t) belongs to the consumption set $\{0, 1\} \times \mathbb{R}$. Individuals with a high θ, the intensity of

Ozer and Alakent (2013) study how a firm's political strategy choices are affected by the ownership structure.

[10] See Appendix B of Rodriguez-Castelan (2015).

preference for quality, are more impacted by quality.

Individuals receive income from three potential sources: the initial endowment e, the wage w if they decide to work, and the dividends if they are shareholders. We further assume that an individual characterized by sensitivity to effort α will have to incur an effort or training cost αv when s/he works in a firm producing quality v. This means that higher qualities require higher effort/training and that higher sensitivity to effort α translates into higher training cost as well. An individual who chooses not to work receives zero wage and is not subject to training cost. Wages are endogenously determined by balancing the supply and demand in the labor market. It is worth noting that the individual incomes are measured in terms of the numeraire good. Thus incomes capture the purchasing power of individuals expressed in terms of the numeraire good.

We consider two market structures: Monopoly and Duopoly, and under Duopoly, quantity and price competition.

Under Monopoly, the firm chooses its quality v_M then its price p_M or its quantity q_M. [11] Under Duopoly, we solve the game under quantity and price competition. More precisely,

- Quantity competition (Cournot): firms choose first their qualities v_1 and v_2, then their quantities q_1 and q_2.
- Price competition (Bertrand): firms choose first their qualities v_1 and v_2, then their prices p_1 and p_2.

Poverty Measure: A poverty line is defined by a level of income sufficient to acquire a given quantity γ of the numeraire good. The poverty line is thus equal precisely to γ. We measure poverty through the size of the population whose income falls below that threshold. The Income Poor Population is thus given by

$$IPP = \{(\alpha, \theta) \in [0, \overline{\alpha}] \times [0, \overline{\theta}] / I(\alpha, \theta) \leq \gamma\}$$

The Poor Population Size (S) at γ is defined as the area of the Income-Poor Population (IPP). We denote it by:

$$S = \int_{(\alpha,\theta)\in IPP} d\alpha \, d\theta.$$

Kahloul et al. (2017) provide the equilibrium outcomes under price competition, for Monopoly and Duopoly, as they do not depend on the ownership structure. The Monopoly outcome is the same for both types of competition, it is reminded in this section in Kahloul et al. Result 1.

Kahloul et al. Result 1 (Monopoly Equilibrium)**.** *At equilibrium, the Monopoly chooses quality $v_M^* = \overline{v}$, charges price $p_M^* = \frac{\overline{\theta}\overline{v}(\overline{\theta}+2\overline{\alpha})}{2(\overline{\theta}+\overline{\alpha})}$ and makes the profit $\pi_M^* = \frac{\overline{\alpha}\overline{\theta}^3\overline{v}}{4(\overline{\theta}+\overline{\alpha})}$. Workers receive a wage $w_M^* = \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\theta}+\overline{\alpha})}$.*

Under each market structure and each type of competition, we determine at equilibrium, the individual incomes then the poor population size (S). We finally compare under each type of competition (quantity and price), the poor population size (S) under Monopoly and under Duopoly, in order to tell under which conditions more competition alleviates poverty.

The result on individual incomes under Monopoly is provided in Lemma 1.

Lemma 1 (Incomes Under Monopoly)**.** *The individual incomes under Monopoly (M) for non-shareholders (NS) and shareholders (S) are respectively given by:*

$$I_{NS}^M(\alpha, \theta) = \begin{cases} e & \text{if } \alpha > \frac{\overline{\alpha}\overline{\theta}}{2(\overline{\theta}+\overline{\alpha})} \\ \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\theta}+\overline{\alpha})} - \alpha\overline{v} + e & \text{if } \alpha \leq \frac{\overline{\alpha}\overline{\theta}}{2(\overline{\theta}+\overline{\alpha})} \end{cases}$$

$$I_S^M(\alpha, \theta) = \begin{cases} \frac{\overline{v}\overline{\theta}^2}{4\mu(\overline{\alpha}+\overline{\theta})} + e & \text{if } \alpha > \frac{\overline{\alpha}\overline{\theta}}{2(\overline{\theta}+\overline{\alpha})} \\ \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\theta}+\overline{\alpha})} - \alpha\overline{v} + \frac{\overline{v}\overline{\theta}^2}{4\mu(\overline{\alpha}+\overline{\theta})} + e & \text{if } \alpha \leq \frac{\overline{\alpha}\overline{\theta}}{2(\overline{\theta}+\overline{\alpha})} \end{cases}$$

All individuals have the same initial endowment e. Individuals with high sensitivity to effort ($\alpha > \frac{\overline{\alpha}\overline{\theta}}{2(\overline{\theta}+\overline{\alpha})}$) prefer not to work and do not receive any wage. Individuals with low sensitivity to effort choose to work and receive the net wage $w_M^* - \alpha\overline{v}$. The income of the shareholders is also composed of their profit share $\frac{\overline{v}\overline{\theta}}{4\mu(\overline{\alpha}+\overline{\theta})}$.

[11] The Monopoly starts by choosing its quality. Results remain the same if the Monopoly then chooses its price or its quantity.

3 Price Competition

In this section, we consider price competition. We aim at determining in this case the effect of competition on poverty through the comparison of the poverty levels at equilibrium under Monopoly and under Duopoly. To do so, under each market structure, we first calculate at equilibrium the individual incomes, then we measure the poor population sizes under Monopoly and Duopoly for each level of poverty line.

The equilibrium outcome under Duopoly is reminded in Kahloul et al. Result 2.

Kahloul et al. Result 2 (Duopoly Equilibrium, price competition)**.** *Under Duopoly, the equilibrium qualities, prices, wages and profits are given by:*

- *Firm 1:* $v_1^* = \frac{4}{7}\overline{v}$; $p_1^* = \frac{\overline{\theta}\overline{v}(\overline{\theta}+8\overline{\alpha})}{14(\overline{\theta}+\overline{\alpha})}$; $w_1^* = \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\theta}+\overline{\alpha})}$ *and* $\pi_1^* = \frac{\overline{\alpha}\overline{\theta}^3\overline{v}}{48(\overline{\theta}+\overline{\alpha})}$,
- *Firm 2:* $v_2^* = \overline{v}$; $p_2^* = \frac{\overline{\theta}\overline{v}(\overline{\theta}+4\overline{\alpha})}{4(\overline{\theta}+\overline{\alpha})}$; $w_2^* = \frac{3\overline{\alpha}\overline{\theta}\overline{v}}{4(\overline{\theta}+\overline{\alpha})}$ *and* $\pi_2^* = \frac{7\overline{\alpha}\overline{\theta}^3\overline{v}}{48(\overline{\theta}+\overline{\alpha})}$.

Lemma 2 provides the individual incomes in the Duopoly case under price competition.

Lemma 2 (Incomes Under Duopoly, price competition)**.** *The individual incomes under Duopoly (D) for non-shareholders (NS) and shareholders (S) are respectively given by:*

$$I_{NS}^D(\alpha,\theta) = \begin{cases} e & \text{if } \alpha > \frac{7\overline{\alpha}\overline{\theta}}{8(\overline{\theta}+\overline{\alpha})} \\ \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\theta}+\overline{\alpha})} - \frac{4}{7}\alpha\overline{v} + e & \text{if } \frac{7\overline{\alpha}\overline{\theta}}{12(\overline{\theta}+\overline{\alpha})} \le \alpha \le \frac{7\overline{\alpha}\overline{\theta}}{8(\overline{\theta}+\overline{\alpha})}, \\ \frac{3\overline{\alpha}\overline{\theta}\overline{v}}{4(\overline{\theta}+\overline{\alpha})} - \alpha\overline{v} + e & \text{if } \alpha < \frac{7\overline{\alpha}\overline{\theta}}{12(\overline{\theta}+\overline{\alpha})} \end{cases}$$

$$I_{S}^D(\alpha,\theta) = \begin{cases} \frac{\overline{v}\overline{\theta}^2}{6\mu(\overline{\theta}+\overline{\alpha})} + e & \text{if } \alpha > \frac{7\overline{\alpha}\overline{\theta}}{8(\overline{\theta}+\overline{\alpha})} \\ \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\theta}+\overline{\alpha})} - \frac{4}{7}\alpha\overline{v} + \frac{\overline{v}\overline{\theta}^2}{6\mu(\overline{\theta}+\overline{\alpha})} + e & \text{if } \frac{7\overline{\alpha}\overline{\theta}}{12(\overline{\theta}+\overline{\alpha})} \le \alpha \le \frac{7\overline{\alpha}\overline{\theta}}{8(\overline{\theta}+\overline{\alpha})}, \\ \frac{3\overline{\alpha}\overline{\theta}\overline{v}}{4(\overline{\theta}+\overline{\alpha})} - \alpha\overline{v} + \frac{\overline{v}\overline{\theta}^2}{6\mu(\overline{\theta}+\overline{\alpha})} + e & \text{if } \alpha < \frac{7\overline{\alpha}\overline{\theta}}{12(\overline{\theta}+\overline{\alpha})} \end{cases}$$

We draw the income functions with respect to α. The relative position of I_{NS}^M, I_S^M, I_{NS}^D and I_S^D depends on μ and $\delta = \frac{\overline{\theta}}{\overline{\alpha}}$: 8 cases appear as shown in Figures 1 and 2, using the following notations:

- $a(\mu) = \frac{\overline{v}\overline{\theta}}{4(\overline{\alpha}+\overline{\theta})}(2\overline{\alpha} + \frac{\overline{\theta}}{\mu}) + e$
- $b(\mu) = \frac{\overline{v}\overline{\theta}^2}{4\mu(\overline{\alpha}+\overline{\theta})} + e$
- $c = \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\alpha}+\overline{\theta})} + e$
- $d(\mu) = \frac{\overline{v}\overline{\theta}}{12(\overline{\alpha}+\overline{\theta})}(9\overline{\alpha} + 2\frac{\overline{\theta}}{\mu}) + e$
- $f(\mu) = \frac{\overline{v}\overline{\theta}}{6(\overline{\alpha}+\overline{\theta})}(\frac{\overline{\theta}}{\mu} + \overline{\alpha}) + e$
- $g(\mu) = \frac{\overline{v}\overline{\theta}^2}{6\mu(\overline{\alpha}+\overline{\theta})} + e$
- $h = \frac{3\overline{\alpha}\overline{\theta}\overline{v}}{4(\overline{\alpha}+\overline{\theta})} + e$
- $k = \frac{\overline{\alpha}\overline{\theta}\overline{v}}{6(\overline{\alpha}+\overline{\theta})} + e$

Under each market structure (Monopoly or Duopoly), the non-shareholders' income curve is below the shareholders' income curve, obviously because the shareholders have an additional source of income (dividends).

For non-shareholders, the individual income curve is lower under Monopoly (I_{NS}^M) than under Duopoly (I_{NS}^D). Indeed their income is composed only of net salaries (equal to wages minus the training costs). The wages are higher under Duopoly thanks to competition on the labor market. Under Duopoly, one firm produces the maximum quality (the same as the Monopoly one) and the other firm produces a lower quality. Thus the training costs under Duopoly are at most equal to those under Monopoly and the net salaries under Duopoly are at least equal to those under Monopoly [12].

Superimposing all the individual income curves, the non-shareholders' income curve under Monopoly (I_{NS}^M) is the lowest (in a broad sense) among the four individual curves. The comparison of the size of the poor populations under each market structure stems from the position of the individual income curve of shareholders under Monopoly (I_S^M) relative to the two individual income curves of shareholders and non-shareholders under Duopoly (I_S^D and I_{NS}^D). This position depends on the workers' type α and on the parameters μ and δ. The difference between the profit shares under Monopoly and Duopoly increases more quickly with δ than the difference between the net salaries. Hence for high enough δ, for shareholders, the individual income curve under Monopoly (I_S^M) is above the individual income curve under Duopoly (I_S^D) (see Figure 2). Moreover, the profit share under Monopoly increases more quickly with δ than the difference between the net salaries under the two market structures. Thus for high enough δ, the shareholders' income curve under Monopoly (I_S^M) is above the non-shareholders' one under Duopoly (I_{NS}^D) (See Figure 1 when $\delta > \mu$ and all cases of Figure 2).

This is why: 1) for high enough δ, the individual income curves are ranked from bottom to top as follows: I_{NS}^M is the lowest, then I_{NS}^D, then I_S^D, and finally I_S^M (the four cases of Figure 2); 2) for low δ ($\delta < 3\mu$), I_S^M always intersects with I_S^D and it also intersects with I_{NS}^D when $\delta < \mu$ as shown in Figure 1.

In each of the 8 cases of Figures 1 and 2, we measure the size of the poor population for each possible poverty line γ. The comparison of these sizes directly delivers the effect of competition on poverty. This comparison allowed us to aggregate the 8 cases into 3 scenarios depending on δ and μ as shown in Proposition 1.

Proposition 1 (Competition effect, price competition)**.** *The competition effect on poverty depends on δ, μ and γ as follows, denoting by S_M and S_D the poor population sizes respectively under Monopoly and under Duopoly:*

1. *Case 1: $\delta \geq \frac{9}{2}\mu$*
   - *i $S_M = S_D = 0$, for $\gamma < e$ (no poor under both regimes).*
   - *ii $S_M > S_D$, for $e \leq \gamma < h$.*
   - *iii $S_M = S_D = (1-\mu)\overline{\alpha}\overline{\theta}$, for $h \leq \gamma < g(\mu)$ (All non-shareholders are poor under both regimes).*
   - *iv $S_M < S_D$, for $g(\mu) \leq \gamma < a(\mu)$.*
   - *v $S_M = S_D = \overline{\alpha}\overline{\theta}$, for $\gamma \geq a(\mu)$ (the whole population is poor under both regimes).*
2. *Case 2: $3\mu \leq \delta < \frac{9}{2}\mu$*
   - *i $S_M = S_D = 0$, for $\gamma < e$ (no poor under both regimes).*
   - *ii $S_M > S_D$, for $e \leq \gamma < g(\mu)$.*
   - *iii $S_M < S_D$, for $g(\mu) \leq \gamma < a(\mu)$.*
   - *iv $S_M = S_D = \overline{\alpha}\overline{\theta}$, for $\gamma \geq a(\mu)$ (the whole population is poor under both regimes).*
3. *Case 3: $\delta < 3\mu$*
   - *i $S_M = S_D = 0$, for $\gamma < e$ (no poor).*
   - *ii $S_M > S_D$, for $e \leq \gamma < g(\mu)$.*
   - *iii $S_M < S_D$, for $g(\mu) \leq \gamma < b(\mu)$.*
   - *iv $S_M > S_D$, for $b(\mu) \leq \gamma < d(\mu)$.*
   - *v $S_M = S_D = \overline{\alpha}\overline{\theta}$, for $\gamma \geq d(\mu)$ (the whole population is poor under both regimes).*

[12] Note that these two curves are merged for sufficiently high α (the individuals preferring not to work under both regimes).

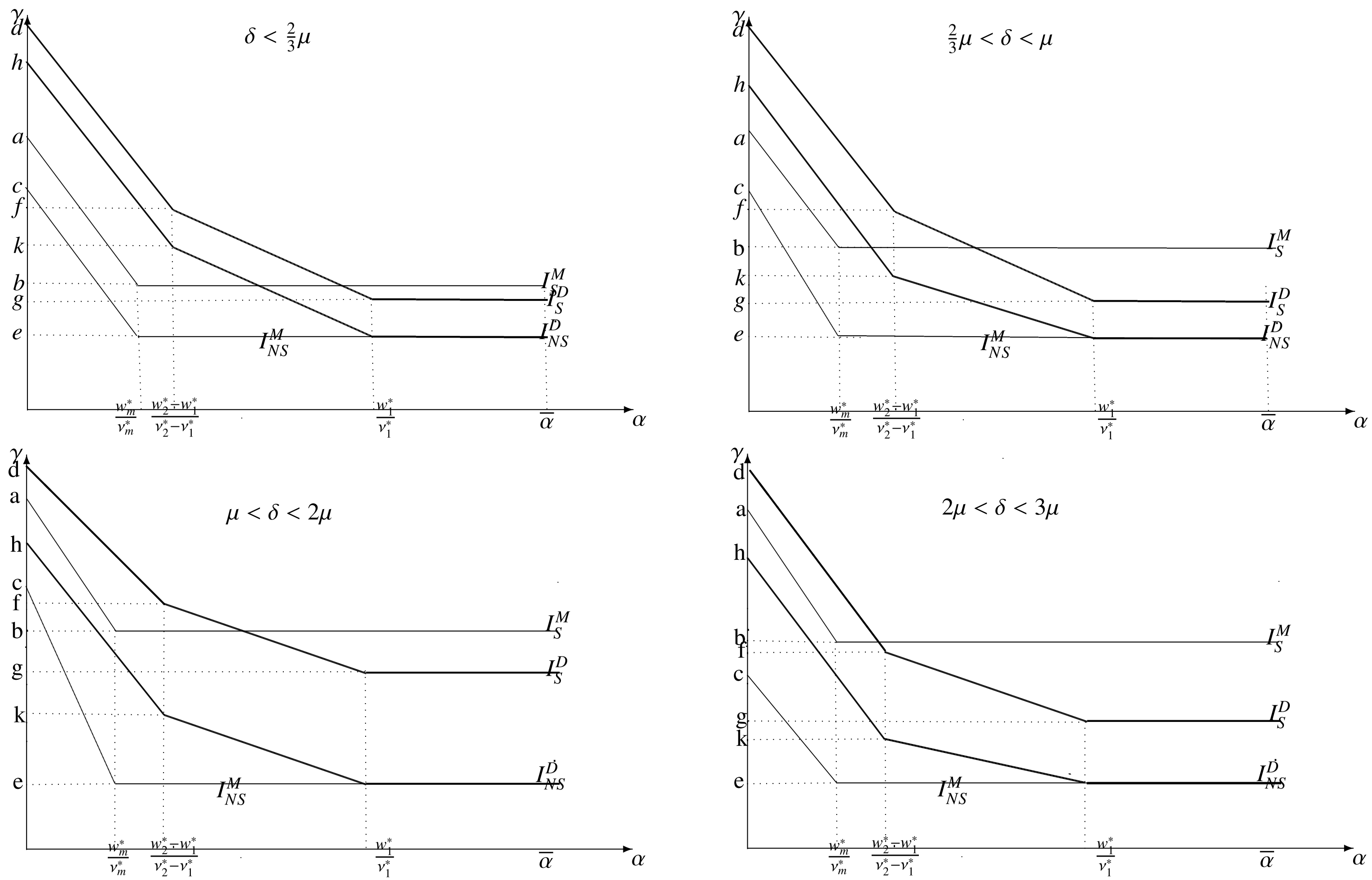

Figure 1 – Income curves positioning under price competition when $\delta < 3\mu$

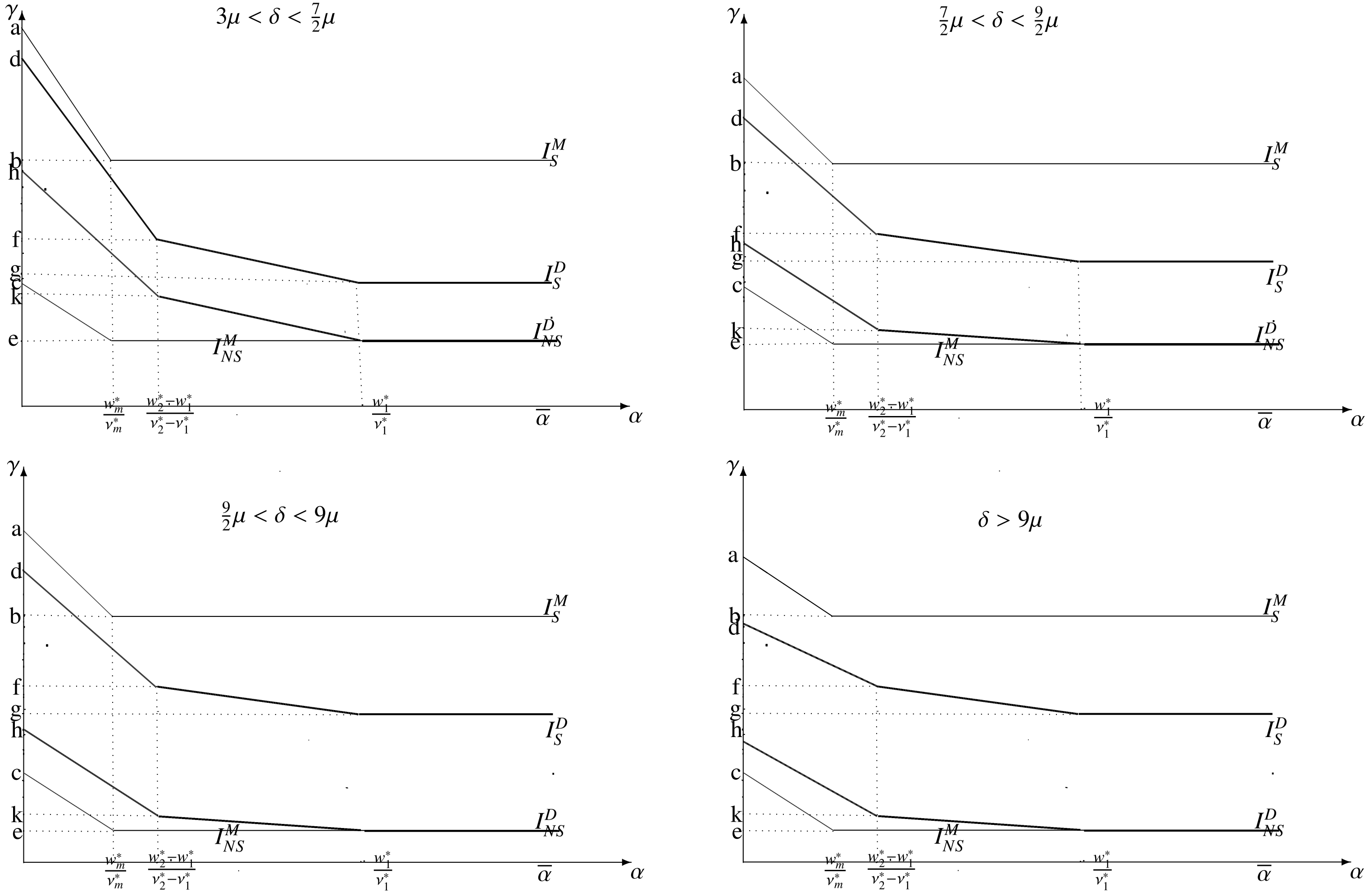

Figure 2 – Income curves positioning under price competition when $\delta > 3\mu$

For a better readability, the results of Proposition 1 can be displayed graphically in the space (μ, γ). As the proportion of shareholders μ is in $[0, 1]$, a discussion on δ is necessary. This leads to three cases:

- if $\frac{2}{9}\delta \geq 1$ (i.e. $\delta \geq \frac{9}{2}$) then for any $\mu \in [0, 1]$, we have $\mu \leq \frac{9}{2}\delta$. Thus, only Case 1 applies and is used to draw Figure 3.
- if $\frac{2}{9}\delta < 1 \leq \frac{1}{3}\delta$ (i.e. $3 \leq \delta < \frac{9}{2}$), then, in Figure 4, Case 1 applies when $\mu \in [0, \frac{2}{9}\delta[$ and Case 2 applies when $\mu \in [\frac{2}{9}\delta, 1]$.
- if $(\frac{2}{9}\delta <)\ \frac{1}{3}\delta < 1$ (thus $\delta < 3$), then in Figure 5, Case 1 applies when $\mu \in [0, \frac{2}{9}\delta[$, Case 2 applies when $\mu \in [\frac{2}{9}\delta, \frac{1}{3}\delta]$ and Case 3 applies when $\mu \in]\frac{1}{3}\delta, 1]$.

In the obtained Figures 3, 4 and 5, the "+" ("-") sign indicates that competition has a positive (negative) effect on poverty i.e. it reduces (increases) the poor population size; i.e. the poor population size under Duopoly is lower (higher) than under Monopoly. The "=" sign indicates that competition has no effect on the size of the poor population; i.e. the poor population size is the same under both market structures.

It is worth noting that $g(\mu) = h$ when $\mu = \frac{2}{9}\delta$ and that $b(\mu) = h$ when $\mu = \frac{1}{3}\delta$. We denote by $z_1(\mu) = \max(a(\mu), d(\mu))$, we have that $z_1(\mu) = a(\mu)$ if $\mu < \frac{1}{3}\delta$ and $z_1(\mu) = d(\mu)$ otherwise.

The effect of competition on non-shareholders is obvious. Indeed, under Duopoly, workers receive a wage at least as high as their wage under Monopoly and their training costs are at most as high as their training costs under Monopoly. Therefore, competition increases the net income of non-shareholders or keeps it unchanged (Individuals who choose not to work under both market structures have an income equal to their initial endowment).

Shareholders have two income sources: their net wage (if they choose to work) and their profits share. Competition has a positive effect on their net wages thanks to more competition on the labor market. However, their profit share under Monopoly is higher than their profit share under Duopoly. The overall impact of competition on shareholders is therefore not obvious. The higher δ, the higher the difference between the profit shares under Monopoly and Duopoly; and the higher δ, the higher the difference between the profit shares under Monopoly and Duopoly compared to the difference between the net wages under both market structures. In other words, the higher δ, the higher the negative effect of competition on profit shares compared to its positive effect on net wages.

Proposition 1 shows three cases depending on μ and δ. The obtained results may be understood using the position of the income curves discussed above. The number of poor depends on the intersection of the poverty line with each type of individual income curve depicted in Figures 1 and 2.

In all Figures 3, 4 and 5, the "=" signs below e is obvious. Indeed, there are no poor as the poverty line is below all individual income curves ($\gamma < e$). This corresponds to cases 1(i), 2(i) and 3(i) in Proposition 1.

As γ increases ($e < \gamma < \min(h, g)$), competition alleviates poverty (the "+" sign for $e < \gamma < \min(h, g)$ in Figures 3, 4 and 5) as only non-shareholders can be poor. All shareholders' incomes are above the poverty line whether under Monopoly or under Duopoly. Indeed, the poverty line intersects both (for the same γ) I_{NS}^{M} and I_{NS}^{D} at a level of α lower for I_{NS}^{M} than for I_{NS}^{D}, meaning that the poor population is higher under Monopoly. This explains sub-cases 1(ii), 2(ii) and 3(ii) in Proposition 1.

When $h < \gamma < g(\mu)$, [13] all non-shareholders are poor under both market structures, while shareholders are not yet concerned with poverty (see the cases $\frac{2}{9}\mu < \delta < 9\mu$ and $\delta > 9\mu$ in Figure 2). This explains the "=" sign when the poverty line satisfies $h < \gamma < g(\mu)$ and case 1(iii) in Proposition 1.

In the zone $g(\mu) < \gamma < z_1(\mu)$ in Figures 3, 4 and 5, some of the shareholders under Duopoly fall below the poverty line (See Figures 1 and 2). This zone can be split in two zones $g(\mu) < \gamma < b(\mu)$ and $b(\mu) < \gamma < z_1(\mu)$.

For $g(\mu) < \gamma < b(\mu)$, only non-shareholders are concerned with poverty under Monopoly (possibly all of them) while both shareholders and non-shareholders are concerned with poverty under Duopoly (possibly all non-shareholders). We prove that the size of the poorest population under Duopoly is larger than under Monopoly [14]. This corresponds to the "-" sign when $g(\mu) < \gamma < b(\mu)$ in Figures 3, 4 and 5 and explains partially cases 1(iv),

[13] The inequality is true only when $\mu \leq \frac{2}{9}\delta$ i.e. $\delta > \frac{9}{2}\mu$ as shown in Figures 3, 4 and 5.

[14] See the details of the calculations for low δ in the proof of Proposition 1.

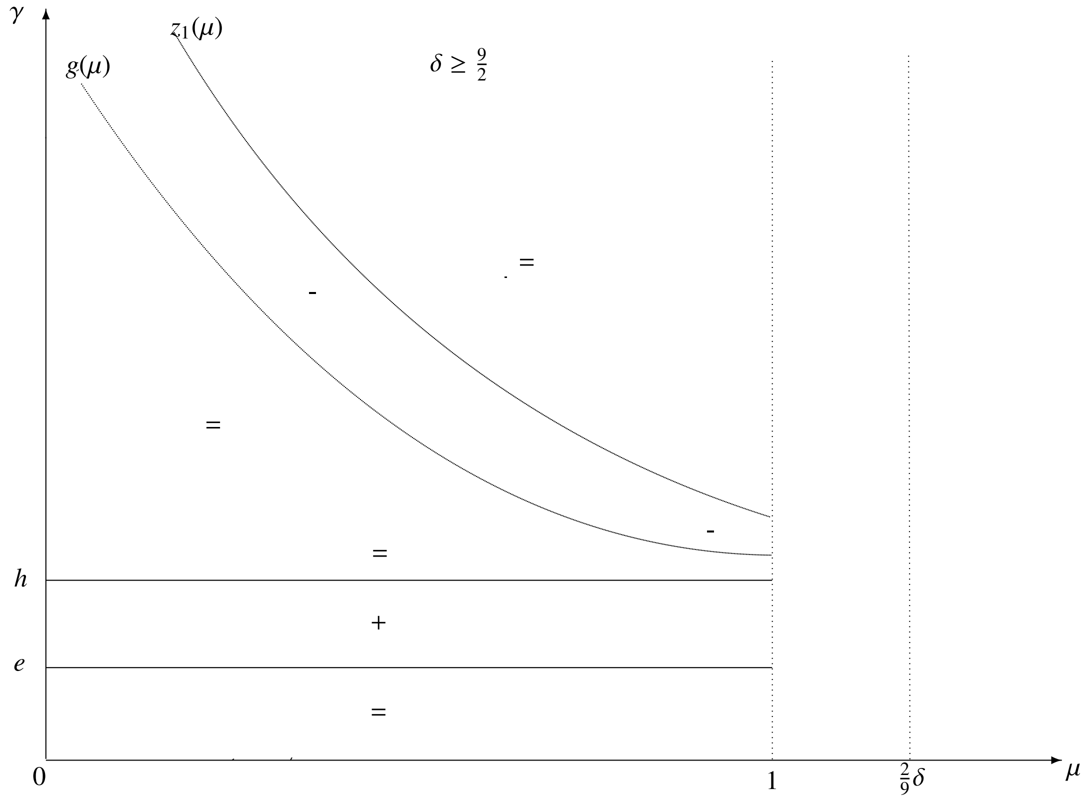

Figure 3 – Effect of competition on Poverty under price competition for $\delta \geq \frac{9}{2}$

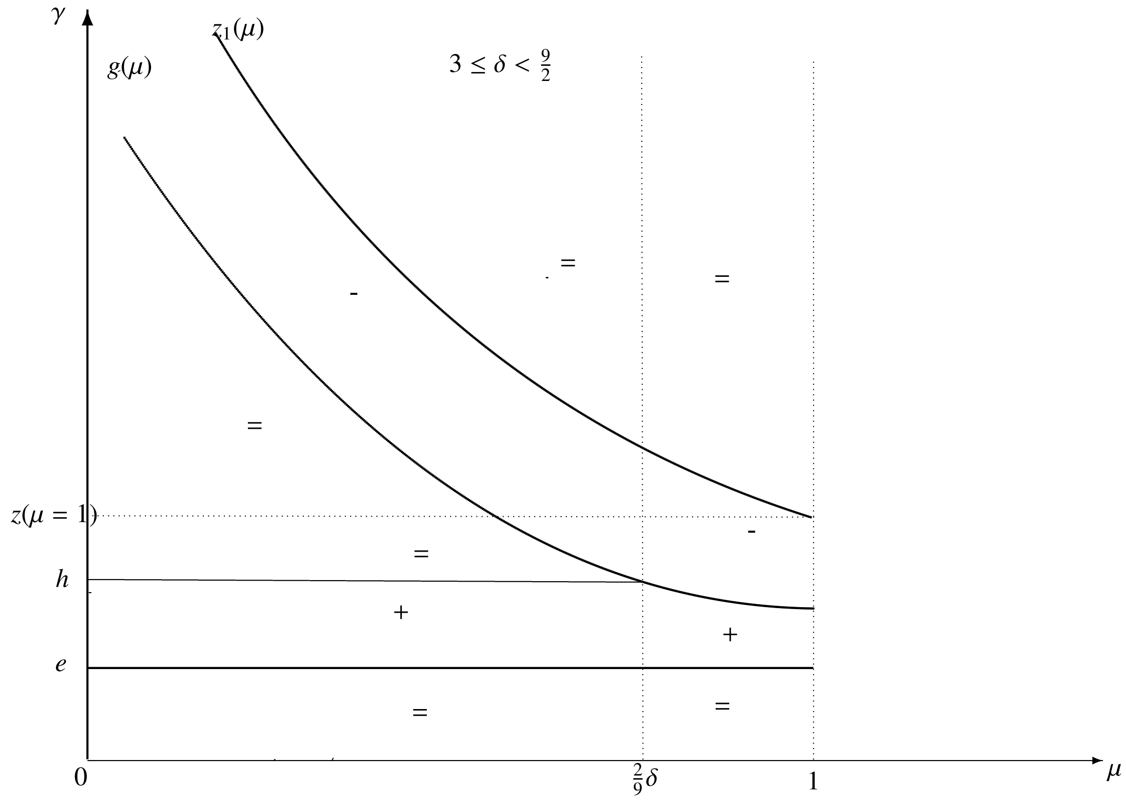

Figure 4 – Effect of competition on Poverty under price competition for $3 < \delta < \frac{9}{2}$

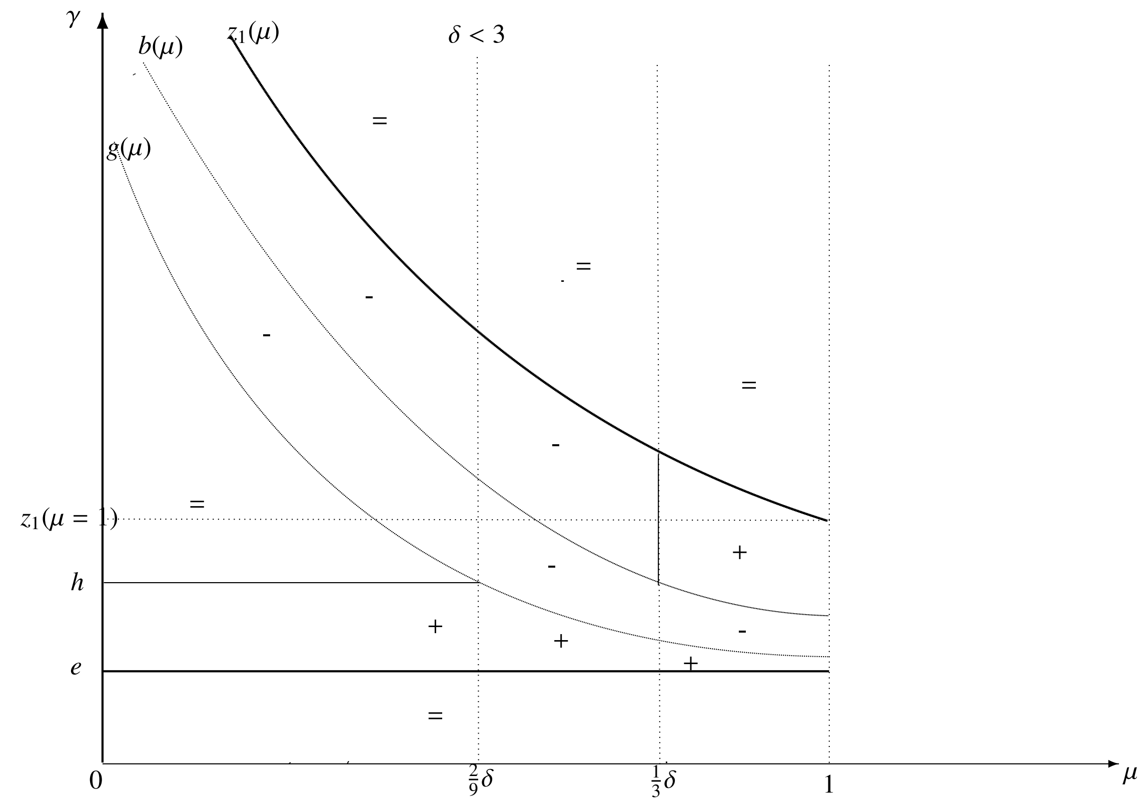

Figure 5 – Effect of competition on Poverty under price competition for $\delta \leq 3$

2(iii) and fully case 3(iii).

For $b < \gamma < z_1(\mu)$, results depend on δ. For high $\delta > 3\mu$ [15], all non-shareholders are poor under both market structures. The comparison of the size of the poorest population depends on the relative position of the shareholders' income curves under both market structures. In this case I_S^D is always below I_S^M as δ is big enough and therefore there are more poor under Duopoly (see all cases of Figure 2). This corresponds to the "-" sign when $b < \gamma < z_1(\mu)$ in Figures 3 and 4 and 5 and finishes explaining cases 1(iv), 2(iii). For small $\delta < 3\mu$ (or equivalently $\mu > \frac{1}{3}\delta$), a positive effect of competition on poverty appears in Figure 5. In this case I_S^M is below I_S^D for small α. Thus, there are more poor shareholders under Monopoly than under Duopoly and (as always) more poor non-shareholders under Monopoly than under Duopoly. Consequently there are more poor under Monopoly. This positive effect of competition on poverty only appears in Figure 5 and only for small δ relative to μ ($\delta < 3\mu$) because the positive competition effect on net wages outweighs its negative effect on profit shares. This explains case 3(iv) in Proposition 1.

In all Figures 3, 4 and 5, the "=" sign above $z_1(\mu)$ is obvious. All population is poor under Monopoly and Duopoly as the poverty line is above all the individual income curves. This corresponds to cases 1(v), 2(iv) and 3(v) in Proposition 1.

4 Quantity Competition

In this section we consider quantity competition. We proceed as in the case of price competition: we first characterize the equilibrium then determine the individual incomes under Duopoly. Finally, for each level of poverty line, we compare the sizes of poor populations under Monopoly and under Duopoly. Lemma 3 provides the Duopoly equilibrium under quantity competition.

Lemma 3 (Duopoly Equilibrium, quantity competition)**.** *Under quantity competition, at Duopoly equilibrium,*

[15] The inequality $\mu < \frac{1}{3}\delta$ is always true in Figures 3 and 4 and not necessarily true in Figure 5.

qualities, quantities, prices, wages and profits are respectively given by: $v_1^* = v_2^* = \overline{v}$; $q_1^* = q_2^* = \frac{\overline{\alpha}\overline{\theta}^2}{3(\overline{\alpha}+\overline{\theta})}$; $p_1^* = p_2^* = \frac{\overline{\theta}\overline{v}(\overline{\theta}+3\overline{\alpha})}{3(\overline{\alpha}+\overline{\theta})}$; $w_1^* = w_2^* = \frac{2\overline{v}\overline{\alpha}\overline{\theta}}{3(\overline{\alpha}+\overline{\theta})}$ *and* $\Pi_1^* = \Pi_2^* = \frac{\overline{\alpha}\overline{\theta}^3\overline{v}}{9(\overline{\alpha}+\overline{\theta})}$

What is worth noting is that, under quantity competition unlike price competition, the two firms produce exactly the same quality, charge the same price, offer the same wage and get the same profit. But as under price competition, profit shares of shareholders decrease under Duopoly, as the sum of the Duopoly profits is lower than the Monopoly profit.

Individual incomes under Duopoly are first calculated in Lemma 4 then we compare the size of the poorest population under each market structure.

Lemma 4 (Incomes Under Duopoly, quantity competition)**.** *The individual incomes under Duopoly (D) for non-shareholders (NS) and shareholders (S) are respectively given by:*

$$I_{NS}^D(\alpha,\theta) = \begin{cases} e & \text{if } \alpha > \frac{2\overline{\alpha}\overline{\theta}}{3(\overline{\theta}+\overline{\alpha})}, \\ \frac{2\overline{\alpha}\overline{\theta}\overline{v}}{3(\overline{\theta}+\overline{\alpha})} - \alpha\overline{v} + e & \text{if } \alpha \leq \frac{2\overline{\alpha}\overline{\theta}}{3(\overline{\theta}+\overline{\alpha})}. \end{cases}$$

$$I_{S}^D(\alpha,\theta) = \begin{cases} \frac{2\overline{v}\overline{\theta}^2}{9\mu(\overline{\theta}+\overline{\alpha})} + e & \text{if } \alpha > \frac{2\overline{\alpha}\overline{\theta}}{3(\overline{\theta}+\overline{\alpha})}, \\ \frac{2\overline{\alpha}\overline{\theta}\overline{v}}{3(\overline{\theta}+\overline{\alpha})} + \frac{2\overline{v}\overline{\theta}^2}{9\mu(\overline{\theta}+\overline{\alpha})} - \alpha\overline{v} + e & \text{if } \alpha \leq \frac{2\overline{\alpha}\overline{\theta}}{3(\overline{\theta}+\overline{\alpha})}. \end{cases}$$

We draw the income functions with respect to α. The relative position of I_{NS}^M, I_S^M, I_{NS}^D and I_S^D depends on μ and $\delta = \frac{\overline{\theta}}{\overline{\alpha}}$: 6 cases appear (See Figures 11 and 12). In each case, we measure the size of the poor population for a specific poverty line γ. The comparison of these sizes directly delivers the effect of competition on poverty. This comparison allowed us to aggregate the 6 cases into 3 scenarios depending on δ and μ as shown in Proposition 2, with the following notations [16]:

- $a(\mu) = \frac{\overline{v}\overline{\theta}^2}{4\mu(\overline{\alpha}+\overline{\theta})} + \frac{\overline{\alpha}\overline{\theta}\overline{v}}{2(\overline{\alpha}+\overline{\theta})} + e.$
- $b(\mu) = \frac{\overline{v}\overline{\theta}^2}{4\mu(\overline{\alpha}+\overline{\theta})} + e.$
- $r(\mu) = \frac{2\overline{v}\overline{\theta}^2}{9\mu(\overline{\alpha}+\overline{\theta})} + \frac{2\overline{\alpha}\overline{\theta}\overline{v}}{3(\overline{\alpha}+\overline{\theta})} + e.$
- $s(\mu) = \frac{2\overline{v}\overline{\theta}^2}{9\mu(\overline{\alpha}+\overline{\theta})} + e.$
- $t = \frac{2\overline{\alpha}\overline{\theta}\overline{v}}{3(\overline{\alpha}+\overline{\theta})} + e.$

Proposition 2 (Competition effect, quantity competition)**.** *The competition effect on poverty depends on δ, μ and γ as follows, denoting by S_M and S_D, the poor population sizes respectively under Monopoly and under Duopoly:*

1. *Case 1: $\delta \geq 6\mu$*
   - i $S_M = S_D = 0$, *for $\gamma < e$.*
   - ii $S_M > S_D$, *for $e \leq \gamma < t$.*
   - iii $S_M = S_D = (1-\mu)\overline{\alpha}\overline{\theta}$ *(the non-shareholders), for $t \leq \gamma < s(\mu)$.*
   - iv $S_M < S_D$, *for $s(\mu) \leq \gamma < a(\mu)$.*
   - v $S_M = S_D = \overline{\alpha}\overline{\theta}$ *(all the population), for $\gamma \geq a(\mu)$.*
2. *Case 2: $3\mu \leq \delta < 6\mu$*
   - i $S_M = S_D = 0$, *for $\gamma < e$.*
   - ii $S_M > S_D$, *for $e \leq \gamma < t$.*
   - iii $S_M = S_D = (1-\mu)\overline{\alpha}\overline{\theta}$, *for $t \leq \gamma < s(\mu)$.*
   - iv $S_M < S_D$, *for $s(\mu) \leq \gamma < b(\mu)$.*

[16] $a(\mu)$ and $b(\mu)$ are just recalled here. They were introduced in section 3.

v $S_M > S_D$, *for* $b(\mu) \leq \gamma < r(\mu)$.

vi $S_M = S_D = \overline{\alpha}\overline{\theta}$, *for* $\gamma \geq r(\mu)$.

3. *Case 3:* $\delta < 3\mu$

i $S_M = S_D = 0$, *for* $\gamma < e$.

ii $S_M > S_D$, *for* $e \leq \gamma < s(\mu)$.

iii $S_M < S_D$, *for* $s(\mu) \leq \gamma < b(\mu)$.

iv $S_M > S_D$, *for* $b(\mu) \leq \gamma < r(\mu)$.

v $S_M = S_D = \overline{\alpha}\overline{\theta}$, *for* $\gamma \geq r(\mu)$.

For a better readability, Proposition 2 may be displayed graphically in the space (μ, δ) (Figures 6, 7 and 8). As the proportion of shareholders μ is in [0,1], a discussion on δ is necessary.

- if $\frac{\delta}{6} \geq 1$ (i.e. $\delta \geq 6$), then for any $\mu \in [0, 1]$, we have $\mu \leq \frac{\delta}{6}$. Thus, only Case 1 applies and is used to draw Figure 6.
- if $\frac{\delta}{6} < 1 \leq \frac{\delta}{3}$ (i.e. $3 \leq \delta < 6$), then in Figure 7, Case 1 applies when $\mu \in [0, \frac{\delta}{6}[$ and Case 2 applies when $\mu \in [\frac{\delta}{6}, 1]$.
- if $(\frac{\delta}{6} <)\ \frac{\delta}{3} < 1$ (i.e. $\delta < 3$), then in Figure 8, Case 1 applies when $\mu \in [0, \frac{\delta}{6}[$, Case 2 applies when $\mu \in [\frac{\delta}{6}, \frac{\delta}{3}]$ and Case 3 applies when $\mu \in]\frac{\delta}{3}, 1]$.

We thus obtain Figures 6, 7 and 8. The"+" ("-") sign indicates that competition has a positive (negative) effect on poverty i.e. it reduces (increases) the poor population size; i.e. the poor population size under Duopoly is lower (higher) than under Monopoly. The "=" sign indicates that competition has no effect on the size of the poor population; i.e. the poor population size is the same under both market structures.

It is worth noting that $s(\mu) = t$ when $\mu = \frac{\delta}{6}$ We denote by $z_2(\mu) = \max(r(\mu), a(\mu))$, we have that $z_2(\mu) = a(\mu)$ if $\mu < \frac{\delta}{6}$ and $z_2(\mu) = r(\mu)$ otherwise.

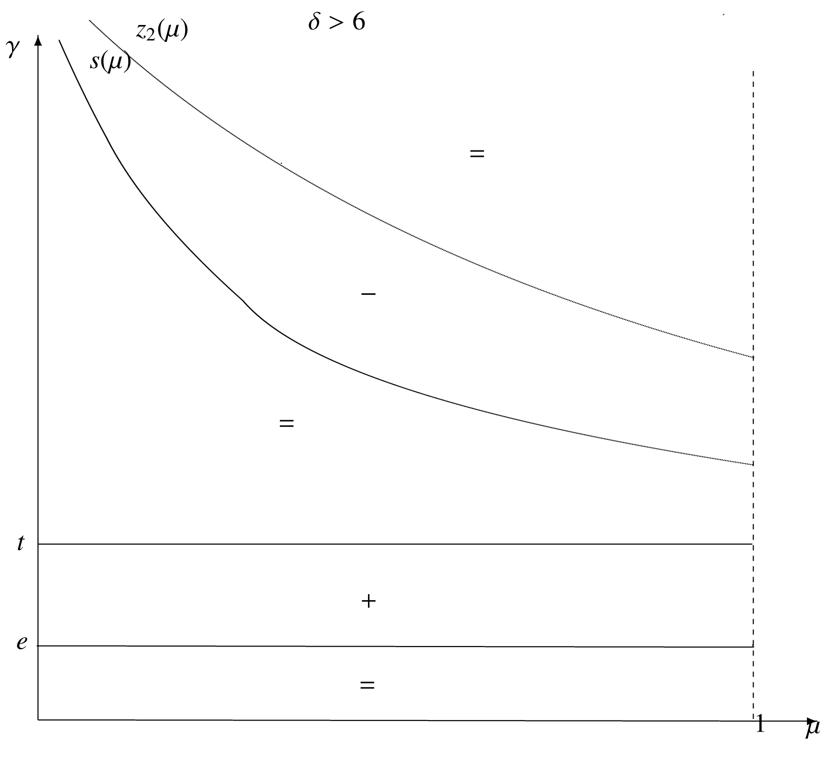

Figure 6 – Effect of Competition on Poverty for $\delta > 6$ under quantity competition.

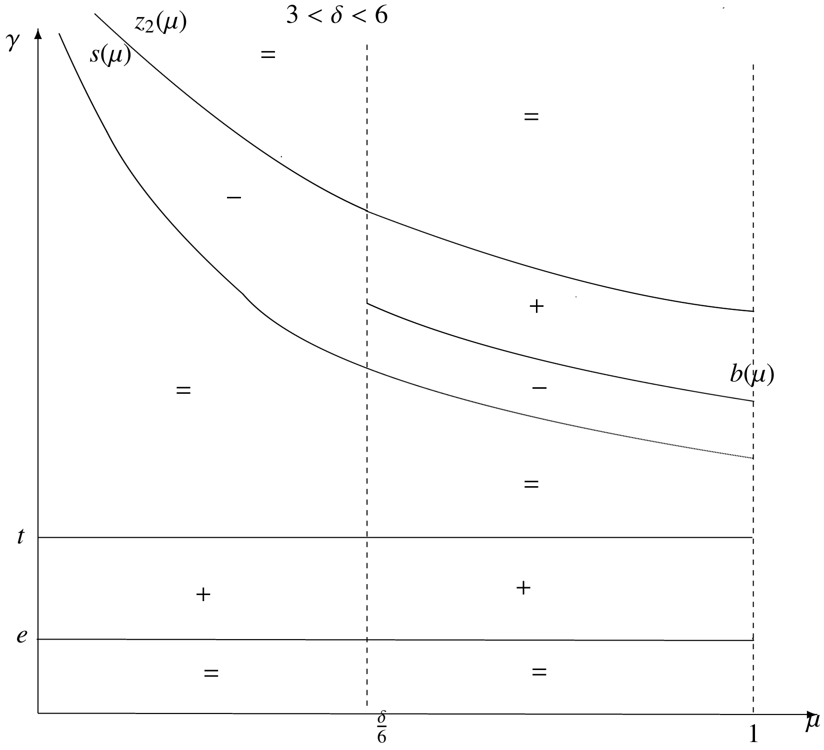

Figure 7 – Effect of Competition on Poverty for $3 < \delta < 6$ under quantity competition.

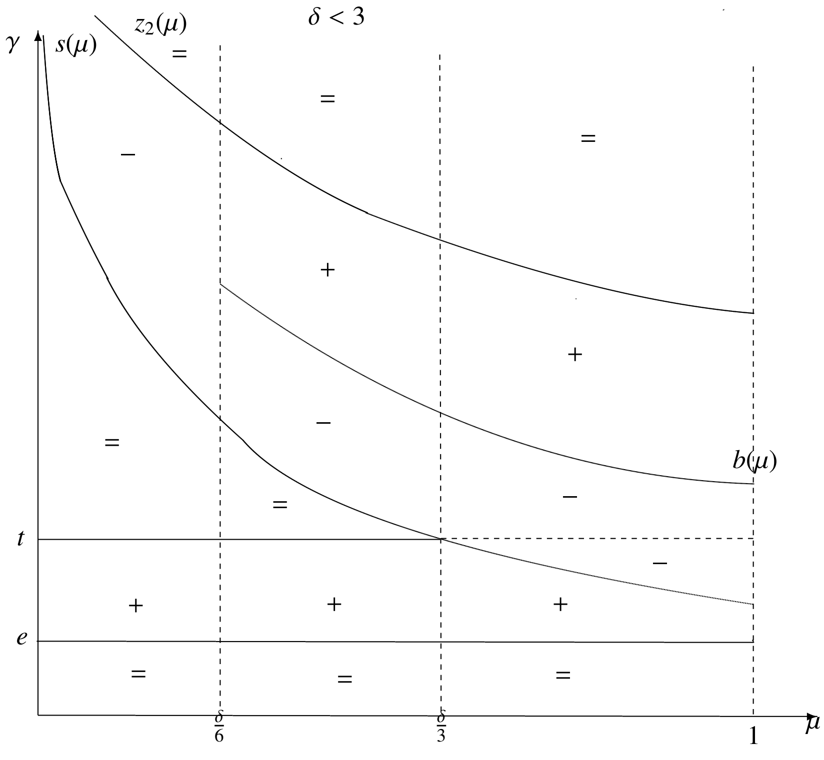

Figure 8 – Effect of Competition on Poverty for $\delta < 3$ under quantity competition.

Qualitatively the results under quantity competition are similar to the results under price competition (in terms of the alternation of the "signs" describing the effect of competition on poverty). The interpretation of the effect of competition on poverty follows thus the same reasoning as under price competition. The main difference between both types of competition comes from the availability of only one quality under quantity competition (the highest one) when two qualities are available under price competition.

To establish the proposition, we follow the same approach as the one undertaken under price competition to compare the sizes of the poor population under Monopoly and Duopoly for different levels of the poverty line. It turns out that the relative position of I_{NS}^{M}, I_{NS}^{D}, I_{S}^{M} and I_{S}^{D} is the same as under price competition with the same explanation.

5 Empirical Analysis

Our objective in this section is to analyze empirically the relationship between competition and poverty taking into consideration our theoretical findings. As there is no simple relation to be tested, we aim in fact at identifying among the two types of competition and the three possible scenarios under each type of competition, the most fit to data.

5.1 Data and Empirical Model

We follow Dixit and Stern (1982) and use the Herfindahl Hirschman market concentration index (HHI) as a proxy for competition. A high value of the HHI indicates a highly concentrated market, thus a low level of competition. The three existing poverty headcount ratios at \$1.90, \$3.2 and \$5.5 a day are used successively to account for poverty. We have used the three poverty lines for which there are data, since the level of the poverty line plays an important role in our theoretical findings on the impact of competition on poverty. We hope this way to find the richest variety of cases possible.

Prior studies [17] highlighted the role of inflation, economic growth and GDP per capita as determinants of poverty, but found non-significant effects of trade openness and government expenses. Kahloul et al (2019) showed however that the two last variables may have significant effects when the ownership structure is taken into account. Therefore we retain their model in our empirical analysis.

$$\begin{aligned}\ln(POV)_{it} = a_1 + a_2 \ln(GDP)_{it} + a_3 \ln(HHI)_{it} + a_4 \ln(G)_{it} + \\ a_5 \ln(TRADE)_{it} + a_6 \ln(INF)_{it} + a_7 GROWTH_{it} + \epsilon_{it}\end{aligned}$$

Where:

- $\ln(POV)$ is the logarithm of the poverty headcount ratio,
- $\ln(GDP)$ is the logarithm of the GDP per capita,
- $\ln(HHI)$ is the logarithm of the HHI,
- $\ln(G)$ is the logarithm of the government expense ratio,
- $\ln(TRADE)$ is the logarithm of the trade ratio,
- $\ln(INF)$ is the logarithm of (1 + inflation rate),
- $GROWTH$ is the growth rate of the GDP.

We use the market capitalization as a proxy for the ownership structure parameter as in Kahloul et al. (2019). Namely, the lower (higher) the value of the market capitalization, the lower (higher) the proportion of shareholders and the more (less) concentrated the ownership structure.

[17] See Rewilak (2017) and Ames et al. (2001), for instance.

Variable	Abbreviation	Definition
Poverty headcount ratio	POV	The percentage of the population living on less than \$1.90 a day at 2011 international prices.
GDP per capita	GDP	Gross Domestic Product per capita converted to international dollars using purchasing power parity rates.
Herfindahl Hirschman Index	HHI	HH market concentration index.
Government expense ratio	G	Expense is cash payments for operating activities of the government in providing goods and services (% of GDP).
Trade openness	TRADE	Trade is the sum of exports and imports of goods and services (% of GDP).
Inflation rate	INF	Inflation as measured by the annual growth rate of the GDP implicit deflator.
Growth	GROWTH	Annual percentage growth rate of GDP at market prices based on constant local currency.
Market capitalization	MCAP	Market capitalization is the share price times the number of shares outstanding for listed domestic companies (% of GDP).

Table 1 – Definition of the variables

The list of the variables and their definitions are given in Table 1. Two databases were merged: the World Integrated Database Solution (WITS) for the Herfindahl Hirschman market concentration index (HHI) and the World Development Indicators for all the remaining variables. Countries for which poverty data and/or market concentration data are not available were removed from the sample. Data covers 80 countries over the period 1991-2017.

5.2 Empirical Results

Our theoretical findings show different cases depending on δ. The empirical study allows to identify the type of competition which approximates better the data and the cases among the ones described in Figures 3, 4 or 5 under price competition and Figures 6, 7 and 8 under quantity competition, which better match our data.

We first run the regression model[18] on the whole sample as shown in column 1 of Table 2 (poverty line \$1.9), Table 3 (poverty line \$3.2) and Table 4 (poverty line \$5.5). The results show that the HH index (and therefore competition) has no significant effect on poverty with the whole sample, when the poverty line is \$1.9 and \$3.2, but has a significant effect with \$5.5. This confirms the result obtained by Kahloul et al. (2019) with the headcount ratio \$1.9 and the whole sample. Even though we find a significant effect with \$5.5, this is not in contradiction with the theoretical results. Indeed, the effect of competition on poverty, according to our theoretical findings, may be of different nature (positive, negative or non-significant), depending on the ownership structure of the countries and on the level of poverty line, which may lead, when the countries are aggregated, to either a significant or a non-significant effect.

To take the ownership structure into account in the light of our theoretical findings, we rank the countries according to their average market capitalization over the period 1991 to 2017 from lowest to highest. Then, we divide the sample in two sub-samples: countries with high market capitalization and countries with low market capitalization. For each level of headcount ratio, we run two separate regressions on each sub-sample to detect the nature (positive, negative or non-significant) of the effect of competition on poverty.

Contrary to Kahloul et al. (2019) who divided the whole sample arbitrarily, we experiment different sub-sample cutoffs and choose the best fit for each level of headcount ratio.[19] Results showing the best cutoffs are reported in the last row of each Table 2, 3 and 4.

[18]We test for autocorrelation and groupwise heteroskedasticity and estimate the model with Feasible Generalized Least Squares (FGLS).

[19]To choose the sub-samples with the best fit, we consider the correlation between the observed and the fitted values of poverty for both groups.

	(1)	(2)	(3)
Variables	Whole sample	Concentrated ownership (low μ)	Dispersed ownership (high μ)
LGDP	-1.6062***	-1.4405***	-1.4687***
LHH	-0.0461	0.3780***	-0.1989***
LG	-0.5635***	-1.3676***	0.0701
LTRADE	-0.3695***	-0.0943	-0.9198***
LINFL	-0.0087	-0.1939***	0.0394
GROWTH	-0.0023	-0.0533***	0.0014
CONSTANT	19.3623***	20.6681***	17.7336***
NUMBER OF COUNTRIES	80	49	31

Note: *** p < 0.01

Table 2 – Impact of market concentration on Poverty with a poverty line γ = \$1.9 a day

	(1)	(2)	(3)
Variables	Whole sample	Concentrated ownership (low μ)	Dispersed ownership (high μ)
LGDP	-1.9568***	-1.9532***	-2.1053***
LHH	-0.0627	-0.0102	-0.3019***
LG	-0.5849***	-0.6266***	-0.2662***
LTRADE	-0.5274***	-0.2904***	-0.6970***
LINFL	0.0264**	-0.0104	0.1312***
GROWTH	0.0038	0.0013	0.0147
CONSTANT	24.0179***	23.3337***	24.5965***
NUMBER OF COUNTRIES	80	49	31

Note: *** p < 0.01, ** p < 0.05

Table 3 – Impact of market concentration on Poverty with a poverty line γ = \$3.2 a day

	(1)	(2)	(3)
Variables	Whole sample	Concentrated ownership (low μ)	Dispersed ownership (high μ)
LGDP	-2.2017***	-1.5188***	-2.5459***
LHH	-0.0749**	-0.2172***	-0.0659
LG	-0.3610***	-0.8364***	-0.2234***
LTRADE	-0.0960*	-0.1458	-0.1201**
LINFL	0.0323***	0.0030	0.0333**
GROWTH	0.0092***	-0.0001	0.0118***
CONSTANT	24.7071***	19.5363***	27.8650***
NUMBER OF COUNTRIES	80	30	50

Note: *** p < 0.01, ** p < 0.05, * p < 0.1

Table 4 – Impact of market concentration on Poverty with a poverty line γ = \$5.5 a day

When the poverty line is \$1.9, competition is shown to have a significant positive impact on poverty for countries with concentrated ownership (see column 2 of Table 2) and has a significant negative impact for countries with dispersed ownership (see column 3 of Table 2). These results are aligned, under price competition, with Figure 4 when the poverty line $g(1) < \gamma < h$, and Figure 5 when $g(1) < \gamma < b(1)$; and under quantity competition, with Figure 8 with $s(1) < \gamma < t$.

When the poverty line is \$3.2, competition has no significant impact on poverty for countries with concentrated ownership (see column 2 of Table 3) and has a significant negative impact for countries with dispersed ownership (see column 3 of Table 3). These results are perfectly aligned under price competition, with Figure 3, when $g(1) < \gamma < z_1(1)$ and Figure 4, when the poverty line $h < \gamma < z_1(1)$; and under quantity competition with Figure 6 when $s(1) < \gamma < z_2(1)$, Figure 7 when $s(1) < \gamma < b(1)$ and Figure 8 when $t < \gamma < b(1)$.

When the poverty line is \$5.5, competition has a significant negative impact on poverty for countries with concentrated ownership (see column 2 of Table 4) and has no significant impact for countries with dispersed ownership (see column 3 of Table 4). Apart from the case of very low μ for which we should have a non-significant effect, for high enough γ the empirical result is in line, under price competition with Figures 3, 4 and 5; and under quantity competition, with Figures 6, 7 and 8. The fact of not obtaining a non-significant effect for very low μ may be explained by the existence, in the theoretical results, for high enough γ, of three segments in terms of μ, which requires to divide the sample into three sub-samples and may yield a sub-sample of low μ too small to give statistically significant results. Put in other words, the empirical results are consistent with the theoretical findings precisely under the condition that $\gamma = \$5.5$ is high enough to make the sub-sample of low μ, on which competition has no effect on poverty, too small to be statistically observable.

The results put together suggest that the cases represented by Figure 4 (price competition and intermediate values of δ) and Figure 8 (quantity competition and low values of δ) are the most consistent with the data for the three levels of poverty lines. We mean that these cases are the best to explain simultaneously the results obtained with the three levels of poverty line.

6 Conclusion

Using a general equilibrium model with vertical preferences, we study the effect of competition on poverty. We consider as the poverty indicator the size of the population whose income is below some given poverty line. Two types of competition are studied in the theoretical part of the paper: price competition and quantity competition. We explicitly take into account the ownership structure to examine the effect of competition on poverty.

Theoretical results reveal the critical role of ownership structure. More competition can alleviate, worsen or have no impact on poverty depending on the ownership structure, the defined poverty line, the relative dispersion of individuals with respect to their intensity of preference for quality and sensitivity to effort, and the competition type.

Empirical results allow to validate our theoretical findings for both price and quantity competition and confirm the decisive role of ownership structure. Namely, for a low poverty line ($1.9 a day), competition lessens poverty under concentrated ownership and worsens it when ownership is dispersed. For an intermediate poverty line ($3.2 a day), competition does not have a significant impact on poverty when ownership is concentrated. However, it intensifies it under dispersed ownership structures. Finally, for a high poverty line ($5.5 a day), while competition worsens poverty when ownership is concentrated, it does not have a significant effect when ownership is dispersed.

Among the cases possible theoretically, we identify two cases corresponding to low enough relative dispersion of individuals with respect to their intensity of preference for quality and sensitivity to effort that fit the empirical findings. The empirical results do not allow to retain one type of competition rather than the other. One case under each type of competition is fit with all the findings. Clearly the three available headcount ratios are not enough for that purpose. If data were available with more poverty headcount ratios, determining the type of competition could have been possible.

Our results can be improved in different ways. First, individuals' incomes were measured in terms of the numeraire good. A possible extension would be to measure the income relatively to the price of the differentiated good. These are two alternative ways to capture the purchasing power of the individuals. Second, instead of examining the effect of competition on poverty, we could consider the effect of competition on the individuals' well-being. To empirically test the results, we need to incorporate well-being indicators. Another interesting question to be addressed in a future work is the effect of competition on inequality taking into account the ownership structure.

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Appendix

Proof of Lemma 1 Table 5 provides, under Monopoly, for each individual his/her consumption and working decisions and indirect utility, depending on his/her characteristics (α, θ), the wage, the product's quality and price.

Working / Consumption	W	$\frac{w_M}{v_M}$	$\overline{W}$	$\overline{\alpha}$
$\overline{C}$	$U_M = w_M + \lambda\pi_M + e - \alpha v_M$		$U_M = \lambda\pi_M + e$	
$\frac{p_M}{v_M}$				
C	$U_M = \theta v_M + w_M + \lambda\pi_M + e - \alpha v_M - p_M$		$U_M = \theta v_M + \lambda\pi_M + e - p_M$	
$\overline{\theta}$				

Table 5 – Consumption/working decisions and indirect utility (U_M) of each individual in the Monopoly case.

Due to the linearity of the utility function, the consumption decision depends only on θ, the quality and the output price; and the working decision depends only on α, the wage and the quality. Thus, 4 groups of individuals appear.

At the Monopoly equilibrium, the marginal worker is characterized by $\frac{w_M^*}{v_M^*} = \frac{\overline{\alpha}\overline{\theta}}{2(\overline{\theta}+\overline{\alpha})}$. All (α, θ) individuals who choose not to work are characterized by $\alpha > \frac{w_M^*}{v_M^*}$. Individuals who choose to work are characterized by $\alpha < \frac{w_M^*}{v_M^*}$

The income of a non-shareholder is the sum of his/her initial endowment e and the net salary $w_M^* - \alpha v_M^*$ if he/she chooses to work . Thus, the income is e when $\alpha > \frac{w_M^*}{v_M^*}$ and $e + w_M^* - \alpha v_M^*$ otherwise.

The income of shareholders is found similarly. In addition to the initial endowment e and the net salary $w_M^* - \alpha v_M^*$ they receive if they choose to work, they also receive their share in the Monopoly profit $\frac{\pi_M^*}{\mu\overline{\alpha}\overline{\theta}}$. Substituting w_M^*, v_M^* and π_M^* by their expressions at equilibrium gives the expressions of the incomes under Monopoly.

Proof of Lemma 2 Table 6 provides, under Duopoly, for each individual his/her consumption and working decisions and indirect utility, depending on his/her characteristics (α, θ), the wage, the product's quality and price.

Working / Consumption	W_2	$\frac{w_2-w_1}{v_2-v_1}$ — W_1	$\frac{w_1}{v_1}$ — $\overline{W}$ — $\overline{\alpha}$
$\overline{C}$	$U = w_2 - \alpha v_2$ $+\lambda(\pi_1 + \pi_2) + e$	$U = w_1 - \alpha v_1$ $+\lambda(\pi_1 + \pi_2) + e$	$U = \lambda(\pi_1 + \pi_2) + e$
$\frac{p_1}{q_1}$			
C_1	$U = \theta v_1 - \alpha v_2 + w_2$ $-p_1 + \lambda(\pi_1 + \pi_2) + e$	$U = v_1(\theta - \alpha) + w_1$ $-p_1 + \lambda(\pi_1 + \pi_2) + e$	$U = \theta v_1 - p_1$ $+\lambda(\pi_1 + \pi_2) + e$
$\frac{p_2-p_1}{v_2-v_1}$			
C_2	$U = v_2(\theta - \alpha) + w_2$ $-p_2 + \lambda(\pi_1 + \pi_2) + e$	$U = \theta v_2 - \alpha v_1 + w_1$ $-p_2 + \lambda(\pi_1 + \pi_2) + e$	$U = \theta v_2 - p_2$ $+\lambda(\pi_1 + \pi_2) + e$
$\overline{\theta}$			

Table 6 – Consumption/working decisions and indirect utility (U_D) of each individual in the Duopoly case.

Under the Duopoly setting, at the equilibrium, the marginal workers are characterized by $\frac{w_1^*}{v_1^*} = \frac{7\overline{\alpha}\overline{\theta}}{8(\overline{\theta}+\overline{\alpha})}$ and $\frac{w_2^*-w_1^*}{v_2^*-v_1^*} = \frac{7\overline{\alpha}\overline{\theta}}{12(\overline{\theta}+\overline{\alpha})}$. Working individuals choose to work for firm 1 if $\frac{w_2^*-w_1^*}{v_2^*-v_1^*} < \alpha < \frac{w_1^*}{v_1^*}$ and for firm 2 if $\alpha < \frac{w_2^*-w_1^*}{v_2^*-v_1^*}$.
The income of a non-shareholder is the sum of his/her initial endowment e and the net salary $w_i^* - \alpha v_i^*$ if he/she chooses to work for firm i. Thus, the income is e when $\alpha > \frac{w_1^*}{v_1^*}$; $e + w_1^* - \alpha v_1^*$ when $\frac{w_2^*-w_1^*}{v_2^*-v_1^*} < \alpha < \frac{w_1^*}{v_1^*}$; and $e + w_2^* - \alpha v_2^*$ otherwise.

The income of shareholders is found similarly. In addition to the initial endowment e and the net salary $w_i^* - \alpha v_i^*$ they receive if they choose to work, they also receive their share in the summed profits $\frac{\pi_1^*+\pi_2^*}{\mu\overline{\alpha}\overline{\theta}}$. Substituting w_1^*, w_2^*, v_1^*, v_2^*, π_1^*, and π_2^* by their expressions at equilibrium gives the expressions of the incomes under Duopoly.
Proof of Lemma 3 We determine the equilibrium prices, quantities and qualities under Duopoly when firms engage in quantity competition. Two cases must be distinguished: (1) $v_1 < v_2$ and (2) $v_1 = v_2 = v_D$.

1. Case 1 $v_1 < v_2$: Given the consumption decisions provided by Table 6, firms' demands are given by

$$\begin{cases} \overline{\alpha}(\frac{p_2-p_1}{v_2-v_1} - \frac{p_1}{v_1}) = q_1 \\ \overline{\alpha}(\overline{\theta} - \frac{p_2-p_1}{v_2-v_1}) = q_2 \end{cases} \tag{2}$$

Firms' prices can be expressed in terms of quantities as follows:

$$\begin{cases} p_1 = v_1(\overline{\theta} - \frac{q_1+q_2}{\overline{\alpha}}) \\ p_2 = v_2\overline{\theta} - \frac{v_2 q_2}{\overline{\alpha}} - \frac{v_1 q_1}{\overline{\alpha}} \end{cases} \tag{3}$$

As the production of one unit of the differentiated good requires one unit of labor and using the working decisions given by Table 6, Equating labor supply and demand gives the following:

$$\begin{cases} \overline{\alpha}(\frac{p_2-p_1}{v_2-v_1} - \frac{p_1}{v_1}) = \overline{\theta}(\frac{w_1}{v_1} - \frac{w_2-w_1}{v_2-v_1}) \\ \overline{\alpha}(\overline{\theta} - \frac{p_2-p_1}{v_2-v_1}) = \overline{\theta}\frac{w_2-w_1}{v_2-v_1} \end{cases} \tag{4}$$

we can therefore express the wages in terms of prices then quantities.

$$\begin{cases} w_1 = -\frac{\overline{\alpha}p_1}{\overline{\theta}} + \overline{\alpha}v_1 = \frac{v_1}{\overline{\theta}}(q_1 + q_2) \\ w_2 = -\frac{\overline{\alpha}p_2}{\overline{\theta}} + \overline{\alpha}v_2 = \frac{v_2}{\overline{\theta}}q_2 + \frac{v_1}{\overline{\theta}}q_1 \end{cases} \tag{5}$$

Firms' profits can now be expressed only in terms of quantities.

$$\begin{cases} \pi_1 = (p_1 - w_1)q_1 = (v_1\overline{\theta} - \frac{v_1(\overline{\alpha}+\overline{\theta})}{\overline{\alpha}\overline{\theta}}(q_1 + q_2))q_1 \\ \pi_2 = (p_2 - w_2)q_2 = (v_2\overline{\theta} - \frac{v_2(\overline{\alpha}+\overline{\theta})}{\overline{\alpha}\overline{\theta}}(q_1 + q_2) - \frac{v_1(\overline{\alpha}+\overline{\theta})}{\overline{\alpha}\overline{\theta}}q_1)q_2 \end{cases} \tag{6}$$

The F.O.C resulting from differentiating the profits with respect to the quantities yield

$$\begin{cases} q_1 = \frac{\overline{\alpha}\overline{\theta}}{\overline{\alpha}+\overline{\theta}}(\overline{\theta} - \frac{\overline{\alpha}+\overline{\theta}}{\overline{\alpha}\overline{\theta}}(q_1 + q_2)) \\ q_2 = \frac{\overline{\alpha}\overline{\theta}}{\overline{\alpha}+\overline{\theta}}(\overline{\theta} - \frac{\overline{\alpha}+\overline{\theta}}{\overline{\alpha}\overline{\theta}}q_2 - \frac{v_1}{v_2}\frac{\overline{\alpha}+\overline{\theta}}{\overline{\alpha}\overline{\theta}}q_1) \end{cases} \tag{7}$$

Solving the system for q_1 and q_2 gives the following equilibrium quantities

$$\begin{cases} q_1^* = \frac{\overline{\alpha}\overline{\theta}^2}{\overline{\alpha}+\overline{\theta}}\frac{v_2}{4v_2-v_1} \\ q_2^* = \frac{\overline{\alpha}\overline{\theta}^2}{\overline{\alpha}+\overline{\theta}}\frac{4v_2-v_1}{4v_2-v_1} \end{cases} \tag{8}$$

Substituting the equilibrium quantities by their expressions in systems 3, 5 and 6 gives the expressions of the equilibrium prices, wages and profits in terms of v_1 and v_2 respectively.

Having the expressions of the profits in terms of the qualities, we differentiate the profit of each firm with respect to its quality.
As $\begin{cases} \pi_1^* = \frac{\overline{\alpha}\overline{\theta}^3 v_1 v_2^2}{\overline{\alpha}+\overline{\theta}(4v_2-v_1)^2} \\ \pi_2^* = \frac{\overline{\alpha}\overline{\theta}^3 v_1 v_2^2}{\overline{\alpha}+\overline{\theta}(4v_2-v_1)^2} \end{cases}$, we easily proof that $\frac{\partial \pi_1^*}{\partial v_1}$ and $\frac{\partial \pi_2^*}{\partial v_2}$ are both positive. Therefore, there exist no equilibrium where $v_1 < v_2$.

2. Case 2 $v_1 = v_2 = v_D$: As firms offer the same quality, consumers may choose either to buy the quality v_D or not. Workers need to choose either to remain idle or to work in one of the two firms and receive a wage w_D. The working and consumption decision are given in Table 7

Consumption \ Working	W (up to $\frac{w_D}{v_D}$)	$\overline{W}$ ($\frac{w_D}{v_D}$ to $\overline{\alpha}$)
$\overline{C}$ (up to $\frac{p_D}{v_D}$)	$U_D = w_D + 2\lambda\pi_D + e - \alpha v_D$	$U_D = 2\lambda\pi_D + e$
C ($\frac{p_D}{v_D}$ to $\overline{\theta}$)	$U_D = \theta v_D + w_D + 2\lambda\pi_D + e - \alpha v_D - p_D$	$U_D = \theta v_D + 2\lambda\pi_D + e - p_D$

Table 7 – Consumption/working decisions and indirect utility (U_D) of each individual in the Duopoly case under quantity competition.

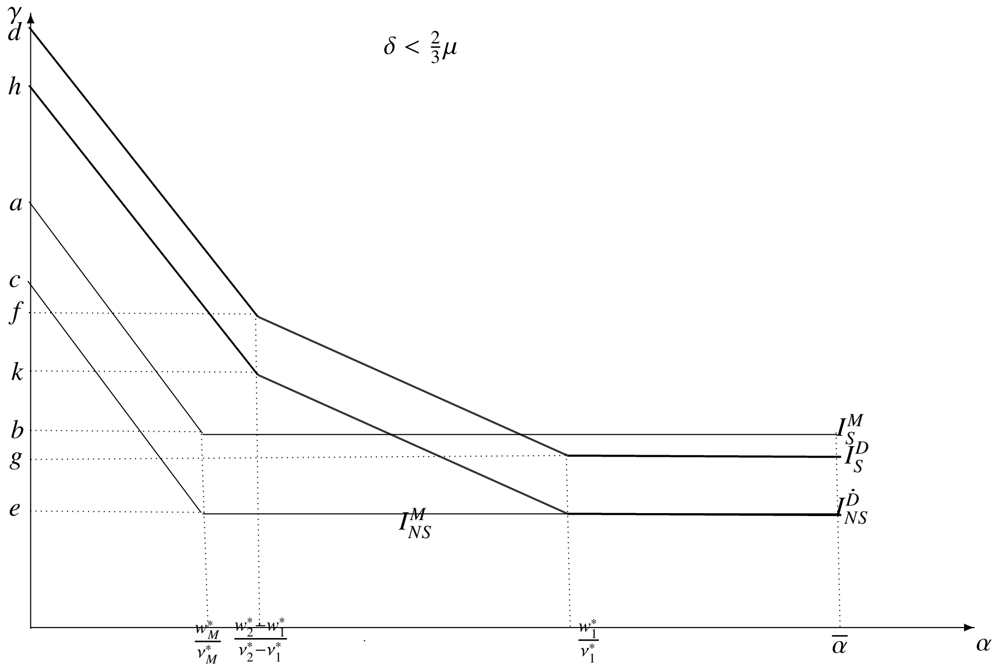

Figure 9 – Incomes curves under Monopoly and Duopoly when $\delta < \frac{2}{3}\mu$, Price Competition.

Equating the demand and supply for the differentiated good implies

$$\overline{\alpha}(\overline{\theta} - \frac{p_D}{v_D}) = q_1 + q_2$$

we can now express the price in terms of the quantities as follows:

$$p_D = (\overline{\theta} - \frac{q_1 + q_2}{\overline{\alpha}})v_D$$

Equating the demand and supply on the labor market allows to write the wage w_D in terms of the quantities:

$$w_D = \frac{v_D}{\overline{\theta}}(q_1 + q_2)$$

As we have that $\pi_1 = (p_D - w_D)q_1$ and $\pi_2 = (p_D - w_D)q_2$, substituting the price and the wage by their expressions in the profit's functions and differentiating the profits with respect to their respective quantities gives that $q_1^* = q_2^* = q_D^* = \frac{\overline{\alpha}\overline{\theta}^2}{3(\overline{\alpha}+\overline{\theta})}$.
We then easily determine the equilibrium prices, wages and quantities.

Proof of Proposition 1 Figure 9 exposes the income curves positioning when $\delta < \frac{2}{3}\mu$. We provide below steps showing the competition effect on poverty in this particular case. The other cases follow the same rationale and their proofs are available upon request. We now consider different levels γ of the poverty line and examine the intersection of the line $I = \gamma$ with the income curves. This determines the size of the poorest population under M and D.
Denote by α_{NS}^M, α_S^M the intersection of the poverty line $I = \gamma$ with I_{NS}^M and I_S^M, respectively.
Similarly, We denote by α_{NS}^D and α_S^D the intersection of the poverty line $I = \gamma$ with I_{NS}^D and I_S^D, respectively.

- If $\gamma < e$, the poverty line is below all income curves. Therefore, there are no poor under both Monopoly and Duopoly.
- If $e < \gamma < g$, the poverty line intersects with I_{NS}^M and I_{NS}^D.
  The size of the poorest population under M is given by $S^M = (1 - \mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^M)$ The size of the poorest population under D is given by $S^D = (1 - \mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^D)$.
  As $\alpha_{NS}^M < \alpha_{NS}^D$ then $S_M > S_D$.

- if $g < \gamma < b$, The poverty line intersects first with I^M_{NS} then with I^D_{NS} and I^D_S. The size of the poorest population under M is given by $S^M = (1-\mu)\bar{\theta}(\overline{\alpha} - \alpha^M_{NS})$ whereas the size of the poorest population under D is given by $S^D = (1-\mu)\bar{\theta}(\overline{\alpha} - \alpha^D_{NS}) + \mu\bar{\theta}(\overline{\alpha} - \alpha^D_S)$.

  The comparison is not straightforward and requires calculations. We can easily show that: $\alpha^M_{NS} = \frac{\overline{\alpha}\bar{\theta}}{2(\overline{\alpha}+\bar{\theta})} - \frac{\gamma-e}{\bar{v}}$, $\alpha^D_{NS} = \frac{7\overline{\alpha}\bar{\theta}}{8(\overline{\alpha}+\bar{\theta})} - \frac{7(\gamma-e)}{4\bar{v}}$, and $\alpha^D_S = \frac{7\overline{\alpha}\bar{\theta}}{48(\overline{\alpha}+\bar{\theta})}(6\overline{\alpha} + 2\frac{\bar{\theta}}{\mu}) - \frac{7(\gamma-e)}{4\bar{v}}$,

  Recall that $\delta = \frac{\bar{\theta}}{\overline{\alpha}}$. Using $\gamma > g$, we prove that $S_D - S_M > \frac{\overline{\alpha}^2\bar{\theta}}{24\mu(\overline{\alpha}+\bar{\theta})}[(3-3\mu)\delta^2 + (12\mu^2 - 9\mu)\delta + 24\mu^2]$.We prove that this quadratic expression in δ is positive for $\mu \in [0,1]$ and $\delta \in [0, \frac{2}{3}\mu]$. This leads to $S_M < S_D$

- If $b < \gamma < c$, the poverty line intersects with all four income curves I^M_{NS}, I^M_S, I^D_{NS}, and I^D_S.The size of the poorest population under M is given by $S^M = (1-\mu)\bar{\theta}(\overline{\alpha} - \alpha^M_{NS}) + \mu\bar{\theta}(\overline{\alpha} - \alpha^M_S)$

  Similarly, the size of the poorest population under D is given by $S^D = (1-\mu)\bar{\theta}(\overline{\alpha} - \alpha^D_{NS}) + \mu\bar{\theta}(\overline{\alpha} - \alpha^D_S)$. Since $\alpha^M_{NS} < \alpha^D_{NS}$ and $\alpha^M_S < \alpha^D_S$ then $S_M > S_D$.

- If $c < \gamma < a$, the poverty line intersects with income curves I^M_S, I^D_{NS}, and I^D_S. The income curve I^M_{NS} is under the poverty line so all non-shareholders under M are poor. The size of the poorest population under M is given by $S^M = (1-\mu)\overline{\theta\alpha} + \mu\bar{\theta}(\overline{\alpha} - \alpha^M_S)$. Similarly, the size of the poorest population under D is given by $S^D = (1-\mu)\bar{\theta}(\overline{\alpha} - \alpha^D_{NS}) + \mu\bar{\theta}(\overline{\alpha} - \alpha^D_S)$. Since $\alpha^M_S < \alpha^D_S$ and all non-shareholders are poor in Monopoly whereas only a fraction of non-shareholders are poor are Duopoly then $S_M > S_D$.

- If $a < \gamma < h$, the poverty line intersects with income curves I^D_{NS} and I^D_S while both I^M_S and I^M_{NS} are under the poverty line. The whole population under M is poor whereas only a fraction of the population is poor under D, so $S_M > S_D$.

- If $h < \gamma < d$, all citizens are poor under M. Under D, all non-shareholders and only a fraction of non-shareholders are poor so $S_M > S_D$.

- If $d < \gamma$, the poverty line is above all income curves. The whole population is poor under both M and D. This gives $S_M = S_D$.

Proof of Lemma 4 As shown in table 7, when firms engage in quantity competition, four categories of individuals appear under Duopoly, depending on their working and consumption choices. We proceed similarly to the proof of Lemma 1. The marginal worker is characterized by $\frac{w^*_D}{v^*_D} = \frac{\overline{\alpha}\bar{\theta}}{3(\bar{\theta}+\overline{\alpha})}$. All (α, θ) individuals who choose not to work are characterized by $\alpha > \frac{w^*_D}{v^*_D}$. Individuals who choose to work are characterized by $\alpha < \frac{w^*_D}{v^*_D}$ The income of a non-shareholder is the sum of his/her initial endowment e and the net salary $w^*_D - \alpha v^*_D$ if he/she chooses to work. Thus, the income is e when $\alpha > \frac{w^*_D}{v^*_D}$ and $e + w^*_D - \alpha v^*_D$ otherwise.

The income of shareholders is found similarly. In addition to the initial endowment e and the net salary $w^*_D - \alpha v^*_D$ they receive if they choose to work, they also receive their share in the Monopoly profit $\frac{2\pi^*_D}{\mu\overline{\alpha}\bar{\theta}}$. Substituting w^*_D, v^*_D and π^*_D by their expressions at equilibrium gives the expressions of the incomes under Duopoly.

Proof of Proposition 2

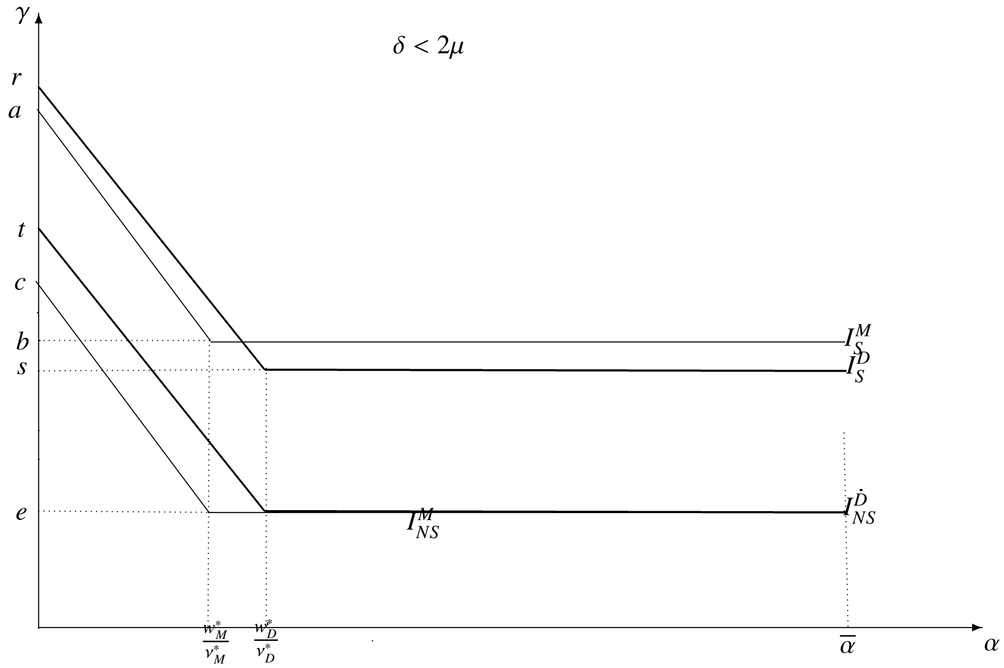

Figure 10 – Income curves under Monopoly and Duopoly when $\delta < 2\mu$, Quantity Competition.

Figure 10 exposes the income curves positioning when $\delta < 2\mu$. We provide below steps showing the competition effect on poverty in this particular case. The other cases follow the same rationale and their proofs are available upon request. We now consider different levels γ of the poverty line and examine the intersection of the line $I = \gamma$ with the income curves. This determines the size of the poorest population under M and D.
Recall that α_{NS}^M, α_S^M the intersection of the poverty line $I = \gamma$ with I_{NS}^M and I_S^M, respectively.
Similarly, We denote by α_{NS}^D and α_S^D the intersection of the poverty line $I = \gamma$ with I_{NS}^D and I_S^D, respectively.

- If $\gamma < e$, the poverty line is below all income curves. Therefore, there are no poor under both M and D.
- If $e < \gamma < s$, the poverty line intersects with I_{NS}^M and I_{NS}^D.
  The size of the poorest population under M is given by $S^M = (1-\mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^M)$ The size of the poorest population under D is given by $S^D = (1-\mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^D)$.
  As $\alpha_{NS}^M < \alpha_{NS}^D$ then $S_M > S_D$.
- if $s < \gamma < b$, The poverty line intersects first with I_{NS}^M then with I_{NS}^D and I_S^D. The size of the poorest population under M is given by $S^M = (1-\mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^M)$ whereas the size of the poorest population under D is given by $S^D = (1-\mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^D) + \mu\overline{\theta}(\overline{\alpha} - \alpha_S^D)$.

  We first show that: $\alpha_{NS}^M = \frac{\overline{\alpha}\overline{\theta}}{2(\overline{\alpha}+\overline{\theta})} - \frac{\gamma-e}{\overline{v}}$, $\alpha_{NS}^D = \frac{2\overline{\alpha}\overline{\theta}}{3(\overline{\alpha}+\overline{\theta})} - \frac{\gamma-e}{\overline{v}}$, and $\alpha_S^D = \frac{2\overline{\theta}^2}{9\mu(\overline{\alpha}+\overline{\theta})} + \frac{2\overline{\alpha}\overline{\theta}}{3(\overline{\alpha}+\overline{\theta})} - \frac{\gamma-e}{\overline{v}}$,

  Recall that $\delta = \frac{\overline{\theta}}{\overline{\alpha}}$. Using $\gamma > s$, we prove that $S_M - S_D < \frac{\overline{\alpha}\overline{\theta}}{6(1+\delta)}(\mu(-3\delta-6)+\delta)$.We prove that this expression in δ is negative for $\mu \in [0,1]$ and $\delta \in [0,2\mu]$. This concludes that $S_M < S_D$
- If $b < \gamma < c$, the poverty line intersects with all four income curves I_{NS}^M, I_S^M, I_{NS}^D, and I_S^D. The size of the poorest population under M is given by $S^M = (1-\mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^M) + \mu\overline{\theta}(\overline{\alpha} - \alpha_S^M)$

  Similarly, the size of the poorest population under D is given by $S^D = (1-\mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^D) + \mu\overline{\theta}(\overline{\alpha} - \alpha_S^D)$.
  Since $\alpha_{NS}^M < \alpha_{NS}^D$ and $\alpha_S^M < \alpha_S^D$ then $S_M > S_D$.
- If $c < \gamma < t$, the poverty line intersects with income curves I_S^M, I_{NS}^D, and I_S^D. The income curve I_{NS}^M is under the poverty line so all non-shareholders under M are poor. The size of the poorest population under M is given by $S^M = (1-\mu)\overline{\theta}\overline{\alpha} + \mu\overline{\theta}(\overline{\alpha} - \alpha_S^M)$. Similarly, the size of the poorest population under D is given by $S^D = (1-\mu)\overline{\theta}(\overline{\alpha} - \alpha_{NS}^D) + \mu\overline{\theta}(\overline{\alpha} - \alpha_S^D)$. Since $\alpha_S^M < \alpha_S^D$ and all non-shareholders are poor in Monopoly whereas only a fraction of non-shareholders are poor are Duopoly then $S_M > S_D$.

- If $t < \gamma < a$, the poverty line intersects with the income curve I_S^M then I_S^D while both I_{NS}^M and I_{NS}^d are under the poverty line.

  The size of the poorest population under M is given by $S^M = (1-\mu)\overline{\theta}\overline{\alpha} + \mu\overline{\theta}\overline{\alpha} - \alpha_S^M)$. Similarly, the size of the poorest population under D is given by $S^D = (1-\mu)\overline{\theta}\overline{\alpha} + \mu\overline{\theta}(\overline{\alpha} - \alpha_S^D)$. Since $\alpha_S^M < \alpha_S^D$ then $S_M > S_D$.

- If $a < \gamma < r$, all citizens are poor under M. Under D, all non-shareholders and only a fraction of non-shareholders are poor so $S_M > S_D$.

- If $r < \gamma$, the poverty line is above all income curves. The whole population is poor under both M and D. This gives $S_M = S_D$.

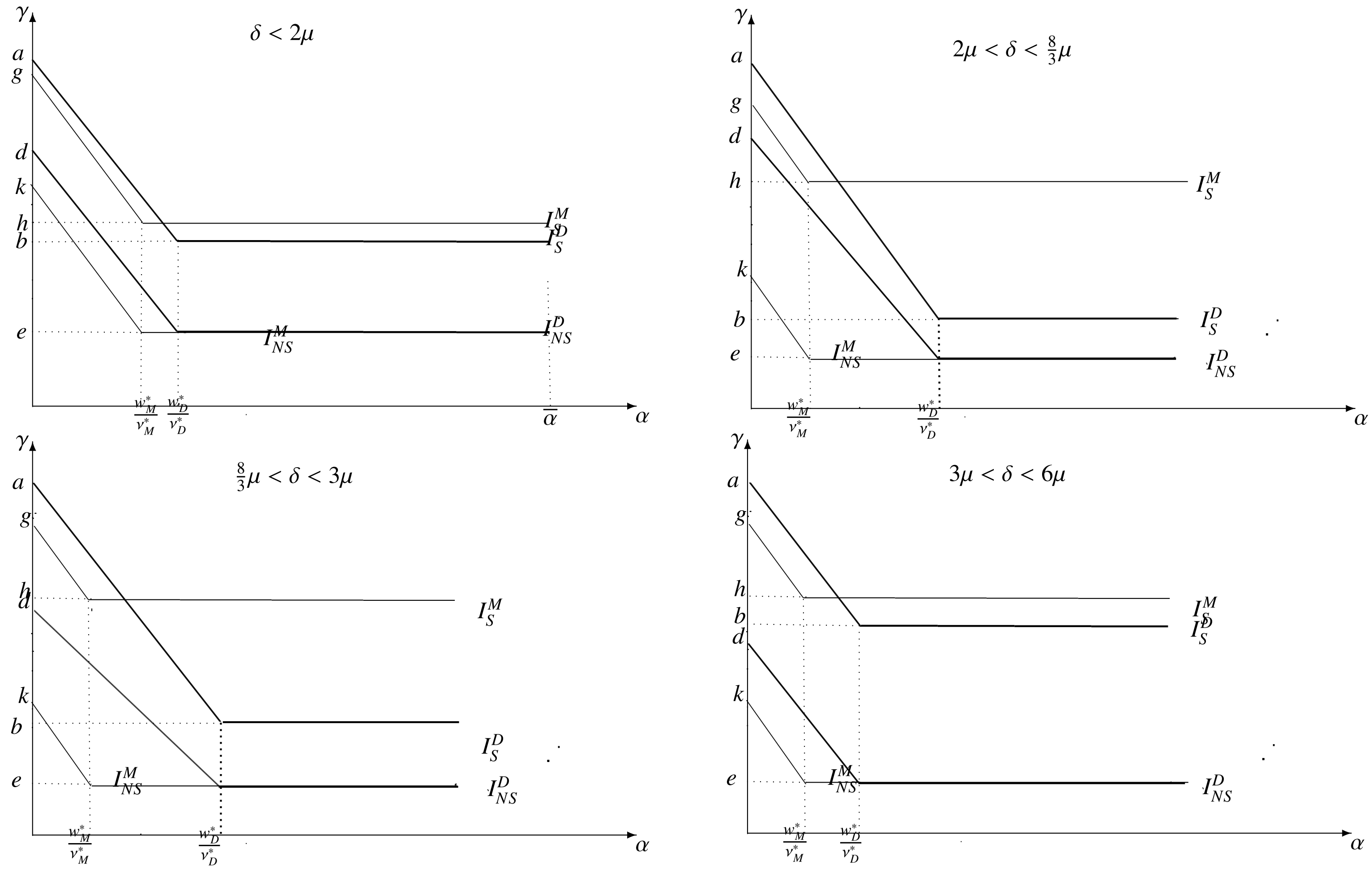

Figure 11 – Income curves positioning under quantity competition when $\delta < 6\mu$

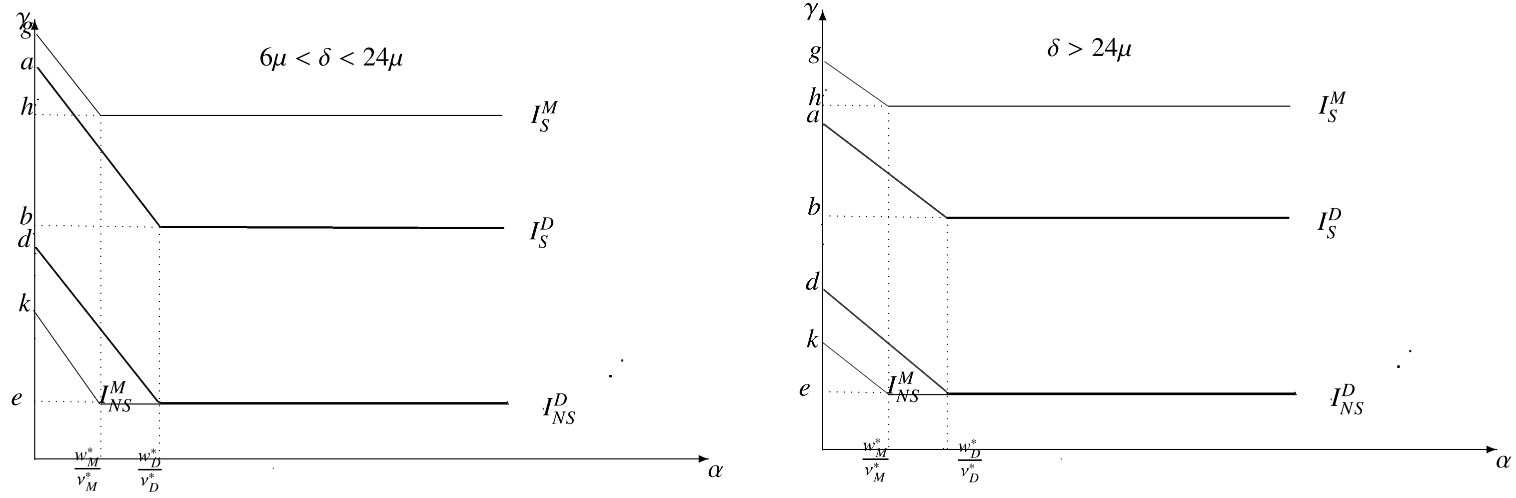

Figure 12 – Income curves positioning under quantity competition when $\delta > 6\mu$