ERF Working Papers Series

Hedging the Risks of MENA Stock Markets with Gold: Evidence from the Spectral Approach

Awatef Ourir, Elie Bouri and Essahbi Essaadi

ECONOMIC RESEARCH FORUM
منتدى البحوث الاقتصادية

Working Paper No. 1511
November 2021

HEDGING THE RISKS OF MENA STOCK MARKETS WITH GOLD: EVIDENCE FROM THE SPECTRAL APPROACH[1]

Awatef Ourir,[2] Elie Bouri,[3] and Essahbi Essaadi[4]

Working Paper No. 1511

November 2021

The authors would like to thank Dr. Raimundo Soto for providing very helpful comments and suggestions that improved the quality of the paper.

Send correspondence to:
Essahbi Essaadi
University of Manouba
Essahbi.Essaadi@gmail.com

[1] This work was sponsored by the Economic Research Forum (ERF) and has benefited from both financial and intellectual support. The contents and recommendations do not necessarily reflect ERF's views.

[2] Laboratory QuARG UR17ES26; University of Jendouba, FSJEG Jendouba, Tunisia. E-mail: awatefourir@gmail.com. https://orcid.org/0000-0003-0002-4531.

[3] Adnan Kassar School of Business, Lebanese American University, Lebanon. https://orcid.org/0000-0003-2628-5027.

[4] Laboratory QuARG UR17ES26; University of Manouba, ESCT, Campus Universitaire Manouba, 2010, Tunisia; https://orcid.org/0000-0003-4770-3785.

First published in 2021 by
The Economic Research Forum (ERF)
21 Al-Sad Al-Aaly Street
Dokki, Giza
Egypt
www.erf.org.eg

Abstract

This paper contributes to the old debate on the dynamic correlation between gold and stock markets by considering a spectral approach within the framework of portfolio hedging. Specifically, we consider eight MENA stock markets (Tunisia, Egypt, Morocco, Jordan, the United Arab Emirates, Saudi Arabia, Qatar, and Oman) and examine the optimal composition between gold and the stock market index, with a minimum portfolio risk and a high expected return. Based on the spectral approach, we propose seven portfolio structures and evaluate them through a comparison with the conventional DCC-GARCH method. The main results show that the spectral-based approach outperforms the DCC-GARCH method. In fact, the optimal gold-stock composition depends on the spectral density of each stock market index, where a stock market index with a stable spectral density requires more investments in gold than a stock market index with an unstable spectral density.

Keywords: Hedge ratio, evolutionary spectral analysis, DCC-GARCH model, gold, stock market index.

JEL Classifications: G11, C10, C61.

ملخص

في هذه الورقة، نساهم في الجدل القديم حول الارتباط الديناميكي بين أسواق الذهب والأسهم من خلال النظر في نهج طيفي في إطار التحوط بالمحفظة. على وجه التحديد، نأخذ في الاعتبار ثمانية أسواق مالية في منطقة الشرق الأوسط وشمال إفريقيا (تونس ومصر والمغرب والأردن والإمارات العربية المتحدة والمملكة العربية السعودية وقطر وعمان) ونفحص التكوين الأمثل بين الذهب ومؤشر البورصة، مع حد أدنى من مخاطر المحفظة وعائد متوقع مرتفع. بناءً على النهج الطيفي، نقترح سبعة هياكل للمحفظة ونقيمها من خلال المقارنة مع الطريقة التقليدية لنماذج الانحدار الذاتي المشروطة بعدم تجانس التباين من خلال تحديد العلاقات المشروطة الديناميكية (DCC-GARCH). تظهر النتائج الرئيسية أن النهج الطيفي يتفوق على طريقة DCC-GARCH. في الواقع، يعتمد التكوين الأمثل لمخزون الذهب على الكثافة الطيفية لكل مؤشر لسوق الأوراق المالية، حيث يتطلب مؤشر سوق الأوراق المالية ذي الكثافة الطيفية المستقرة استثمارات في الذهب أكثر من مؤشر سوق الأوراق المالية ذي الكثافة الطيفية غير المستقرة.

1. Introduction

Stock markets play a key role in economic activity and mirror the health of an economy. When businesses are growing and the economy is expanding, investors return to the stock markets to take advantage of this expediency. Not only does the economy grow; its fundamental structure changes across time. The changes in several economies and their opening to international markets supported the increase in the frequency of shocks and crises in such markets. In looking for opportunities, many investors devote attention to the potential risks of these actions. Therefore, in order to hedge, they diversify their portfolios and/or purchase derivative products that limit their losses.

After the double COVID-19 crisis (supply and demand crisis) on the oil market and the fall in prices below zero, gold seems to be a safe haven that still retains its function as a storage of value. Besides, due to a lack of a futures market in Middle East and North Africa (MENA)[5] economies, we suggest gold as hedge alternative in the stock market. As a part of their culture, gold occupies an important part of life in MENA countries. The volatility and correlation dependency and interdependency for financial series among different markets are assumed to be time-varying due to the presence of shocks and structural changes. Under these circumstances, we investigate the dynamic relationship between two financial indicators (gold and stock markets) for selected MENA countries. While including gold in the portfolio reduces volatility, we can conclude that gold can provide diversification and hedging for investors. Using an evolutionary spectral analysis, we can examine the time-varying relationship at different frequencies.

The novelty of this study is two-fold. To our knowledge, we are the first to study the interdependence between gold and the stock market index for the MENA region as a whole. Our study also expands the related literature by using a non-parametric measurement based on the definition of the spectrum introduced by Priestley (1965, 1996) and respecting the properties emphasized by Loynes (1968). However, previous studies that introduced frequency analysis use wavelet theory. In the present study, we analyze the dynamic dependence between series. Our measure of time-varying coherence provides not only the dynamics of the correlation process, but also the frequencies at which they comove. Therefore, we can determine the nature of the dynamic correlation process for the short run (high frequencies) and/or long run (low frequencies). The advantage of the frequency approach is that it detects the variability in the dependence process at different time scales. With this additional information, we can know which cycles are more relevant for each market. The use of frequency-domain allows us to distinguish between the properties of each cycle and estimate the dynamic hedge ratio in each cycle. Using a nonparametric approach of the spectral approach, we use seven possibilities to construct a

[5] In this paper, we consider a sample of eight MENA countries: Tunisia, Morocco, Egypt, Jordan, Oman, Qatar, the United Arab Emirates (UAE), and Saudi Arabia.

hedged portfolio for each frequency. These frequencies correspond to a specific period T, which we calculate by the ratio of $2\pi/w$, where w is the studied frequency.

The remainder of the paper is organized as follows. Section 2 presents the literature review for different hedging strategies and related analyses. Section 3 focuses on the methodology adopted in this paper. Section 4 presents data and analysis of the empirical results. Section 5 concludes.

2. Related literature

Gold is one of the most relevant commodities in the global economy. It performs several key functions in various markets; it is used as a store of value, a means of exchange, and a complement to international reserves. It has long been recognized as a good hedge and safe haven for financial assets, particularly stocks. Compared to other safe-haven assets, the dependence between gold and stocks has attracted much attention, as this relationship has direct implications on portfolio composition and risk management. However, the notion of modeling and understanding the degree and structure of the dependence between these two asset classes has attracted the attention of several studies.[6] The role of gold in equity portfolios has been confirmed by some and denied by others.

The first strand of studies uses the GARCH family to estimate the variance-covariance relation between different assets and construct a hedged portfolio. Some authors, such as Iqbal (2017), have not provided clear-cut results. He confirmed that the hedging role of gold against the adverse movement of stock is more complex than indicated in a linear relationship. Using the EGARCH model applied to daily and monthly gold price data along with stock prices in India, Pakistan, and the United States (US), he found that the hedging potential of gold is not uniformly strong; rather, it is dependent on the state of the gold market. Ku et al. (2007) compared various hedging portfolio strategies to investigate the optimal hedge ratios of British (BP) and Japanese (JY) currency futures markets. Firstly, they applied the DCC and CCC-GARCH models to estimate the time-varying hedge ratio. Then, the OLS and the Error Correction Model are used to presume a time-invariant hedge ratio. Their main finding proved that the DCC-GARCH model yields the best hedging performance in both the JY and BP markets. Although it is well known that the time-varying hedging ratio is more realistic, they confirmed that dynamic conditional correlations can better capture the frequent fluctuations. In the same vein, and by relying on the DCC-GARCH model, Creti et al. (2013) investigated the links between price returns for 25 commodities and stocks by paying particular attention to gold. They showed that the correlations between gold and stock markets evolve through time and are highly volatile, particularly since the financial crisis. They also found evidence of a negative correlation between the S&P 500 stock market index and gold price, allowing gold to serve as a refuge value function.

[6] For summary literature, see Tiwari et al. (2019).

Basher and Sadorsky (2016) enriched previous studies by using the DCC, ADCC, and GO-GARCH to model volatilities and conditional correlations between 23 emerging markets stock prices and gold prices. They found that hedge ratios estimated from the GO-GARCH are the most effective for hedging stock prices with gold, but only after the 2008-2009 recessions. They suggested that their results are reasonably robust to the choice of model refits, forecast length, and distributional assumptions. Further, Kumar (2014) reviewed portfolio designs and the hedging effectiveness of gold in the Indian stock market. Using the VAR-ADCC-BVGARCH model, negative values of estimated time-varying conditional correlations are mainly observed during periods of market turbulence and crisis. This result indicates the scope of portfolio diversification and hedging during these critical periods. They also estimated optimal weights, hedge ratios, and hedging effectiveness for the stock-gold portfolios. Their findings suggest that a stock-gold portfolio provides better diversification benefits than stock portfolios. The same result has been confirmed by Chkili (2016) for BRICS[7] countries: gold is a refuge value.

The second strand of literature proposes several techniques for measuring the ability of gold to cover a portfolio of the stock market index. With a sample of major emerging and developing countries over 30 years, Baur and McDermott (2010) used a specific econometric approach to measure the degree of dependence between gold price and stock market evolution. They showed that gold is both a hedge and safe haven for major European stock markets and the US, but not for Australia, Canada, Japan, and large emerging markets such as the BRIC countries. Thereafter, using the Generalized Method of Moments (GMM), Ziaei (2012) found a strong adverse correlation, giving gold a convenient hedging statue for ASEAN+3[8] countries with quarterly data. As confirmed by Ciner et al. (2013), they tried to examine a time variation in conditional correlations to determine if gold acts as a hedge of stock portfolios. They investigated the dynamic correlations between gold and the stock markets of the US and the United Kingdom (UK) using quantile regression methods. They found that gold can be regarded as a safe haven against exchange rates for both countries, highlighting its monetary asset role. Otherwise, both Gokmenoglu and Fazlollahi (2015) and Bouri et al. (2017) examined the inter-relationship between gold and financial markets through the co-integration test and nonlinear causality. They found a strong relationship between gold price and Indian and American stock markets, respectively. Raza et al. (2016) reported different results depending on the markets. According to a non-linear ARDL approach, they found that the price of gold has a positive impact on the stock market prices of large emerging BRICS economies and a negative impact on stock markets in Mexico, Malaysia, Thailand, Chile, and Indonesia. Gold volatility harms the stock markets of all emerging economies in both the short and long run. The results indicate that stock markets in emerging economies are

[7] BRICS countries include Brazil, Russia, India, China, and South Africa.
[8] The members of the Association of Southeast Asian Nations along with China, Japan, and South Korea.

more vulnerable to bad news and events that result in uncertain economic conditions. The dynamic relationship among gold and USD exchange rates is examined by Dong et al. (2019), and a negative association is shown while indicating the significance of structural breaks on the relationship. Using the quantile-on-quantile approach, they argued that dependence between stock and gold is not uniform and that this relationship is country- and market state-specific. Likewise, using the bivariate cross- quantilogram in the US market, Baumöhl and Lyócsa (2017) showed that the safe-haven properties of gold have a changing nature. Before and after the financial crisis, gold can be considered a safe haven, but in periods of stress, there are a few sectors for which gold checked the relation. Tiwari et al. (2019) studied the dependence between gold and the stock market for seven emerging economies during 2002-2018. The study combined the bivariate cross-quantilogram with quantile-on-quantile regression (QQR) approaches. They suggested that gold may be a hedge for stocks during the pre-crisis compared to the post-crisis period. Further, international risk factors should be considered in optimal investment decisions between domestic and global market assets (stocks and gold).

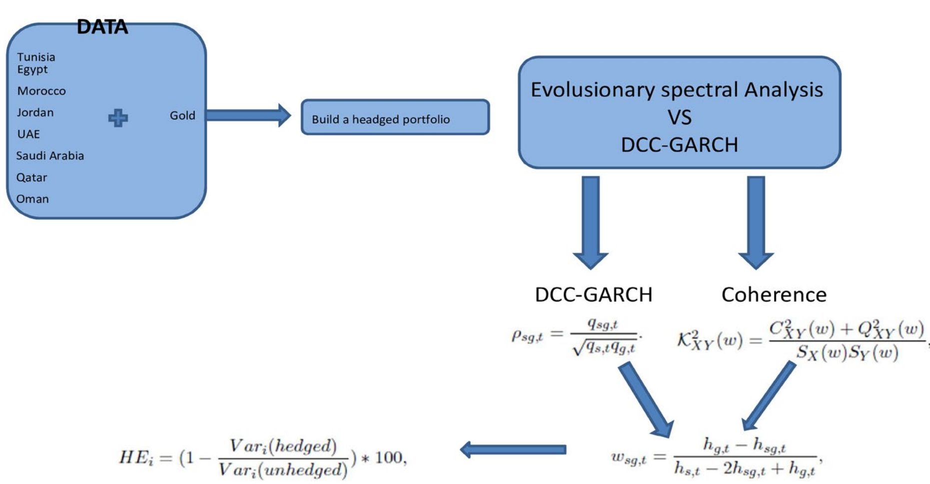
The third strand of literature uses the frequency approach to measure the relevance of gold in stock market portfolios. By employing a time-frequency approach with a wavelet methodology, Baruník et al. (2016) analyzed dynamic correlations, especially between gold and stocks, and they used the volatility and DCC-GARCH approaches for comparison. For the daily and intra-daily US data, they checked heterogeneity correlations between gold and stocks. Bouri et al. (2017) applied a frequency approach to check causality dynamics between gold and Chinese and Indian stock markets. They found significant bi-directional effects between both series in both high and low frequencies, suggesting an unstable feature of gold as refuge value. However, using a hybrid wavelet-based Dynamic Conditional Correlation (DCC) approach, Bhatia et al. (2020) investigated the dynamic relationship between precious metals and stock markets in the time and frequency domain for major developed (G7) and emerging (BRICS) nations. Their results suggest that the DCC between series varies with timescales in terms of the dynamism, persistence, and strength of the relationship. Some related studies consider continuous wavelet transformation in the oil-stock nexus (Ghosh et al., 2021) and provide evidence of contagiousness in the financial markets (Ftiti et al., 2015), especially during the US subprime crisis (Zhou, 2017). Also, with a large sample of 34 emerging and developing economies, Bulut and Rizvanoghlu (2020) affirmed the hedge property of gold in stock markets. Using a GARCH-copula approach, they found that the safe-haven properties vary from one country to another.

Most previous studies have verified the dependency relationship between the price of gold and the stock market index to confirm the hedging property and refuge character of gold. This relationship varies over time in terms of sign and intensity. Given this variability, the composition of an optimal portfolio (gold/ stocks) also varies according to the degree of existing dependence between them. To detect this dynamic relationship, we use an evolutionary spectral analysis and propose a statistic of coherence and its components based on a frequency approach. Our study allows us to decompose movements according to their component cycles and allows those cycles to vary in importance and characteristics over time. Considering these properties and the specificity of the MENA stock markets, this paper expands on previous studies by examining the hedging character of gold at the expense of the derivatives market using the evolutionary spectral analysis defined by Priestley (1965, 1996).

3. Empirical methodology

There are two well-defined approaches to analyzing time series: (1) the Spectral Approach (frequency approach) and (2) the Temporal Approach (dynamic approach). The interest of the spectral analysis is the simplicity of the periodicty's visibility within the series. Therefore, it does not require upstream processing, unlike the temporal approach, which relies on the assumption of stationarity in mean, variance, and covariance after the elimination of trends. In this paper, we compare hedge performance for the two different approaches using data on MENA countries. On the one hand, relying on the presence of heteroscedasticity and dynamics in the interdependence between returns in the financial market, we use a widely employed DCC-GARCH model to estimate time-varying correlations and their components. On the other hand, we propose a frequency alternative for correlation and hedge estimation. Figure 1 illustrates the main methodological steps that we follow in our empirical analysis.

Figure 1. Methodological steps



3.1. DCC-GARCH model

Introduced by Engel (2002), the DCC-GARCH model is widely used in financial time series modeling as presented in the previous section. The model relies on a time-varying variance-covariance matrix decomposition. The multivariate DCC-GARCH model is presented as follows:

$$r_t = \mu + \varepsilon_t,\ \ \varepsilon_t \sim N(0, H_t),$$

where $r_t = (r_{s,t}, r_{g,t})'$ is the vector of log returns of stock market s and the second for the gold log returns at time t, $\mu = (\mu_s, \mu_g)'$ is the vector of the expected value of the conditional r_t, $\varepsilon_t = (\varepsilon_{s,t}, \varepsilon_{g,t})'$ is the vector of conditional residual with $E(\varepsilon_t) = 0$ and $Cov(\varepsilon_t) = H_t$, the variance covariance matrix.

$$H_t = D_t R_t D_t,\ (2)$$

where H_t is a 2×2 matrix of conditional variance of mean-corrected k-return series (r_t) at time t, $E(r_t) = 0$ and $Cov(r_t) = H_t$. D_t is a 2×2 diagonal matrix of time varying standard deviations from 2 univariate GARCH models at time t.

$$D_t = \begin{bmatrix} \sqrt{h_{st}} & 0 \\ 0 & \sqrt{h_{gt}} \end{bmatrix},$$

The R_t matrix represents the time varying conditional correlation matrix.

$$R_t = (diag(Q_t))^{-1/2} Q_t (diag(Q_t))^{-1/2},\ (4)$$

where Q_t has the GARCH (1,1) specification

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha z_{t-1} z'_{t-1} + \beta Q_{t-1},$$

The parameters α and β are nonnegative with a sum less than unity. When $\alpha = \beta = 0$, we find a CCC-GARCH model.

$$\overline{Q} = \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} z_{s,t}^2 & z_{s,t} z_{g,t} \\ z_{g,t} z_{s,t} & z_{g,t}^2 \end{bmatrix} = \begin{bmatrix} \overline{\rho}_s & \overline{\rho}_{sg} \\ \overline{\rho}_{sg} & \overline{\rho}_g \end{bmatrix},$$

where $z_{i,t} = \frac{\varepsilon_{i,t}}{\sqrt{h_{i,t}}}$, Q_t is a 2×2 symmetric positive definite matrix.

where $z_{i,t} = \frac{\varepsilon_{i,t}}{\sqrt{h_{i,t}}}$, Q_t is a 2×2 symmetric positive definite matrix.

$$\begin{aligned} q_{s,t} &= \overline{\rho}_s(1-\alpha-\beta) + \alpha(z_{s,t-1}z_{s,t-1}) + \beta(q_{s,t-1}), \\ q_{g,t} &= \overline{\rho}_g(1-\alpha-\beta) + \alpha(z_{g,t-1}z_{g,t-1}) + \beta(q_{g,t-1}), \\ q_{sg,t} &= \overline{\rho}_{sg}(1-\alpha-\beta) + \alpha(z_{sg,t-1}z_{sg,t-1}) + \beta(q_{sg,t-1}). \end{aligned}$$

The conditional correlation R_t at time t is defined as follows:

$$\rho_{sg,t} = \frac{q_{sg,t}}{\sqrt{q_{s,t}q_{g,t}}}. \tag{8}$$

According to Engel (2002), we estimate the model with a two-step maximum likelihood method using the following equation:

$$L = -\frac{1}{2}\sum_{t=1}^{T}(2log(2\pi) + 2log|D_t| + log|R_t| + z_t' R_t^{-1} z_t).$$

However, Dong and Yoon (2018) revealed that the estimation of the DCC-GARCH model changes if we introduce a structural break date in the model. For this reason, we propose an alternative to estimate the time-varying conditional correlation matrix without being affected by a structural change or other events.

3.2. Frequency approach: Evolutionary spectral analysis

The spectral approach presented by Priestley (1965, 1996) is the reference for frequency analysis because it respects the properties of the ideal spectrum, namely unicity, positivity, and the estimate from one realization (Loynes, 1968). The frequency-domain approach improves our analysis in six ways. First, it does not depend on any particular detrending technique. Second, we do not have an "end-point problem"; no future information is used, implied, or required as in band-pass or trend projection methods. Third, there is no deletion of short or long cycles, so their importance relative to cycle frequencies remains clear. Fourth, the coherence measure generalizes on simple correlation or concordance measures. Fifth, this approach can be applied to stationary or non-stationary processes. Finally, it performs our time-varying analysis at different frequencies simultaneously. This allows us to separate the different dynamic components of the co-movement.

We denote $\{X_t\}$ the observable time series. The evolutionary aspect of the spectrum is related to the non-stationarity in this series which follows an oscillatory process.

$$X_t = \int_{-\pi}^{\pi} A_X(w,t)e^{iwt}dZ_X(w), \tag{10}$$

where for each w, the sequence $\{A_X(w,t)\}$, as function of t admits a maximum Fourier transform (in module) in zero with $\{Z_X(w)\}$ an orthogonal process on$[-\pi,\pi]$, $E[dZ_X(w)] = 0$, $E[|dZ_X(w)|^2] = d\mu_X(w)$ and $\mu_X(w)$ a measure. The evolutionary spectral density of $\{X_t\}$ is the function $S_X(w,t)$ defined as follows:

$$S_X(w,t) = \frac{dH_X(w,t)}{dw}, -\pi \leq w \leq \pi, \tag{11}$$

where $dH_X(w,t) = |A_X(w,t)|^2 d\mu_X(w)$. The variance of $\{X_t\}$ at time t is given by:

$$\sigma^2_{X,t} = Var(X_t) = \int_{-\pi}^{\pi} S_X(w,t)dw \tag{12}$$

3.2.1. Presentation of the coherence function

Coherence is interpreted as the linear squared correlation coefficient for each frequency of the spectra of two series. The time approach gives the instantaneous coherent peaks between two series and describes their patterns over time. Therefore, for our case, it is crucial to know whether coherence has been increased or not between the cycles of some assets, so we can conclude if the assets are suitable to be included in the portfolio.

In the frequency domain, we define the correlation between two components in frequency as the coherence (K). We consider two zero-mean stochastic processes (X_t; Y_t), and let $S_X(w)$ and $S_Y(w)$, $-\pi \leq w < \pi$, be the spectral density functions and $S_{XY}(w)$ be the cospectrum. Each of the two processes can be written as:

$$X_t = \int_{-\pi}^{\pi} A_X(w)e^{iwt}dZ_X(w), \qquad Y_t = \int_{-\pi}^{\pi} A_Y(w)e^{iwt}dZ_Y(w). \tag{13}$$

where

$$\begin{aligned} E[dZ_X(w_1)\overline{dZ_Y(w_2)}] &= 0 \qquad w_1 \neq w_2 \\ &= S_{XY}(w)dw, \qquad w_1 = w_2 = w. \end{aligned} \tag{14}$$

where $\overline{Z}$ denotes the complex conjugate of Z. It is well-known that

$$S_{XY}(w) = C_{XY}(w) - iQ_{XY}(w), \tag{15}$$

where

$$\begin{aligned} C_{XY}(w) &= \Re\{S_{XY}(w)\} \ and \\ Q_{XY}(w) &= -\Im\{S_{XY}(w)\} \end{aligned} \tag{16}$$

are the Real Cospectrum (the gain) and the Quadrature Spectrum (the phase) respectively. $\Re$ and $\Im$ are the real and the imaginary parts of the cross-spectrum.

The coherence $\mathcal{K}_{XY}(w)$ at frequency w is given by

$$\mathcal{K}^2_{XY}(w) = \frac{C^2_{XY}(w) + Q^2_{XY}(w)}{S_X(w)S_Y(w)}, \tag{17}$$

Eq(15) verifies this inequality coherence, $C^2_{XY}(w)+Q^2_{XY}(w) \leq S_X(w)S_Y(w)$. Therefore $\mathcal{K}^2_{XY}(w)$ cannot be greater than one. This statistic shows the degree of co-movement between two series at the frequency w and it is analogous to the coefficient of the correlation between the two samples in the time domain. The coherence measure provides more detailed information than the conventional correlation and concordance measures.

3.2.2. A Time-Varying Coherence Function (TVCF)

In the time domain, a dynamic Pearson correlation can give us some crucial responses to this issue. However, the choice of window can seriously affect the dynamic correlation pattern (Essaadi et al., 2009). To overcome this limitation, we propose to measure the co-movement variability by the frequency approach. The time-varying coherence function estimates not only a degree of co-movement over time, but also the behavior of its co-movement in each frequency. Through the TVCF, we can calculate the relationship between two financial data series and their time-varying changes at each frequency. Our goal is to locate the estimated time-varying variance-covariance statistic of the series to estimate the optimal dynamic hedge ratio.

We introduce a new method to estimate the TVCF for financial portfolio design. We aim to present a new portfolio composition that outperforms the DCC-GARCH proposition by reducing risk and/or increasing returns.

The TVCF is a valuable tool for investigating dynamic interdependence problems and reviewing the short- and long-run dynamic properties of multiple time series. This time-varying relationship will conclude whether we should change our position by investing in financial assets or keep the money in a storage investment.

Priestley (1988) extends the theory of evolutionary spectra to the case of bivariate non-stationary processes. Consider two oscillatory component processes, $(X_t; Y_t)$: we can write

$$X_t = \int_{-\pi}^{\pi} A_X(w,t)e^{iwt}dZ_X(w), \qquad Y_t = \int_{-\pi}^{\pi} A_Y(w,t)e^{iwt}dZ_Y(w). \tag{18}$$

Where

$$
\begin{aligned}
E[dZ_X(w_1)\overline{dZ_X(w_2)}] &= E[dZ_Y(w_1)\overline{dZ_Y(w_2)}] \\
&= E[dZ_X(w_1)\overline{dZ_Y(w_2)}] = 0 \quad when \;\; w_1 \neq w_2. \\
E[|dZ_X(w)|^2] &= d\mu_X(w), \;\; E[|dZ_Y(w)|^2] = d\mu_Y(w) \;\; and \\
E[dZ_X(w)\overline{dZ_Y(w)}] &= d\mu_{XY}(w). \qquad (19)
\end{aligned}
$$

According to Priestley (1988), in the non-stationary case, the cross-spectrum is time-varying and defined as $dH_{XY}(w, t)$. By virtue of the Cauchy-Schwarz inequality, we have

$$
|dH_{XY}(w,t)|^2 \leq dH_X(w,t)dH_Y(w,t), \quad for\ all\ t\ and\ w. \qquad (20)
$$

We can write

$$
dH_{XY}(w,t) = S_{XY}(w,t)\ dw, \qquad (21)
$$

where $S_{XY}(w, t)$ is the evolutionary cross-spectrum density function.

$$
S_{XY}(w_j,t) = C_{XY}(w_j,t) - iQ_{XY}(w_j,t), \qquad (22)
$$

where

$$
\begin{aligned}
C_{XY}(w_j,t) &= \Re\{S_{XY}(w_j,t)\}\ and \\
Q_{XY}(w_j,t) &= -\Im\{S_{XY}(w_j,t)\} \qquad (23)
\end{aligned}
$$

are the Real Time-Varying Co-spectrum (the gain) and the Time-Varying Quadrature Spectrum (the phase) respectively. $\Re$ and $\Im$ are the real and the imaginary parts of the time-varying cross-spectrum.

3.2.3. Estimate of the evolutionary spectrum and cross-spectra

Estimation of $S_X(w,t)$ is performed by use of two windows $\{g_u\}$ and $\{w_v\}$

$$
\widehat{S_X}(w,t) = \sum_{v\in Z} w_v |U_{t-v}(w)|^2, \qquad (24)
$$

where $U_t(w) = \sum_{u\in Z} g_u X_{t-u} e^{-iw(t-u)}$. We choose $\{g_u\}$ and $\{w_r\}$ as follows:

$$
g_u = \left\{ \begin{array}{cc} 1/(2\sqrt{h\pi}) & if\ |u| \leq h \\ 0 & if\ |u| > h \end{array} \right| \qquad (25)
$$

$$
w_v = \left\{ \begin{array}{cc} 1/T' & if\ |v| \leq T'/2 \\ 0 & if\ |v| > T'/2 \end{array} \right| \qquad (26)
$$

Here $h = 7$ and $T' = 100$. According to Priestley (1988) we have $E(\widehat{S_X}(w)) \approx S_X(w,t)$, $var(\widehat{S_X}(w))$ decreases when T' increases and $\forall(t_1,t_2), \forall(w_1,w_2)$, $cov(\widehat{S_X}(w_1,t_1), \widehat{S_X}(w_2,t_2)) \approx 0$. If one of the two conditions (j) and (jj) is satisfied.

$$(j) \quad |t_1 - t_2| \geq T', \qquad (jj) \quad |w_1 \pm w_2| \geq \pi/h \tag{27}$$

Let $S_X^{iw} = log(S_X(w,t_i))$ and $\Lambda_X^{iw} = log(\widehat{S_X}(w,t_i))$. From Priestley (1988), we have:

$$\Lambda_X^{iw} \approx S_X^{iw} + e_X^{iw}, \tag{28}$$

where the sequence $\{e_X^{iw}\}$ is approximately normal, uncorrelated and identically distributed.

Following Priestley (1965, 1996), in an evolutionary spectral theory, Essaadi and Boutahar (2010) propose a time-varying cross-spectrum estimator using spectral density. Estimation of $S_{XY}(w,t)$ is obtained by use of the 'double window technique' $\{g_u\}$ and $\{w_v\}$ given in (25) and (26).

$$\widehat{S_{XY}}(w,t) = \sum_{v \in Z} w_v U_X(w, t-v) U_Y(w, t-v), \tag{29}$$

where $U_X(w,t) = \sum_{u \in Z} g_u X_{t-u} e^{-iw(t-u)}$ and $U_Y(w,t) = \sum_{u \in Z} g_u Y_{t-u} e^{-iw(t-u)}$.

Here $h = 7$ and $T' = 100$. According to Priestley (1988), we have $E(\widehat{S_{XY}}(w)) \approx S_{XY}(w,t)$, $var(\widehat{S_{XY}}(w))$ decreases when T' increases and $\forall(t_1,t_2), \forall(w_1,w_2)$, $cov(\widehat{S_{XY}}(w_1,t_1), \widehat{S_{XY}}(w_2,t_2)) \approx 0$, if one of the two conditions (j) and (jj) of (27) is satisfied.

To respect the (j) and (jj) conditions we choose $\{t_i\}$ and $\{w_j\}$ as follows:

$$\{t_i = 100i\}_{i=1}^{I} \;\; where \;\; I = [\frac{T}{100}] \;\; and \;\; T \;\; the \;\; sample \;\; size, \tag{30}$$

$[x]$ denotes the integer part of x.

$$\{w_j = \frac{\pi}{20}(1 + 3(j-1))\}_{j=1}^{7}. \tag{31}$$

Following (jj) condition, we inspect instability in these frequencies: $\pi/20$, $4\pi/20$, $7\pi/20$, $10\pi/20$, $13\pi/20$, $16\pi/20$ and $19\pi/20$.

3.3. Variance reduction hedging strategy

The most optimal hedging portfolio strategy is to minimize variance by an optimal hedge ratio. We consider a portfolio composition that reduces risk without sacrificing return. We design our respective portfolios by calculating the weighting of the two assets as follows:

$$w_{sg,t} = \frac{h_{g,t} - h_{sg,t}}{h_{s,t} - 2h_{sg,t} + h_{g,t}}, \tag{32}$$

and

$$w_{sg,t} = \begin{cases} 0, & if \quad w_{sg,t} < 0 \\ w_{sg,t}, & if \quad 0 \leq w_{sg,t} \leq 1, \\ 1, & if \quad w_{sg,t} > 1. \end{cases} \tag{33}$$

Where $w_{sg,t}$ is the weight of the S asset in a one-dollar gold/metal portfolio at time t, $h_{sg,t}$ is the conditional covariance between the stock market index and gold, $h_{s,t}$ is the conditional variance of the stock market index and $h_{g,t}$ is the conditional variance of the gold. The weight of the gold in a dollar's portfolio is $1 - w_{sg,t}$. This strategy was adopted by several previous studies (Hammoudeh et al., 2010; Chkili, 2016) to examine conditional volatility and correlation dependency for major precious metals and to develop portfolio hedging strategies. In the same line, Khalfaoui et al. (2015) propose a portfolio composed of oil and stock market based on equations 32 and 33. To compare different portfolio propositions, we estimate a hedging effectiveness ratio. Following Chkili (2016), hedging effectiveness is defined by the reduction in the variance of the hedge portfolio compared to the unhedged one.

$$HE_i = (1 - \frac{Var_i(hedged)}{Var_i(unhedged)}) * 100, \tag{34}$$

where i denote country i. In our case, the unhedged portfolio corresponds to the one where the investment share in the stock market is 100 percent as $Var_i(unhedged) = Var_i(S)$[9].

4. Empirical study

4.1. Data presentation

We use gold prices in USD and a selection of stock market indices for eight countries of the MENA region:

- Tunindex (Tunisia) from 02/01/1998 to 30/03/2020 (5,360 observations)
- EGY 30 (Egypt) from 05/01/1998 to 30/03/2020 (4,290 observations)
- MI (Morocco) from 03/01/2002 to 30/03/2020 (4,442 observations)
- Amman (Jordan) from 11/01/2000 to 16/03/2020 (3,881 observations)
- Dubai (the UAE) from 05/01/2004 to 30/03/2020 (3,230 observations)
- Tadawul (Saudi Arabia) from 20/10/1998 to 30/03/2020 (3,856 observations)
- Doha index (Qatar) from 14/03/2001 to 30/03/2020 (3,781 observations)
- MSM 30 (Oman) from 21/12/2000 to 30/03/2020 (3,754 observations)

This study used daily data over the longest period available[10] and only common price observations between gold and each stock index are considered. The return rate of series

[9] We summarize in Tables 3, 4 and 5 the results of the dynamic hedge ratio performance for a portfolio composed by the stock market index and gold.

is calculated by the first difference of the logarithm of daily gold prices and stock indexes, as follows:

$$r_{i,t} = \Delta(log(X_{i,t})) = log(\frac{X_{i,t}}{X_{i,t-1}}), \quad (35)$$

where $r_{i,t}$ is returns series of i, $i = s$ for stock market index and $i = g$ for gold price in USD. X_t denotes the selected time series of gold price or stock indices. The descriptive statistics of the return series are presented in Table 1.

We restrict our analysis to two asset portfolios (gold-stock index). Note that for the need of an evolutionary spectral estimation (Figures 11-18), we lose 100 observations at the beginning. Therefore, we apply a hedging ratio to $T - 100$.

4.2. Descriptive analysis

The descriptive statistic of the return series (Table 1) gives various results. The gold series is considered a reference series and its mean return and standard deviation are relatively small (0.0325 and 0.011518, respectively), reflecting the stable properties of the underlying assets. Some countries offer higher average stock market returns than gold, such as Tunisia, Egypt, Saudi Arabia, and Qatar, at the expense of others. Thus, investing in these stock markets is more profitable than gold. However, the study of the return variable remains incomplete without the market risk measure. Under this second indicator, a higher standard deviation of the stock market index (of Egypt, Saudi Arabia, Qatar, Jordan, and Dubai) relative to gold is synonymous with price instability. These markets are volatile. Investing in these markets is profitable but remains risky compared to gold. These are the main MENA markets affected by several economic and financial crises. Contrary to Morocco, Oman, and Tunisia, which are small developing markets, they are less risky and more stable due to their diversified economy. The rejection of the normality test for all these series is verified through the Jarque-Bera statistic, which is far from the critical value. This brings us to the question of the effectiveness of gold as a hedging instrument for these markets.

4.3. Results

The graph of the evolutionary spectral density of TUNINDEX (Tunisia) (Figure 2) reveals an intense change in the short cycle in 2005 as the stock market achieved its second best double-digit performance (21 percent) since its creation. The second long cycle change occurred between late 2010 and early 2011, reflecting the Arab Spring revolution.

Qatar (Figure 8) and Oman (Figure 9) share a common graphical pattern illustrated by a change in late 2008 and early 2009, marking the contagion of the US subprime crisis. This

[10] Data retrieved from https://www.investing.com/indices/

crisis has affected both low (long- term) and high (short-term) frequencies. We find the same conclusion as well for Jordan. The end of 2005 and the beginning of 2006 constitute a second short-term date for the Jordanian stock market (Figure 5). The subprime crisis is revealed also in the spectral density pattern of gold (Figure 10),

thereby affecting both the short and long term. Nevertheless, the latest recession at the end of 2019 seems to only affect the short term for the time being.

For the Moroccan stock exchange (Figure 3), early 2006 and mid-2007, as well as the date of the last observations (end of 2019 and beginning of 2020), seem to affect the short- and long-term frequencies. These dates correspond to the main events that characterized the Moroccan market: the increase in the market capitalization to GDP ratio jumped from 23.8 percent in 2002 to 86.1 percent in 2006 following multiple initial public offerings (IPOs); the 2007 global financial crisis that damaged the prices of traded securities; and the global recession and COVID-19 effect at the end of 2019 and the beginning of 2020. These factors have an important short- and long-term effect on the Moroccan stock market. The three most volatile markets (Dubai, Egypt, and Saudi Arabia) have undergone frequent fluctuations and share the same dates of change in their spectral density (notably 2006 and 2008). For this period, the subprime crisis was the major event that severely hit these financial markets. Moreover, for the Dubai stock market, a remarkable growth of 150 percent in 2004 and 190 percent in 2005, followed by a correction of 24 percent in 2006, affected the whole Gulf region. Besides, in 2014, the MSCI[11] included the UAE (which includes Dubai) in the emerging markets category. The evolutionary spectral density shows both short- and long-term fluctuations for this date. For the Egyptian stock market, a second fluctuation appears in 2011. This period was critical for all of Egypt following the Arab Spring revolution. The market capitalization to GDP ratio fell from 72 percent in 2006 to 21 percent in 2011. The effect of the end of 2019 also appears in all these markets, especially for the low frequencies (in the long term).

Figures 11 to 18 show a common feature in the fluctuation of the co-movement between the stock market indexes and the price of gold, with an equal range between zero and a bit more than 0.6. Notably, we notice some differences between the co-movements in the short-, medium-, and long-term. Specifically, the short-term co-movement shows more peaks whereas the long-term co-movement is relatively more stable. Furthermore, we provide in Table 2 the Pearson correlation between gold and stock market returns. Gold is negatively correlated with Saudi Arabia, Jordan, and Qatar and positively correlated with the rest of the MENA countries. However, in all cases, the correlation coefficients are very low and too close to zero, which suggests potential diversification benefits.

[11] Morgan Stanley Capital International.

Once the dynamic variance-covariance is estimated using both the DCC-GARCH model and the spectral method, the weight for each asset is determined and portfolios are constructed (Equations 32 and 33). As the spectral method discloses seven series of consistency, we have seven alternative proposals, each one corresponding to a specific frequency. Then, the return of each portfolio will be computed as well as its variance (Table 3). For each stock market, the variances relating to portfolios composed in the presence of gold are lower than those relating to their stock market indices. Our conclusion indicates that a portfolio with gold can help reduce investors' risk. The analysis of Table 3 is based on a comparison of the (return, variance) couple between our seven portfolios estimated by the spectral analysis and the famous DCC-GARCH model. The model providing the largest stock return and the smallest standard deviation is considered the most effective in measuring gold coverage. The results are mixed. The main results prove that our approach announces lower variances for all countries than that proposed by the famous DCC-GARCH model (at least for a portfolio among the seven). However, by referring to these two indicators, the taken sample can be split into two subgroups. The first includes Tunisia, Oman, Qatar, Dubai, and Saudi Arabia. The results prove a total superiority of the spectral analysis in the construction of hedging portfolios compared to the DCC-GARCH model. With a higher return and lower volatility, choosing an optimal portfolio is obvious. The second includes Morocco, Egypt, and Jordan. For these three stock markets, the statistical results of the spectral analysis are inconclusive. Despite its superiority through the variance measure (lower variance), the return of constructed portfolios is also lower than that of the DCC-GARCH. Therefore, we cannot determine the superiority of our approach.

To do this, we used the hedging performance ratio through variance reduction, widely adopted in the literature by Hammoudeh et al. (2010) and Chkili (2016), among others (Equation 34, array 4). The mean value of w_{sg} of the seven portfolios built according to the spectral analysis and the DCC-GARCH model for each country is reported in Table 5. The table clearly shows the outperformance of our approach over the second. A total superiority of this ratio (for all portfolios) is marked for the stock markets in Tunisia, Dubai, and Jordan. For the rest, the hedging performance ratio of the built portfolios exceeds that of the DCC-GARCH model for at least one case. We could then decide about the superiority of the spectral analysis on the stock markets in Morocco, Egypt, and Jordan. Therefore, our new approach outperforms the old measure for all the selected MENA countries. We will focus, thereafter, on the w_{sg} optimal portfolio values for each country corresponding to the largest hedge ratio (Table 5) and the hedging performance (Table 4). The optimal gold weighting varies from 28.81 percent for Egypt (corresponding to the best hedging performance ratio of around 73.91) to 82.74 percent for Tunisia (corresponding to the best ratio hedging performance of around 56.09). These results coincide with the spectral density properties of each market. Countries with the most stable density (Tunisia, Oman, Morocco, and Jordan) have the highest gold share

(82.74 percent, 74.95 percent, 69.14 percent, and 70.09 percent, respectively) in the portfolios, thus minimizing the total variance. In countries with a widely fluctuating spectral density (Egypt and Dubai), the use of gold in their investment portfolio is the lowest (28.81 percent and 39.32 percent, respectively). Qatar and Saudi Arabia constitute their optimal portfolios with a relatively balanced composition between stocks and gold, with a gold average percentage in the order of 46.23 percent and 45.73 percent, respectively.

However, gold continues to be a reliable hedge, especially during the COVID-19 crisis. The optimal allocation between gold and stocks depends primarily on the stock market and its ability to absorb shocks. The results revealed that the more volatile the market and the more frequent occurrence of adverse circumstances, the more investors switch to gold. Therefore, gold is a good tool to hedge against the risk of a crash in stock prices for stable markets with a low spectral density.

5. Conclusion

In this paper, we introduce a new method for estimating the hedge ratio. We propose seven portfolio structures based on spectral analysis and then evaluate them through a comparison with the classical DCC-GARCH method. As such, we distinguish the properties of hedging cycles for high and low frequencies. By applying the frequency method to some countries of the MENA region, we demonstrate its ability to reduce the estimated risk by the coverage efficiency ratio.

This work led to several empirical results:

- By using the DCC-GARCH model and the spectral analysis, both methods reduce variance compared to the unhedged portfolio. Gold always keeps its hedging characteristics.
- By using the hedge effectiveness ratio based on variance reduction, the spectral analysis outperforms the DCC-GARCH model in the construction of hedge portfolios.
- The use of gold in investment portfolios can reduce the variance by more than 70 percent for the stock market index of Egypt, Dubai, Jordan, and Saudi Arabia, and by at least 35 percent of the rest of the countries studied, especially for the newly suggested methodology.
- The optimal composition between gold and stocks depends essentially on the spectral density of each market. A stock market index with a stable spectral density requires more investments in gold than a stock market index with an unstable spectral density.

Future research can consider a more detailed comparison with recent frequency-based techniques, including wavelets, to refine the portfolio implications. Another extension can cover emerging countries from other regions, such as South America, Europe and the Asia-Pacific.

References

Baruník, J., E. Kočenda, and L. Vácha (2016). Gold, oil, and stocks: Dynamic correlations. *International Review of Economics & Finance 42*, 186–201.

Basher, S. A. and P. Sadorsky (2016). Hedging emerging market stock prices with oil, gold, vix, and bonds: A comparison between dcc, adcc and go-garch. *Energy Economics 54*, 235–247.

Baumöhl, E. and Š. Lyócsa (2017). Directional predictability from stock market sector indices to gold: A cross-quantilogram analysis. *Finance Research Letters 23*, 152–164.

Baur, D. G. and T. K. McDermott (2010). Is gold a safe haven? international evidence. *Journal of Banking & Finance 34*(8), 1886–1898.

Bhatia, V., D. Das, and S. B. Kumar (2020). Hedging effectiveness of precious metals across frequencies: Evidence from wavelet based dynamic conditional correlation analysis. *Physica A: Statistical Mechanics and its Applications 541*, 123631.

Bouri, E., A. Jain, P. Biswal, and D. Roubaud (2017). Cointegration and nonlinear causality amongst gold, oil, and the Indian stock market: Evidence from implied volatility indices. *Resources Policy 52*, 201–206.

Bouri, E., D. Roubaud, R. Jammazi, and A. Assaf (2017). Uncovering frequency domain causality between gold and the stock markets of China and India: Evidence from implied volatility indices. *Finance Research Letters 23*, 23–30.

Bulut, L. and I. Rizvanoghlu (2020). Is gold a safe haven? The international evidence revisited. *Acta Oeconomica 70*(4), 531–549.

Chkili, W. (2016). Dynamic correlations and hedging effectiveness between gold and stock markets: Evidence for BRICS countries. *Research in International Business and Finance 38*, 22–34.

Ciner, C., C. Gurdgiev, and B. M. Lucey (2013). Hedges and safe havens: An examination of stocks, bonds, gold, oil and exchange rates. *International Review of Financial Analysis 29*, 202–211.

Creti, A., M. Joëts, and V. Mignon (2013). On the links between stock and commodity markets' volatility. *Energy Economics 37*, 16–28.

Dong, M. C., C. W. Chen, S. Lee, and S. Sriboonchitta (2019). How strong is the relationship among gold and USD exchange rates? analytics based on structural change models. *Computational Economics 53*(1), 343–366.

Dong, X. and S.-M. Yoon (2018). Structural breaks, dynamic correlations, and hedge and safe havens for stock and foreign exchange markets in greater China. *The World Economy 41*(10), 2783–2803.

Engel, R. (2002). Dynamic conditional correlation-a simple class of multivariate garch models. *Journal of Business and Economic Statistics 20*(3), 339–350.

Essaadi, E. and M. Boutahar (2010). A measure of variability in comovement for economic variables: a time-varying coherence function approach. *Economics Bulletin 30*(2), 1054–1070.

Essaadi, E., J. Jouini, and W. Khallouli (2009). The Asian crisis contagion: A dynamic correlation approach analysis. *Panoeconomicus 56*(2), 241–260.

Ftiti, Z., A. Tiwari, A. Belanès, and K. Guesmi (2015). Tests of financial market contagion: Evolutionary cospectral analysis versus wavelet analysis. *Computational Economics 46*(4), 575–611.

Ghosh, I., M. K. Sanyal, and R. Jana (2021). Co-movement and dynamic correlation of financial and energy markets: An integrated framework of nonlinear dynamics, wavelet analysis and dcc-garch. *Computational Economics 57*(2), 503–527.

Gokmenoglu, K. K. and N. Fazlollahi (2015). The interactions among gold, oil, and stock market: Evidence from S&P 500. *Procedia Economics and Finance 25* (Supp. C), 478–488.

Hammoudeh, S. M., Y. Yuan, M. McAleer, and M. A. Thompson (2010). Precious metals–exchange rate volatility transmissions and hedging strategies. *International Review of Economics & Finance 19*(4), 633–647.

Iqbal, J. (2017). Does gold hedge stock market, inflation and exchange rate risks? an econometric investigation. *International Review of Economics & Finance 48*, 1–17.

Khalfaoui, R., M. Boutahar, and H. Boubaker (2015). Analyzing volatility spillovers and hedging between oil and stock markets: Evidence from wavelet analysis. *Energy Economics 49*, 540–549.

Ku, Y. H. H., H. C. Chen, and K.H. Chen (2007). On the application of the dynamic conditional correlation model in estimating optimal time-varying hedge ratios. *Applied Economics Letters 14*(7), 503–509.

Kumar, D. (2014). Return and volatility transmission between gold and stock sectors: Application of portfolio management and hedging effectiveness. *IIMB Management Review 26*(1), 5–16.

Loynes, R. M. (1968). On the concept of the spectrum for non-stationary processes. *Journal of the Royal Statistical Society: Series B (Methodological) 30*(1), 1–20.

Priestley, M. (1965). Evolutionary spectra and non-stationary processes. *Journal of the Royal Statistical Society. Series B (Methodological) 27*(2), 204–237.

Priestley, M. (1996). Wavelets and time-dependent spectral analysis. *Journal of Time Series Analysis 17*(1), 85–103.

Priestley, M. B. (1988). *Non-linear and non-stationary time series analysis*, Volume 223. Academic Press London.

Raza, N., S. J. H. Shahzad, A. K. Tiwari, and M. Shahbaz (2016). Asymmetric impact of gold, oil prices and their volatilities on stock prices of emerging markets. *Resources Policy 49*, 290–301.

Tiwari, A. K., A. O. Adewuyi, and D. Roubaud (2019). Dependence between the global gold market and emerging stock markets (e7+ 1): Evidence from granger causality using quantile and quantile-on-quantile regression methods. *The World Economy 42*(7), 2172–2214.

Zhou, W. (2017). Dynamic and asymmetric contagion reactions of financial markets during the last subprime crisis. *Computational Economics 50*(2), 207–230.

Ziaei, S. M. (2012). Effects of gold price on equity, bond and domestic credit: Evidence from asean+ 3. *Procedia-Social and Behavioral Sciences 40*, 341–346.

Appendices

A. Figures

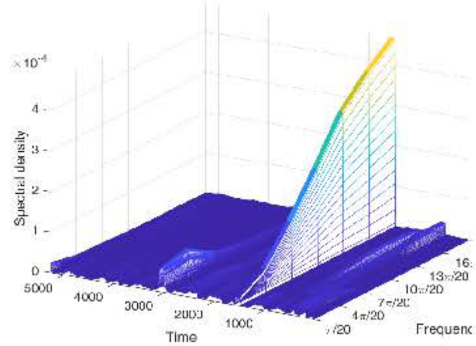


Figure 2: Evolutionary spectral density of Tunisian Stock returns

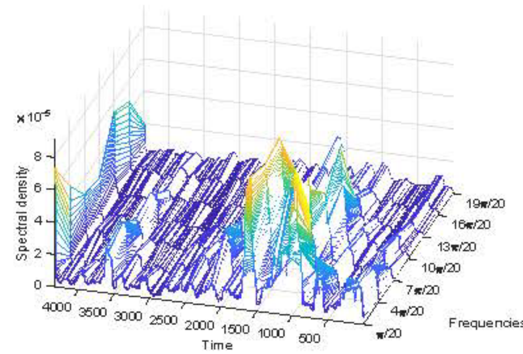


Figure 3: Evolutionary spectral density of Morocco Stock returns

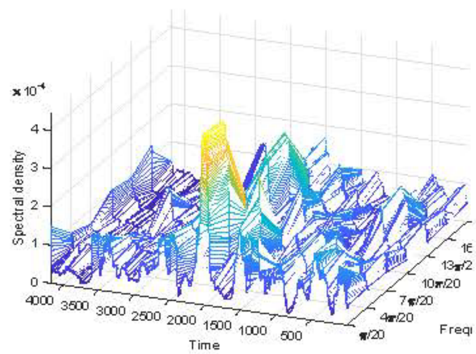


Figure 4: Evolutionary spectral density of Egypt Stock returns

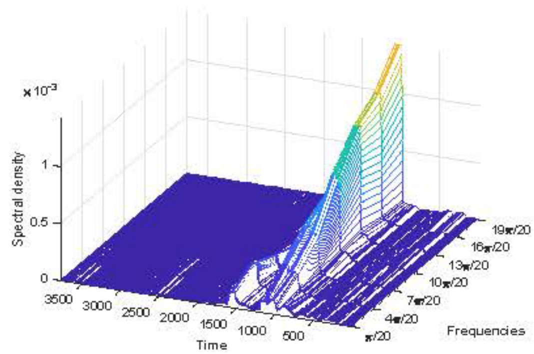


Figure 5: Evolutionary spectral density of Jordan Stock returns

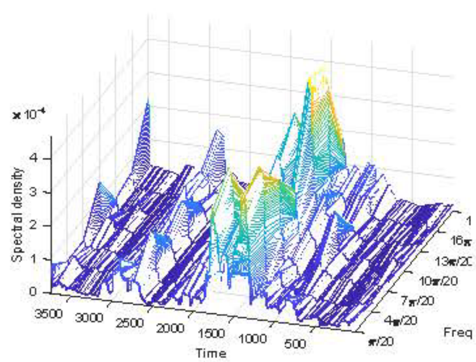


Figure 6: Evolutionary spectral density of Saudi Arabi Stock returns

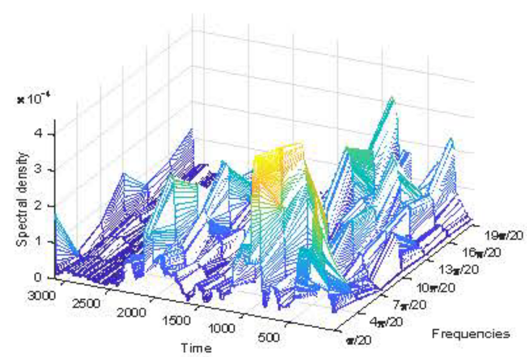
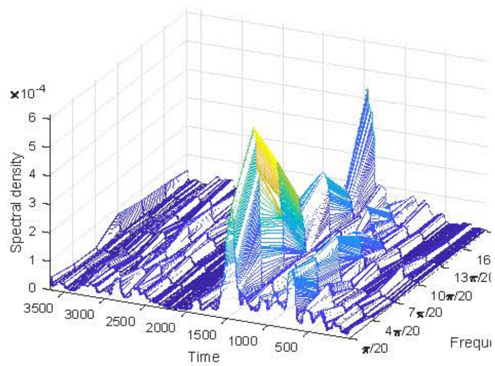


Figure 7: Evolutionary spectral density of Dubai (UAE) Stock returns



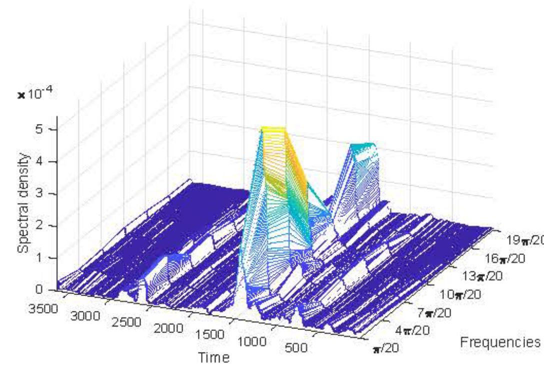
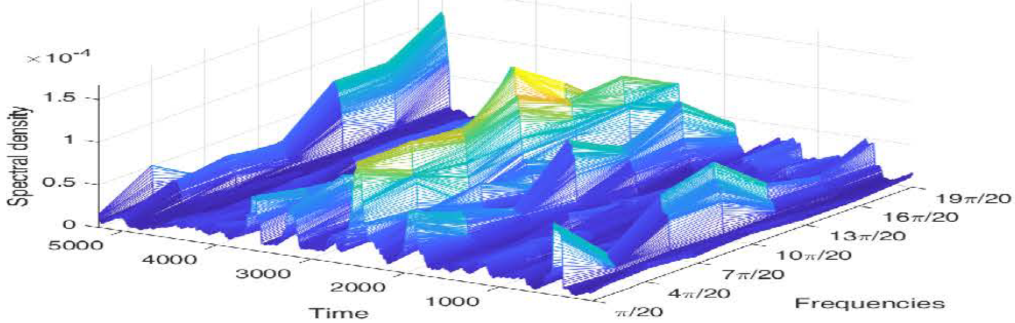
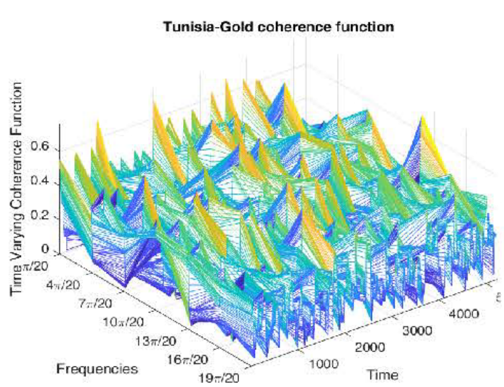


Figure 8: Evolutionary spectral density of Qatar Stock returns

Figure 9: Evolutionary spectral density of Oman Stock returns

Figure 10: Evolutionary spectral density of gold returns





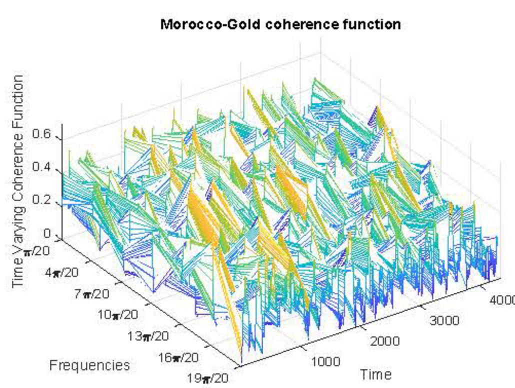


Figure 11: Tunisian Stock and Gold prices coherence dynamics

Figure 12: Morocco Stock and Gold prices coherence dynamics

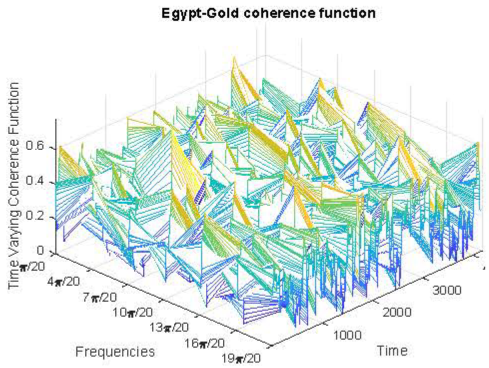


Figure 13: Egytian Stock and Gold prices coherence dynamics

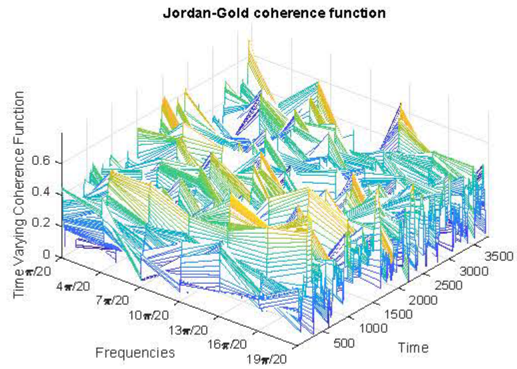


Figure 14: Jordan Stock and Gold prices coherence dynamics

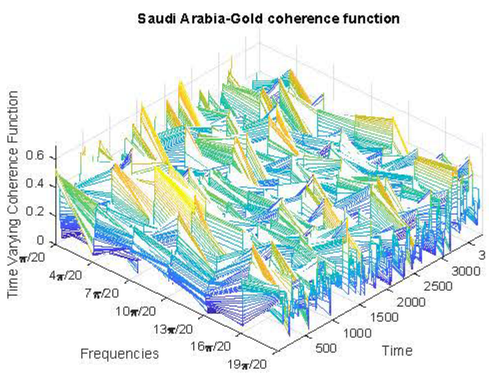


Figure 15: Saudi Arabi Stock and Gold prices coherence dynamics

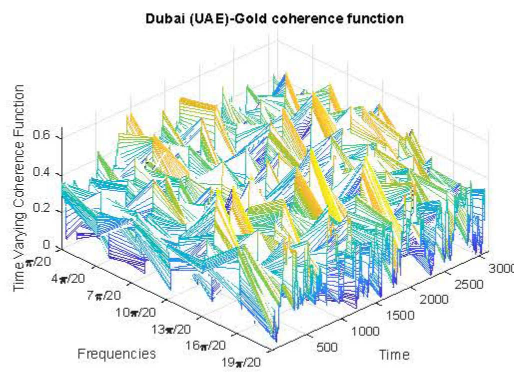


Figure 16: Dubai (UAE) Stock and Gold prices coherence dynamics

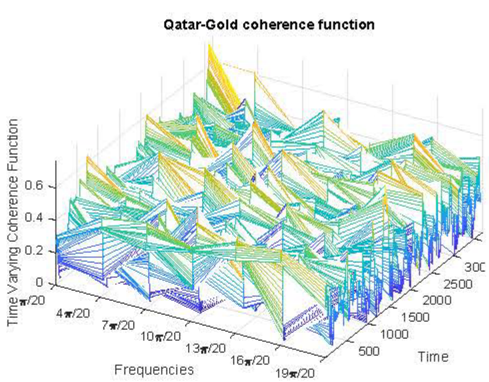


Figure 17: Qatar Stock and Gold prices coherence dynamics

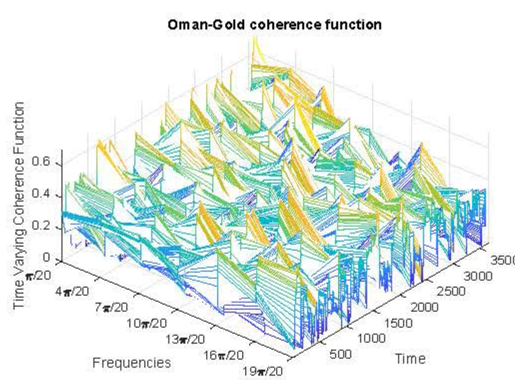


Figure 18: Oman Stock and Gold prices coherence dynamics

B. Tables

Table 1. Descriptive statistic of return series

	Gold	Tunisia	Morocco	Egypt	Jordan	Saudi Arabia	Dubai	Qatar
Mean (%)	0.0325	0.0348	0.0223	0.0526	0.0137	0.0377	0.0176	0.0513
Median (%)	0.0290	0.0186	0.0320	0.0944	0.0208	0.1152	0.0572	0.0661
Maximum (%)	8.8902	26.5408	5.3054	11.1799	43.8263	16.5902	13.1420	11.0893
Minimum (%)	-9.8206	-26.6943	-9.2317	-17.9916	-41.2833	-20.9680	-16.8532	-13.6797
Std. Dev.	0.011518	0.007687	0.007870	0.019399	0.015418	0.017165	0.019346	0.015470
Skewness	0.001321	-0.462192	-0.858326	-0.588632	1.011545	-1.522374	-0.800418	-0.851283
Kurtosis	9.277214	544.7939	15.34535	10.28216	372.3867	22.01469	13.19301	16.19314
J-B	8798.456	65545368	28747.06	9724.563	22053836	59564.41	14323.33	27870.82
Prob.	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Obs.	5359	5359	4441	4289	3879	3855	3229	3780

Note: The critical value of the Jarque-Bera (J-B) test at the five percent level is 5.99.

Table 2. Correlation between gold and stock market returns

	Pearson correlation with Gold
Tunis	0.0041
Morocc	0.0088
Egy	0.0180
Qat	-0.0014
Saudi	-0.0500
Dubai	0.0049
Jorda	-0.0344
Oma	0.0241

Table 3: Mean and Variance of portfolio (Spectral and DCC-GARCH approach) ($\times 10^{-4}$)

		$P_{\pi/20}$	$P_{4\pi/20}$	$P_{7\pi/20}$	$P_{10\pi/20}$	$P_{13\pi/20}$	$P_{16\pi/20}$	$P_{19\pi/20}$	$P_{DCC-GARCH}$
Tunisia	r_t	3,2886	3,6565	3,5256	3,0960	3,2555	3,1212	3,2523	3,3787
	var	0,3459	0,2804	0,2595	0,2605	0,2635	0,2622	0,2677	0,3814
Morocco	r_t	2,5633	2,3316	2,4556	2,2452	2,3589	2,5463	2,5493	2,6252
	var	0,3920	0,3710	0,3689	0,3728	0,3723	0,3869	0,3888	0,3697
Egypt	r_t	3,7390	3,8878	3,1733	3,5157	3,6479	3,6194	3,4291	4,0537
	var	1,0579	1,0577	1,0043	0,9820	0,9911	0,9912	1,1034	0,9881
Oman	r_t	2,5689	3,0277	2,8086	2,5772	1,7490	2,0709	1,9528	2,4883
	var	0,5655	0,5861	0,5793	0,5371	0,6015	0,5708	0,5949	0,5827
Jordan	r_t	1,6525	1,9818	2,1761	1,7012	1,9711	1,9367	1,7578	2,1962
	var	0,6774	0,5853	0,5394	0,5636	0,5530	0,5712	0,5950	0,6799
Qatar	r_t	4,2819	4,3268	4,4206	5,0541	4,3285	5,0821	4,6451	4,7944
	var	0,8117	0,7798	0,7910	0,7146	0,7696	0,7424	0,8291	0,8128
Dubai	r_t	2,7628	2,6558	2,4535	2,0254	2,1013	2,4964	1,6440	2,5359
	var	0,9089	0,8482	0,8835	0,8607	0,8498	0,8267	0,8860	0,9249
Saudi	r_t	4,8288	4,5892	4,4861	3,9741	4,5431	4,1550	3,1018	4,4503
Arabia	var	0,9065	0,8430	0,8236	0,8527	0,8006	0,8685	0,9117	0,8836

Table 4: Hedging performance in terms of variance reduction

	Tunisia	Morocco	Egypt	Oman	Qatar	Dubai	Saudi Arabia	Jordan
$P_{\pi/20}$	41,46	36,71	71,89	50,75	66,09	75,72	69,23	71,51
$P_{4\pi/20}$	52,56	40,10	71,89	48,96	67,42	77,34	71,39	75,38
$P_{7\pi/20}$	56,09	40,43	73,31	49,55	66,95	76,40	72,05	77,31
$P_{10\pi/20}$	55,92	39,81	73,91	53,22	70,14	77,00	71,06	76,29
$P_{13\pi/20}$	55,42	39,89	73,66	47,61	67,84	77,29	72,83	76,74
$P_{16\pi/20}$	55,63	37,53	73,66	50,29	68,98	77,91	70,52	75,97
$P_{19\pi/20}$	54,70	37,23	70,68	48,19	65,36	76,33	69,06	74,97
$P_{DCC-GARCH}$	35,45	40,31	73,74	49,25	66,04	75,29	70,01	71,40

Table 5: Mean weight level in percentage

	Tunisia	Morocco	Egypt	Oman	Qatar	Dubai	Saudi Arabia	Jordan
$P_{\pi/20}$	25,53	36,08	82,03	42,36	64,56	74,19	66,00	35,71
$P_{4\pi/20}$	22,77	32,23	83,77	34,04	60,17	67,73	60,68	36,34
$P_{7\pi/20}$	17,26	30,86	75,04	28,61	55,05	62,60	54,28	29,91
$P_{10\pi/20}$	15,30	21,91	71,19	25,05	53,77	63,74	55,82	29,58
$P_{13\pi/20}$	13,79	16,62	71,72	22,72	46,50	63,88	54,27	27,98
$P_{16\pi/20}$	13,05	16,64	64,71	21,05	45,29	60,68	48,34	24,90
$P_{19\pi/20}$	11,83	15,23	62,63	19,63	45,09	57,62	50,28	25,67
$P_{DCC-GARCH}$	27,33	28,77	67,49	30,01	50,39	61,08	52,26	33,34