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Abstract

Variety of models and estimation techniques have been proposed for electricity price forecasting in the literature. We contribute by introducing a dynamic forecasting model for hourly electricity prices based on nonlinear excess demand specification. Our modelling framework depends on the neoclassical price adjustment equation that necessitates prices adjust toward equilibrium at a rate that is proportional to the excess demand. We approximate the adjustment as a nonlinear (cubic) function of the excess demand which itself is modeled as a latent factor. We show that nonlinear relation of excess demand and price leads to a more accurate description of price evolution toward equilibrium and with this framework, the equilibrium forecast for the price is given by a nonlinear equation of the excess demand that can be modeled as a function of important variables of supply and demand. This generates an advantage to forecasters in employing all the information on supply and demand functions in price pre-diction, rather than simply modelling the price on an ad-hoc manner. We further develop a maximum likelihood estimator with excess demand defined as a normally distributed random variable conditional on observables. We demonstrate our likelihood estimator by using data from Turkish electricity market. Our modelling framework implies time varying volatility for prices which along with the nonlinear mean function, brings two important features of time series modelling dynamics together in parsimonious model.

Keywords: Electricity price, Energy forecasting, Excess Demand, Nonlinear Modelling.
JEL Classifications: C01, C13, C51, C53, Q47.

1 Introduction

The last few decades have witnessed substantial deregulation practices and transformation processes in electricity markets. Traditionally, these markets have been monopolistic government-controlled industries and the price was set without much referencing to the market conditions. The recent transformation is reshaping the market structure all over the world into a more market-based environment, by introducing competitive market applications. What lies behind, for this overhaul of electricity markets through deregulated new structure, is the agenda to provide long run competition and efficiency gains and yet to establish a sustainable energy infrastructure. As a result, worldwide we observe that electricity now trades under market rules with significant use of spot markets and contracts.

Electricity market has unique characteristics that make electricity distinct from other traded commodities. Lack of satisfactory storage technologies, stringent requirement of meeting the demand during the day and finally seasonality at various frequencies are three dominant characteristics of electricity markets (Weron (2006), Weron (2014)). Firstly, electricity is economically non-storable. Unlike many other commodities or goods, substantial storage of electricity is not easy and generally not affordable. This is fundamentally different from many other consumption goods, i.e. food, clothes, furniture, gas, medicines, etc., which can be stored for later consumption. That, of course, does not mean that there is no available technology to store electrical energy. In fact, the technological development on that front have increased the performance and storage capacity of the battery systems in recent years (Baker (2008)). Yet, the amount of electrical energy that can be stored by this kind of system is limited, and it remains as a fact that, at large, electricity must be consumed as it is being produced. Secondly, power system stability requires a constant balance between production and consumption of electricity (Kaminski (2012), Shahidehpour et al. (2002)). Also, electricity demand like other energy markets exhibits seasonality. Most commonly monthly, weekly, daily, and hourly seasonality can be observed. Electricity demand strongly relies on weather conditions, i.e. temperature changes, the business congestion and daily routine practices like on-peak or off-peak hours, and weekdays or weekends (Koopman et al. (2007)). Therefore, these characteristics should be considered by researchers in the development of forecasting methods for the electricity prices (Weron (2014)).

Forecasting electricity prices is an important part of many economic planning activities, including long-term and short-term objectives. Spot price and day-ahead price prediction falls into short term objectives and more related to hedging concerns, primarily to be proactive against price volatility which can be substantial in the electricity market (Misiorek et al. (2006)). Some part of the volatile nature of electricity market is mechanical as such real time balancing of electrical supply and demand is an integral part of a market-based system and therefore prices will fluctuate compared to a fixed pricing system. However, this brings extra modeling issues for price forecasting [1]. Understanding the factors that affect the supply and demand balance becomes crucial as they play an important role in price volatility, and thus that can be reflected in the electricity prices through uncertainty. Such factors include but not limited to power station interruption, imperfect transmission grid reliability, weather conditions changes, related commodity price changes including fuel prices (Weron (2013), Aggarwal et al. (2009b)). Besides, strategic interactions emerge as another important factor, especially when considering the number of participants as suppliers and distributors with differential roles in the supply-chain of the electricity market. This makes forecasting of short-term electricity price critical for all participants in the market.

Different types of models and estimation techniques have been suggested for electricity prices in the literature. Weron (2014) offers a recent survey on various methods proposed in short term price prediction in the literature among others [2]. Following the taxonomy in Weron (2006) and Weron (2014), those can be classified into five main groups of models as; multi-agent simulation, equilibrium, game theoretic models [3]; structural models [4]; reduced form models [5]; statistical/econometric models [6] and artificial intelligence-based, non-parametric, non-linear statistical models [7]. Obviously, many models developed in the literature falls into multiple categories as they are hybrid solutions, combining techniques from two or more of the groups (Aggarwal et al. (2009a), Weron and Misiorek (2005), Weron (2006)). In this paper, we contribute to the litera-

[1] Garcia et al. (2005), Bowden and Payne (2008), Gianfreda and Grossi (2012), Hickey et al. (2012) and Bowden and Payne (2008) used GARCH type conditional volatility modelling to predict spot electricity prices; among others.

[2] For other recent review articles that fully or partially addressing the various models used in the literature, see Bunn (2000), Batlle and Barquin (2005), Ventosa et al. (2005), Amjady (2006), Aggarwal et al. (2009a), Aggarwal et al. (2009b). Also Conejo et al. (2005) reviews forecasting techniques for 24 hourly prices and compares their forecasting performance.

[3] Papers employed ANN (Artificial Neural Network) models for spot electricity price forecasting include, Pao (2007) for Leipzig Power Market and Catalao et al. (2007) for California and Spain Power Markets and Ghodsi and Zakerinia (2012) forecasted spot electricity prices for Ontario electricity market. Also Zhang et al. (1998) provides one of the early surveys for ANN, Amjady and Keynia (2009) uses cascaded neural networks, Zhang et al. (1998), employed sophisticated applications and Zhang and Luh (2005) neural network-based market clearing price prediction and confidence intervals.

[4] Liebl (2013), Karakatsani and Bunn (2008), (Carmona and Coulon (2014) and the literature cited therein and also in Weron (2014) and Aggarwal et al. (2009a) are some of the papers in this strand of the literature. One particular paper Coulon and Howison (2009) develops a fundamental model for spot electricity prices, based on stochastic processes for the underlying factors that relate the bid ask prices to the drivers of the prices, for instance.

[5] Some papers are; particularly in regime switching models Weron et al. (2004), Weron and Misiorek (2006), Weron and Misiorek (2008), Janczura and Weron (2010), regression-based models, Bessec et al. (2016), Robinson (2000), Ghosh and Das (2002).

[6] Among others, Contreras et al. (2003) for Spain and Californian energy markets, Crespo Cuaresma et al. (2004) for Leipzig Power Exchange (LPX) of Germany, Ming Zhou et al. (2004) for California power markets, Kristiansen (2012) for Nord Pool power market employed ARIMA type models for spot electricity price forecasting. Weron and Misiorek (2005) employed ARMA and ARMAX Model for California power markets spot electricity price. Also, Che and Wang (2010) uses support vector regression (SVR) model, and Panagiotelis and Smith (2008) employs time series autoregressive models with Bayesian techniques; among other papers in the literature.

[7] Hong and Wu (2012), Aggarwal et al. (2009a), Weron et al. (2004), Weron (2014) and the references therein.

ture by introducing a dynamic forecasting model for hourly electricity prices based on nonlinear excess demand specification. We demonstrate our model to produce price forecast in Turkish electricity market. Turkey is an emerging economy with growing energy needs and its electricity market recently has been transformed into a competitive structure from a traditionally monopolistic government-controlled system. Our modelling framework depends on the neoclassical price adjustment equation that necessitates that prices adjust toward equilibrium at a rate that is proportional to the excess demand [8]. We approximate the adjustment as a nonlinear (cubic) function of the excess demand which itself is modeled as a latent factor in the model. As in Caginalp (2005), nonlinear excess demand captures the fact that the demand and supply are generally nonlinear functions of price. We show that nonlinear relation of excess demand and price leads to a more accurate description of price evolution toward equilibrium (Watson and Getz (1981)). With this framework, the equilibrium forecast for the price is given by a nonlinear equation of the excess demand that can be modeled as a function of important variables of supply and demand. This generates an advantage to forecasters in employing all the information on supply and demand functions in price prediction, rather than simply modelling the price on an ad-hoc manner. Often the other available information on excess demand (demand and supply) included in price prediction models. However, in that respect our framework reveals the rationale of including those variables and particularly demonstrates the final functional form of the price prediction equation in which those variables take part nonlinearly.

Our approach is based on a nonlinear price adjustment equation where daily electricity prices change in response to unobserved excess demand which is itself modeled as a stochastic linear function of observable variables in the electricity market. This estimation strategy helps us to capture;

(i) the nonlinearity in the reaction of the price changes to the excess demand, i.e. we observe when price changes are small, a linear relationship of the price change with the excess demand adequately presents the market behavior, however for large price changes this is no longer the case;

(ii) the process of the demand more realistically, i.e.: the excess demand function depends on an unobserved component as well as the observables, therefore the model is flexible to absorb sudden price changes and also it implies a nonconstant variance for the price process.

In this respect, our modelling approach can be classified within the parsimonious structural models in the literature (Carmona and Coulon (2014)) where our flexible structural framework for day-ahead prices incorporates excess demand while allowing a closed-form prediction equations for price and price volatility. Advantages and disadvantages as well as strengths and weakness, and degree of complexity of various approaches in the electricity price forecasting literature are

[8] i.e., the difference between the demand and supply divided by the demand (at that price).

addressed in Weron (2006) and Weron (2014). The models which take into consideration the non-linearity have generally been superior to linear models except maybe the work done by Misiorek et al. (2006). Our study contributes to the nonlinear pricing models as well. However, our approach with non-linear excess demand specification differs from the pure statistical techniques. Like the models utilizing neoclassical price adjustment equation, we rely on the excess demand to take the prices adjust toward equilibrium at a rate that is proportional to it (Caginalp (2005)). Excess demand models in price prediction were earlier considered by other economists to address different problems. Hendry and Richard (1982) and Richard and Zhang (1996) constructed a nonlinear excess demand model for the UK housing market. More recently Giarratani et al. (2015) applied the excess demand framework for the US scrap steel prices. The latent excess demand structure is proved to be a useful tool to understand the price dynamics in a volatile market environment. In this paper, like Giarratani et al. (2015), a latent excess demand function is estimated and its movements along with the price prediction is demonstrated. The daily prices for the electricity market in Turkey is predominantly determined by matching the sell and buy orders from the previous day on the day-ahead market, that is obtained as the equilibrium price based on fitting supply and demand curves (proposed for the next day). With this argument, the short run price fluctuations can be approximated by the discrepancy in demand and supply curves (Carmona and Coulon (2014), Liebl (2013), Karakatsani and Bunn (2008)). Therefore, while we introduce a latent factor as the excess demand in our model, we restrict it to be positive whenever the price movement is upward, and to be negative when the price movement is downward. Therefore, implicit in this assumption, we expect the prices not to change if the excess demand is zero. Therefore, we could predict the direction of the price movement by observing the excess demand, yet excess demand itself is latent and can only be retrieved with the model assumptions and restrictions. Moreover, the reaction of price in general will depend nonlinearly on the excess demand, and models assuming a linear dependency of price changes will generally fall short of capturing the actual dynamics. Linearity might prove useful when the change in the demand (supply) is relatively small from one day to the other, however it is likely to perform poor whenever the excess demand increases or decreases significantly over a day. In this case, the proportional change in price assumption is very much likely to be violated.

In this paper, the nonlinear reaction to excess demand is proposed as the price adjustment equation for the electricity price. Subsequently, a maximum likelihood estimator is developed based on the nonlinear (cubic) specification of the price reaction to the excess demand. Our estimator is then demonstrated using data from Turkish Electricity market (EXIST). Our model encompasses the linear regression-based models as it reduces to the linear regression framework when a linear reaction function of the price to excess demand is assumed [9]. However, with a nonlinear reaction,

[9]Time series regression models, frequently used in the empirical literature including AR/ARMA models, ARDL

the price prediction equation becomes non-trivial and is required to be derived based on distributional assumptions. We show the steps of this derivation and demonstrate the maximum likelihood estimator. The aim of our study is not a full forecast comparison of alternating models, but rather to introduce the nonlinear excess demand specification as a forecasting tool. The former remains as future research. Finally, our modelling framework implies time varying volatility for prices which along with the nonlinear mean function, brings two important features of time series modelling dynamics together in parsimonious model.

The rest of the paper is organized as follows. Section 2 briefly discusses the market structure and price formation in the Turkish electricity market. Section 3 introduces the excess demand framework, derives the price forecast based on nonlinear excess demand specification, and subsequently develops the maximum likelihood estimator. Section 4 demonstrates the maximum likelihood estimator using data from Turkish electricity prices. Finally, section 5 concludes. Details and proofs are given in the appendix of the paper.

2 Market Structure

The price dynamics during the day is predominantly determined by a matching mechanism coming from a Day-Ahead market[10]. In this structure, market participants notify the market operator (EXIST), with their day-ahead market offers for the following day through the submission system[11]. The match-up transaction is conducted by the generation of the supply-demand curves through the assessment of 24 hour offers via an optimization tool, and a unique price is determined for each hour for the following day. Developing those frontier mechanisms could be challenging and costly and probably more complex than the old system of state-controlled prices and distribution channels. However, there is the potential for a significant role for production and distribution companies, that operate for their own best interest in the new mechanism. These would open the floor for new companies and subsequently new investment projects for which local and global partners would be sought, especially the large companies with their capabilities to execute projects on that scale. As a growing emerging economy, Turkey strategically transformed its electricity market, having in mind to foster the industry and therefore encourage the market participants to undertake the new initiatives on infrastructure investments and large production projects. Of course, under-

models and even Markov Switching type models fall into the linear reaction case in this framework. Similar and richer models can be obtained with the extension of these time series models in the general nonlinear framework.

[10]In EXIST, there are two price series in a particular day. One is the price series determined in the Day-Ahead Market which is the series taken into consideration in this paper. The other is the spot price during the day which can change in response to unforeseen hourly mismatches during the day in the proposed electricity demand/supply from the previous day.

[11]The details of the market making process can be found at the Energy Exchange Istanbul (EXIST) website at www.exist.com.

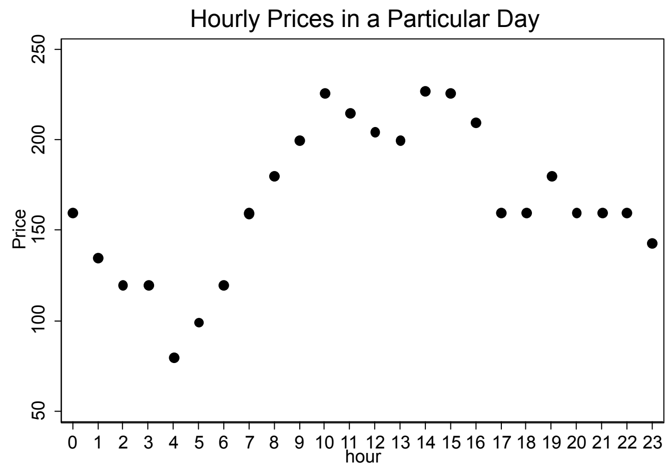
taking them would require considerable confidence on both sides. The prospect can be expected to be a long-term relationship where the state steps back from the market entirely and the new market design facilitates the competitive structure. Yet, from a practical perspective, that transition time should be measured not in years, but maybe decades, until full impact would likely be felt. Then after, the EXIST will be serving as the market maker, and its sole role will be ensuring the smooth operation of the market in terms of price formation.

Table 1: Variables

	Variable	Explanation
1	weekday	day of the week
2	hour	hour of the day
3	day	day information
4	month	month information
5	year	year information
6	price	price level
7	volume	volume level

Table 1 presents the information available from the market in a particular day. Data from daily transactions are handled and then shared by the Energy Exchange Istanbul (EXIST). Hour, day, month, and year information of transaction volume and prices are available for transparency as well as research purposes. The matching in the day-ahead market produces 24 prices from 12:00 A.M. to 11:00 P.M. in 24 hourly increments. In Figure 1, the daily prices are presented for a particular day. We immediately observe from the figure that the hourly prices vary substantially during the day. This is a general characteristic of the electricity market in which large differences in demand during the day are common. Besides, the production at different hours can be affected by different natural factors. The lack of an efficient storage technology also is a major characteristic of this market which otherwise would be expected to be used for smoothing the hourly prices during the day. However, to alleviate the uncertainty for distributers, the demand side can determine the amount that it will be liable to in accordance with the price level. This allows the demand side of the market to protect itself against the price that will be formed in the matching. Other safety mechanisms are also in place for smooth operation of the system such as the portfolio-balancing for firms and daily reconciliation of the debts and receivables of the participants emanating from the commercial transactions following the trade. These are similar rules and regulations that are widely used in energy markets in post transition to a competitive market structure.

Figure 1: Hourly Prices



Note: Hourly price averages over the sample.

3 Model

3.1 Linear Model

The linear model with the excess demand structure can be described as follows. Let the log differenced price Δp_t be defined as:

$$\Delta p_t = g(E_t; \phi) \tag{1}$$

where $g(E_t; \phi)$ is a linear reaction function of the excess demand E_t for the electricity market and let E_t be defined as a linear function of the observable explanatory variables Z_t and the unobserved variable v_t that affect the excess demand in the electricity market. The coefficient vector γ collects the impact of each variable in Z_t on the excess demand.

$$g(E_t; \phi) = E_t, \;\; E_t = \gamma' Z_t + v_t \tag{2}$$

This specification implies that the differenced price Δp_t can be written as a function of the variables that determine the excess demand. We can put the definition for the function $g(E_t; \phi)$ in equation (1) using equation (2) for the excess demand.

$$\Delta p_t = \gamma' Z_t + v_t \tag{3}$$

The specification given in equation (3) has been used as a benchmark for modelling price change in the electricity as well as many other commodity markets. The linear specification and the explanatory variables in the vector Z_t are chosen either depending on theory or as it is done most often in the literature, they are chosen based on theoretical considerations which provide the necessary guideline to the selection of variables, but the linear specification is used as an approximation to the true process. However linear modelling can be short of capturing the true dynamics in the price changes. This might especially be an issue for the electricity market where the demand response to prices can be sensitive to the magnitude of the price change. For this purpose we propose a more flexible reaction function $g(E_t;\phi)$ to capture the nonlinear price change effects of the excess demand. This specification though comes with a cost in terms of estimation of the model. The linear model in equation (3) can be easily estimated using regression based methods by paying attention to the time series dynamics through error-correction frameworks and/or ARMA modelling. However a different specification of the function $g(E_t;\phi)$ than a linear one requires nontrivial derivation of the estimation equations which eventually requires a maximum likelihood estimation.

3.2 Linear Model Extensions

Obviously the specification given in equations (1), (2) and (3) can be easily generalized to account for AR and ARMA specifications. The flexible structure presented in equation (3) in the case of AR or ARMA will include lag values of the log differenced price Δp_t and/or v_t in the vector Z_t.

3.3 Nonlinear Excess Demand Model

The dynamics of the nonlinear model differentiates itself from the linear model in terms of the functional form of the reaction function. Log differenced price Δp_t can still be defined as:

$$\Delta p_t = g(E_t;\phi) \tag{4}$$

whereas the function $g(E_t;\phi)$ is a now a nonlinear function of the excess demand E_t for the electricity market[12]:

$$g(E_t;\phi) = E_t + \phi_2 E_t^2 + \phi_3 E_t^3 \tag{5}$$

$$E_t = \gamma' Z_t + v_t \tag{6}$$

[12] A cubic form is choosen for the excess demand as in Richard and Zhang (1996), and Giarratani et al. (2015). Obviously any other nonlinear function can also be applied. Cubic function has the attractiveness since the cubic term captures the disproportionate response of the price to large changes in excess demand. Also the quadratic term gives a natural free parameter to test the asymmetric response to positive and negative excess demand.

where the coefficients in the nonlinear reaction function are required to satisfy the following restrictions:

$$\phi_2 > 0, \phi_3 > 0, \phi_3 > \frac{\phi_2^2}{3} \quad (7)$$

The constraints are set in order to ensure that the increase in excess demand is associated with a positive price movement. This is $\frac{\partial g(E_t;\phi)}{\partial E_t} > 0$. To obtain an estimation equation from this specification is not a trivial task since replacing the nonlinear reaction function for the excess demand from equation (5) into equation (4) is no longer an option. The equation that would be obtained this way will include higher order terms in the unobserved component of the excess demand v_t, and the interaction terms that include both v_t and the observables Z_t. Therefore the linear model in equation (3) can only be obtained and subsequently can be estimated using regression based methods under the linear reaction function. We will describe the procedure that we will follow to estimate the parameters of the above model in the nonlinear case. There are several steps to obtain the likelihood function in this model and it requires derivation of the conditional distribution of the log differenced price Δp_t.

3.4 Derivations

The estimation of the nonlinear models as the one we presented above generally starts with initial selection of regressors to be used in the estimation. For instance Hendry (1984) uses a similar modelling of the excess demand to estimate the housing demand in UK, but determined the set of regressors to be used from a extensive specification test in an earlier step. The main difficulty for the nonlinear models is the cost of applying a recursive framework to eliminate the set of regressors. This will require multiple solutions to the model under the given constraints and requires in general a numeric solution to the objective function (for instance: the likelihood function). In this section, we develop the estimation framework based on the nonlinear specification in equation (5). Then we demonstrate the construction of a maximum likelihood estimator that is derived based on the model specifications and distributional assumptions about unobservables. For the specification test, a likelihood ratio test is proposed for eliminating the set of regressors.

In our empirical application we use variables that prove to be useful in predicting the differenced price in the literature in an ad-hoc manner. Our aim is to demonstrate the maximum likelihood estimator in estimation and to identify the key parameters of the model (like the nonlinear price reaction curve). Therefore, we do not perform a specification test while selecting the list of regressors. Yet it should be kept in mind that it will be ineffcient and also misleading to use the same set of explanatory variables in the excess demand model and the other models if the focus is the forecasting performance. A specification test using the model recursicely to eliminate the list of regressors is required. Since the focus is not the forecasting performance of the nonlinear excess demand specification in this paper, that is left as future research.

3.4.1 Inversion of the Price Equation I

We propose the following inversion of equation (4)

$$E_t = h(\Delta p_t; \phi) \tag{8}$$

Substituting this into (6)

$$h(\Delta p_t; \phi) = \gamma' Z_t + v_t \tag{9}$$

Equation (9) is fairly straightforward least squares (LS) problem. It is linear in γ given ϕ, and nonlinear in ϕ. Combining analytical and numerical methods, we can find Maximum Likelihood (ML) estimates of (γ, ϕ).

3.4.2 Reparametrization of the Excess Demand

The first step consists of reparametrization of equation (4) in a way which simplifies the inversion in (8). Note that under (5), the cubic equation has a unique real solution in E_t. One can employ a solution to the cubic equation in a number of different ways. We proceed with Cardano's method (Nickalls (1993)). However the key is to introduce a linear transformation to E_t which sets the second order coefficient of the cubic equal to 0. Let

$$E_t = \phi_3^{-\frac{1}{3}} y_t - \frac{\phi_2}{3\phi_3} \tag{10}$$

Then equation (5) becomes

$$\Delta p_t = y_t^3 + a y_t + b \tag{11}$$

and the coefficients of the new transformation is related to the coefficients ϕ as follows:

$$\begin{aligned} a &= \phi_3^{-\frac{1}{3}} \left(1 - \frac{\phi_2^2}{3\phi_3}\right) > 0 \\ b &= \frac{1}{3}\frac{\phi_2}{\phi_3}\left(\frac{2}{9}\frac{\phi_2^2}{\phi_3} - 1\right) < 0 \end{aligned} \tag{12}$$

The parametrization in (a, b) is in one-to-one correspondence with (ϕ_2, ϕ_3) under the constraints in (7) and (12). The construction of the model in terms of (a,b) or (ϕ_2,ϕ_3) is observationally equivalent. However it proves to be more convenient to proceed with (a,b).

3.4.3 Inversion of the Price Equation II

With $a > 0$, the unique real root for the inverse of (11) can be written as[13]:

$$y_t = h(\Delta p_t; a, b) = A(\Delta p_t; a, b) + B(\Delta p_t; a, b) \tag{13}$$

where

$$\begin{aligned} A(\Delta p_t; a, b) &= \left[-\frac{b - \Delta p_t}{2} + C(\Delta p_t; a, b) \right]^{\frac{1}{3}} \\ B(\Delta p_t; a, b) &= \left[-\frac{b - \Delta p_t}{2} - C(\Delta p_t; a, b) \right]^{\frac{1}{3}} \end{aligned} \tag{14}$$

and $C(\Delta p_t; a, b)$ is

$$C(\Delta p_t; a, b) = \left[\left(\frac{b - \Delta p_t}{2} \right)^2 + \left(\frac{a}{3} \right)^3 \right]^{\frac{1}{2}} \tag{15}$$

3.4.4 Conditional Density of the Differenced Price

We need obtain the conditional density of $\Delta p_t \parallel Z_t$ to relate our estimator to the data. Since y_t is a linear transformation of E_t, which is itself a linear function of Z_t, we can transform (6) into:

$$y_t = c' Z_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2) \tag{16}$$

where c and σ^2 can be obtained by $c = p\gamma$ and $\sigma^2 = p^2 \sigma_v^2$. We will write down the likelihood function and conduct a joint specification search in terms of $\theta = (a, b, c, \sigma^2)$. Once the search is concluded. we should retrieve the original parameters $(\phi_2, \phi_3, \gamma, \sigma_v^2)$ from the estimated parameter values for $\hat{\theta} = (\hat{a}, \hat{b}, \hat{c}, \hat{\sigma}^2)$. As will be explained below. this is not a completely trivial task.

Using (16), we have the following density function for the transformed variable y_t:

$$f(y_t | Z_t; \theta) \propto \frac{1}{\sigma} \exp\left[-\frac{1}{2\sigma^2} (y_t - c' Z_t)^2 \right] \tag{17}$$

Next we can apply the inverse transformation from y_t to Δp_t as given in (13) with Jacobian:

[13] This solution is obtained by the Cardano's method. There are alternative ways of obtaining the real root.

$$J(\Delta p_t; a, b) = \frac{\partial h(\Delta p_t; a, b)}{\partial \Delta p_t} > 0 \tag{18}$$

to obtain the density for the Δp_t:

$$f(\Delta p_t | Z_t; \theta) \propto \frac{J(\Delta p_t; a, b)}{\sigma} \exp\left[-\frac{1}{2\sigma^2}(h(\Delta p_t; a, b) - c' Z_t)^2\right] \tag{19}$$

3.4.5 Log-Likelihood Function of the Model

Let

$$\dot{p}(a,b) = \begin{pmatrix} h(\Delta p_1; a, b) \\ \vdots \\ \vdots \\ h(\Delta p_T; a, b) \end{pmatrix} \quad Z = \begin{pmatrix} Z_1 \\ \vdots \\ \vdots \\ Z_T \end{pmatrix}$$

The log-likelihood function is given by:

$$\ln L(\theta) = -\frac{T}{2} \ln \sigma^2 + \sum_{t=1}^{T} J(\Delta p_t; a, b) - \frac{1}{2\sigma^2}(\dot{p}(a,b) - Zc')(\dot{p}(a,b) - Zc) \tag{20}$$

The log-likelihood function can be maximized analytically in (c, σ^2) given (a,b) by OLS estimation as follows:

$$\tilde{c}(a,b) = (Z'Z)^{-1} Z' \dot{p}(a,b) \tag{21}$$

$$\tilde{\sigma}^2(a,b) = \frac{1}{T} \dot{p}(a,b)' M_Z \dot{p}(a,b) \tag{22}$$

and

$$M_Z = I - Z(Z'Z)^{-1} Z' \tag{23}$$

Ignoring additive constants, the corresponding concentrated log-likelihood is given by:

$$\ln L_c(a,b) = -\frac{T}{2} \ln\left[\hat{\sigma}^2(a,b)\right] + \sum_{t=1}^{T} J(\Delta p_t; a, b) \tag{24}$$

which will be maximized numerically in (a,b).

3.4.6 Jacobian of the Transformation

The Jacobian can be obtained from equations (13), (14) and (15) For ease of notation we ignore the arguments of the related functions (i.e.: $A_t = A(\Delta p_t; a, b)$). Therefore the Jacobian is:

$$J_t = \ln(3\Delta p_t^2 + a) \tag{25}$$

3.4.7 Maximization

The computation of the maximization of log-likelihood is programmed in Gauss programming language and conducted in a Intel Core-7 laptop computer. A single calculation of the optimum parameters given Z_t while depending on the number of iterations and the size of Z_t approximately takes 1-5 minutes on average. The procedure calculates the optimum parameters $(\hat{a}, \hat{b})$ as a result of the values that maximize the concentrated log-likelihood given in equation (24). Once we obtain the ML estimates of $(\hat{a}, \hat{b})$, they are substituted in equations (21) and (22) to obtain the ML estimates of (c, σ^2) as $\hat{c} = \tilde{c}(\hat{a}, \hat{b})$ and $\hat{\sigma}^2 = \tilde{\sigma}^2(\hat{a}, \hat{b})$.

3.4.8 Retrieving the Original Parameters

y_t represents a linear transformation of the latent variable E_t. Easiest way to retrieve (ϕ_2, ϕ_3) from (a, b) is to introduce the inverse transformation

$$y_t = pE_t + q \tag{26}$$

and find (ϕ_2, ϕ_3) as a function of (a, b). Substituting (26) into (11) and imposing the constraints in (7) produces the following:

$$b + aq + q^3 = 0 \tag{27}$$

with $a > 0$, this produces a unique real root $q > 0$. Using this real valued solution, we can solve for p and from that we can obtain (ϕ_2, ϕ_3):

$$\begin{aligned} p &= \frac{1}{a + 3q^2} > 0 \\ \phi_2 &= 3p^2 q \qquad \phi_3 = p^3 \end{aligned} \tag{28}$$

which corresponds effectively to the inverse of (10) as a function of (a, b).

4 Empirical Application

4.1 Data

We use data from the Istanbul Energy Exchange (EXIST) for the Turkish hourly electricity prices and trade volumes between 2010 and 2016[14]. The electricity prices are for each hour and recorded from 12:00 A.M. to 11:00 P.M. in 24 increments. We use the hourly prices for 00:00 A.M., 08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M., and estimate the excess demand models for each of these series separately. Table 2 reports the main descriptive statistics of the data used in the analysis. The price exhibits substantial variation during the day, with different price levels are observed at particular hours. This is expected since the price is actually formed in the previous day by a price matching mechanism that takes into consideration the bid and ask prices raised by the suppliers and the distributors. Therefore different levels are mainly related to the cost of producing electricity at different parts of the day corresponding to the given demand level. We also observe higher price variation during the hours where the price level is high. The standard deviation of the price peaks at 11:00 A.M. and continues to be high at 14:00 P.M. and gradually decreases during the later hours in the day through midnight. The log price change series have mean zero on average and exhibit a standard deviation that is higher for the morning hours than the rest of the day.

Table 2: Descriptive Statistics

Hour	Price		Log Price Change		Log Volume Change	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
00:00 A.M.	145.75	28.09	0.00046	0.1337	0.00173	0.3402
08:00 A.M.	153.13	33.78	-0.01302	0.1682	-0.02540	0.4333
11:00 A.M.	174.38	38.20	-0.00044	0.1471	-0.00173	0.4691
02:00 P.M.	168.48	36.35	-0.00495	0.1451	-0.01445	0.4334
06:00 P.M.	153.79	37.42	-0.00265	0.1466	-0.00554	0.3764
09:00 P.M.	146.53	31.64	-0.00032	0.1371	0.00069	0.3370

The time period used is January 2010 to January 2016. Price is the price for one MGW of electricity reported in Turkish Lira.

The reaction of prices to volume changes does not follow a linear trend. Figure 2 shows the change in the volume from the previous day drawn against the change in the respective price. On the top left picture, the result for the midnight is given. The reaction seems to be depicted by a

[14] Including only first month of 2016.

linear reaction well for small changes in the volume. Whereas the linearity becomes insufficient in explaining the larger volume. Similarly for 8 A.M. in the morning the electricity price change and the change in the volume exhibit the nonlinear relation. For 11:00 A.M. and 2:00 P.M. given in the mid part of the Figure 2, we observe that the reaction of the price starts to diverge from linearity more severely. The same kind of trend is observed in the graphs given in the bottom panel for the hours 6:00 P.M. and 09:00 P.M.

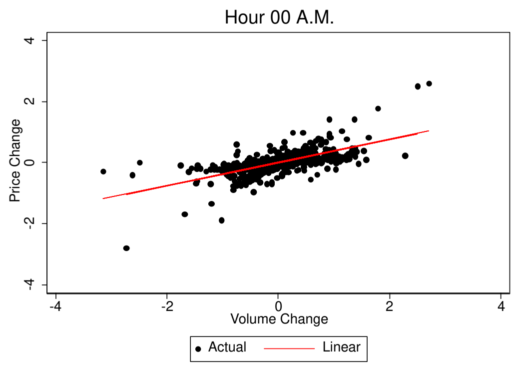
Also the evening hours (called the puant hours) are the time periods where the deviation from the linearity is more severe. This is of course partly related to the fact that magnitude of change in the volume is not exogenous. The relative demand changes occurs in the periods where the demand is already high (as reported with higher standard deviation of these hours in Table 2) .

4.2 Estimation Results

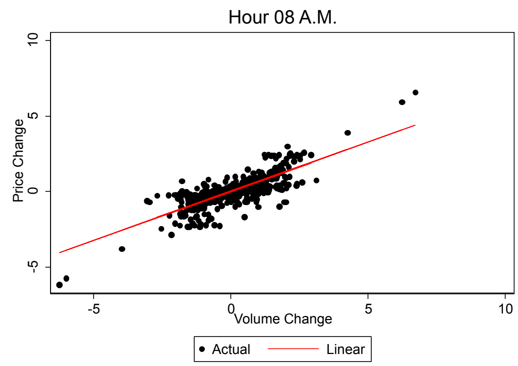
All variables used in the estimation are constructed from the main variables reported in Table 1. The dynamic relationship between the price and the latent excess demand is modeled through the explanatory variables used in the excess demand specification and the random component. The excess demand specification includes lags of the reported variables. As customary in this literature (reasons concerning stationarity of the variable in levels), we are forming a prediction model for the price changes, and our dependent variable is the first differenced log prices. The explanatory variables used includes 10 lags of the dependent variable, 5 lags of the first differenced log volume, first differenced log prices of the nearest 10 hours from the previous day and first differenced log volume chages of the nearest 10 hours from the previous day . For instance, for the hour 02 P.M., the differenced log prices and volumes of hours: 01:00 P.M., 12:00 P.M., 11:00 A.M., 10:00 A.M. and 09:00 A.M. from the previous day are included as well as 03:00 P.M., 04:00 P.M., 05:00 P.M., 06:00 P.M. and 07:00 P.M.. These are to capture the environemental effects (both for demand and supply) that are peculiar to the specific parts of the day. As stated before, these variables are not based on a specification test for the nonlinear model as this is not in the current scope of the paper. Rather, same set of explanatory variables (with addition of square and cubic terms of each variable, other than the dummy variables for obcious reasons) are used to estimate a linear version using the specification for log price changes given in equation (3). Aim is to demonstrate the likelihood estimation and provide a naive comparison to a linear excess demand specification. We collect the right hand side variables in a big vector of Z_t where, Z_t has 42 variables in it including a constant and dummy variables for the day of the week with Monday as the excluded class.

The estimation results are given in Table 3. We immediately see from the log-likelihoods that the nonlinear model fits the data much better compared to the linear excess demand specification. For all hours, the log-likelihood values that are reported in the first row of Table 3 are considerably

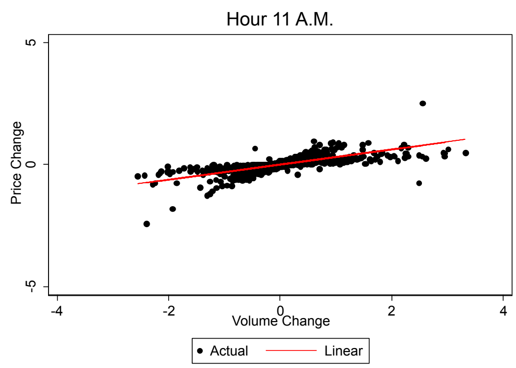
Figure 2: Nonlinearity in the Reaction Function



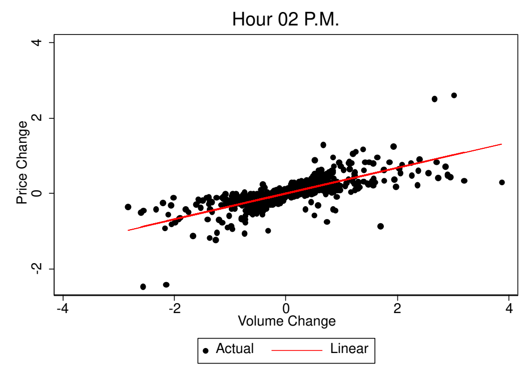
(a) Midnight



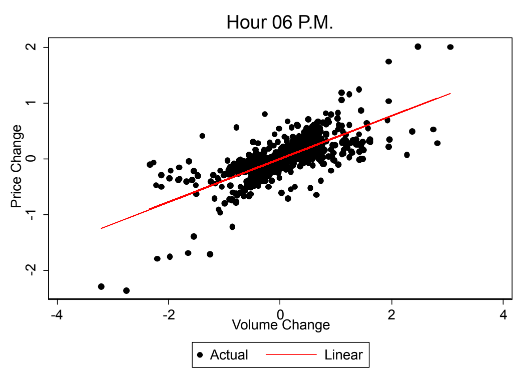
(b) Early morning



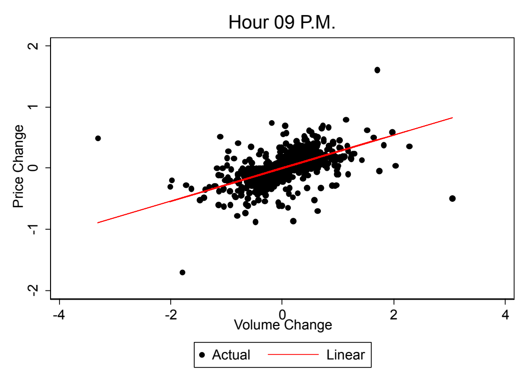
(c) Late morning



(d) Early afternoon



(e) Late afternoon



(f) Night

Note: Linear line is obtained from a regression of price changes on volume change.

higher than the linear model results (under the assumption of normal errors). To understand the causes for the better performance of the nonlinear model, we report the RMSE in the second row. The RMSE results indicate that, the predicted values for the price changes from the excess demand model are very close to those obtained from the linear model, however our constructed prediction intervals are quite different. In particular, prediction intervals exhibit significant heteroskedasticity with greater uncertainty in the particular periods, where the price volatility is higher. Furthermore we report, AIC, BIC statistics in rows three and four. The nonlinear model is performing better according to this criteria as well. Finally the R^2 measure from the linear model and the pseudo-R^2 from the nonlinear excess demand model is reported in the final row[15] in the top panel in Table 3. In all hour specifications, the nonlinear model produces a better fit measure.

The estimated parameter values from the nonlinear model are given in the second panel of Table 3. The coefficient estimates from the transformed model (a,b) are reported first. Following those estimates, the coefficient estimates for the main parameters of interest, ϕ_2 and ϕ_3 are reported. We find significant nonlinearity where the coefficient ϕ_3 of the cubic term E_t^3 in equation (5) is large for all hours. However, the coefficient estimate for ϕ_2 is close to zero in all specifications. This, we interpret as the symmetric response behavior of the price to the change in the excess demand. The rest of the tables report the estimate for the conversion factor p, estimates for the coefficients c and σ^2 of the transformed model along with the original models' coefficients (γ, σ_v^2). As explained in the model section, the derived coefficients $(\hat{a}, \hat{b}, \hat{c}, \hat{\sigma}^2)$ from the transformed model are estimated using the likelihood function based on the search for these parameters. The original coefficients are then retrieved using the conversion factor p and solving equation (27) for the unique root.

Next we consider the predictive performance of the nonlinear model. It is important to note that we expect the mean prediction performance of the nonlinear model not to differ much from the linear counterpart[16]. Also there are various alternatives for the distribution of the stochastic component v_t in equation (6). We considered v_t is distributed normal with $N(\,(0,v))$ for demonstrating our likelihood estimator. Obviosuly this might not be the best choice as we know electricity prices can exhibit large spikes and threfore a fat tailed distribution can better capture these effects

[15]Since we use the same dependent variable Δp_t for both linear and nonlinear models, this measure is comparable across models.

[16]In this paper, we are not interested in the selection process for the explanatory variables in the linear (OLS) and the nonlinear models. The same set of explanatory variables are used in the Z_t vector for both specifications. In general, it is not trivial to choose the model variables in a nonlinear model. The model is needed to be estimated for the each new specification again. Given that restriction, researchers often rely on some set of initial explanatory variables from a simplified model first to estimate the nonlinear model. The most appealing candidate is the OLS estimator's explanatory variables which proves to be significant in a simple OLS estimation. This basic idea however misses the opportunity of nonlinear dynamics that are not significant and as a result not captured in the OLS, but may happen to be significant and important in the nonlinear specification. Those considerations in a unified framework for the choice of set of explanatory variables in a recursive procedure which could be potentially different than a linear model are addressed in a subsequent paper.

and yet perform better. However, independent of the distbution assumption our model's structure allows the prediction intervals to vary, therefore accounting the heteroskedasticity in the price process. It is especially important to account for the volatility in the short term price prediction in the electricity market where the price can exhibit substantial heteroskedasticity.

Table 3: Estimation Results

	Hour = 00 A.M.		Hour = 08 A.M.		Hour = 11 A.M.	
	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear
Log Likelihood	2525.39	1606.56	2141.55	1319.92	3489.82	1922.26
RMSE	0.1130	0.1144	0.1223	0.1217	0.1032	0.0995
AIC	-4960.79	-3127.11	-4193.11	-2553.85	-6889.63	-3758.53
BIC	-4706.25	-2883.89	-3943.64	-2315.46	-6634.59	-3514.82
Pseudo R^2	0.2925	0.2683	0.5625	0.4965	0.6660	0.5411
Estimated parameter values of the excess demand (Non-linear) model						
	a	b	a	b	a	b
	0.26766	-1.00E-13	0.2739	-1.00E-13	0.16175	-2.88E-04
	ϕ_2	ϕ_3	ϕ_2	ϕ_3	ϕ_2	ϕ_3
	1.56E-11	52.149**	1.46E-11	48.665***	0.204*	236.276***
p		3.73606		3.65095		6.18215
	σ	σ_v	σ	σ_v	σ	σ_v
	0.0562**	0.0040**	0.0526***	0.0039***	0.0526**	0.0014**

	Hour = 02 P.M.		Hour = 06 P.M.		Hour = 09 P.M.	
	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear
Log Likelihood	3495.57	1761.47	2438.29	1458.77	2601.04	1531.61
RMSE	0.1096	0.1073	0.1235	0.1241	0.1198	0.1207
AIC	-6901.13	-3436.93	-4786.59	-2831.54	-5112.08	-2977.22
BIC	-6646.07	-3193.20	-4531.23	-2587.54	-4856.35	-2732.86
Pseudo R^2	0.5652	0.4586	0.3340	0.3017	0.2627	0.2383
Estimated parameter values of the excess demand (Non-linear) model						
	a	b	a	b	a	b
	0.14393	-2.62E-04	0.26236	-3.55E-04	0.23176	-1.00E-13
	ϕ_2	ϕ_3	ϕ_2	ϕ_3	ϕ_2	ϕ_3
	0.263*	335.274***	0.059	55.366**	2.41E-11	80.323**
p		6.94704		3.81137		4.31466
	σ	σ_v	σ	σ_v	σ	σ_v
	0.0635**	0.0013**	0.0622*	0.0043*	0.070***	0.0038***

Note: Superscripts *, ** and *** represent the significance at 10%, 5%, and 1% levels, respectively. ϕ_2 and ϕ_3 are retrieved by transforming the estimated parameters (a,b) according to their relation given in equation (28). The standard errors are calculated using the delta method. The standard deviation of the error term in the transformed model is denoted by σ and the original model's error standard deviation is obtained by using the relation: $\sigma^2 = p^2\sigma_v^2$. The significance of the parameters is reported only for the original model parameters as they directly follow from the transformed model.

4.3 The nonlinear reaction

In Figure 3, the reaction functions from the nonlinear model are displayed. The cubic reaction function is fitted using the parameter estimates from Table 3. The reaction of the price change to small changes in the excess demand is well captured by the linearity, however as the magnitude of the excess demand gets larger in absolute value, the deviations start to display nonlinearity. This was the main descriptive evidence for a need of nonlinear modelling. One may argue that this nonlinearity should be captured by the linear specification by including higher order polynomial terms. However, remember that the nonlinear model is estimated with the same set of variables used in the OLS estimation of the linear excess demand specification, yet still it produces a better fit and better forecast performance (will be detailed below). Also the excess demand specification is methodologically a convenient way to conceptualize the economic mechanism in price formation in the electricity market. The nonlinearity is more pronounced in some hours than others, although the nonlinear model fits the data better for any particular hour considred. For instance the reaction function estimates for the hours 08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. graphically seems to better capture the price behavior than the hours 00:00 A.M. and 09:00 P.M..

We further observe from Figure 3 that the reactionof prices is symmetric due to the very small coefficient estimated for the squared term in the specification. This is true for all hours considred, and can be interpreted as the symmetric response of the price change to changes in the excess demand. The black dots in the figure represent the actual log price changes that corresponds to the estimated excess demand on the horizontal axis. As expected, most of the price changes are small and accumulated around the origin. However the main difference from a linear specification is the different characteristics of the larger changes, as a linear model would be over predicting the effect of small changes and underpredicting the larger ones.

The expected value of the log price change, Δp_t will depend on higher moments of the deterministic and stochastic parts of excess demand function in the nonlinear specification and also the conditional variance will exhibit heteroskedasticity with a specific functional form. Those expectations are needed to be derived for the nonlinear model. We show below the specific form of the mean and the variance equations for log price change in the excess demand model.

Excess Demand and Mean Prediction:

The conditional mean of the nonlinear model can be estimated using the below functional form:

$$E(\Delta p_t|Z_t) = \gamma' Z_t + \phi_2(\gamma' Z_t)^2 + \phi_3(\gamma' Z_t)^3 + (\phi_2 + 3\phi_3\gamma' Z_t)\sigma_v^2 \quad (29)$$

where $\gamma' Z_t$ is the expected value of the excess demand, i.e.: $E(E_t \mid Z_t) = \gamma' Z_t$.[17]

Conditional Variance:

The conditional variance can be estimated by the below formula[18]:

$$\begin{aligned} Var(\Delta p_t|Z_t) &= (1 + 2\phi_2\gamma' Z_t + 3\phi_3(\gamma' Z_t)^2)^2\sigma_v^2 + (2\phi_2^2 + 6\phi_3 + 24\phi_2\phi_3\gamma' Z_t \\ &\quad + 36\phi_3^2(\gamma' Z_t)^2)\sigma_v^4 + 15\phi_3^2\sigma_v^6 \end{aligned} \quad (30)$$

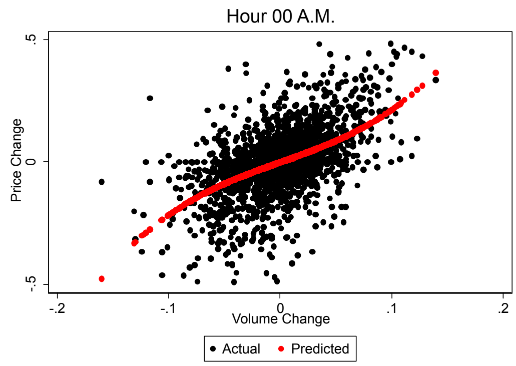
4.3.1 Price Prediction and Prediction Intervals

One of the important outcomes of the model developed and estimated in the previous sections is the capacity to produce price predictions. This is relatively easy with the specification given in equation (29) with all the explanatory variables are in lag forms in Z_t. Therefore, at any point in time t, we may produce an estimate for the $t+1$. In this section, we will produce the price predictions from both the linear model and the nonlinear model and also report some statistics regarding the prediction performance. Time series behavior of the latent factor excess demand (E_t) and the conditional variance will be graphically presented along with the price predictions. Note that we did not perform any specification test for the selection of variables in the linear and nonlinear models in this paper and just use the same set of explanatory variables in both. Already reported by the RMSE statistics in Table 3 and 4, the estimated mean price changes are not different among the linear versus nonlinear model, however the nonlinear model specifies a time varying variance for the predictions which is essential in terms of the capturing the increase in the uncertainty in particular periods.

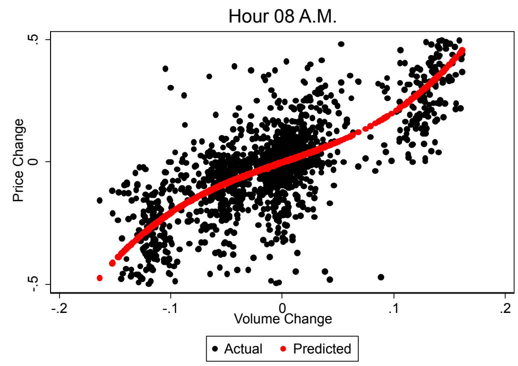
Table 5 shows the performance of the excess demand model in producing price predictions. The analysis is conducted for the whole sample period, and then only for the period from October 2015 to January 2016. In the later period log price changes exhibits more volatility and conditional variance produced by the nonlinear model can be especially important. The first statistics in the table reports the percentage of the time that actual log price change remains in the 1.65 standard deviation band around the predicted price (90% prediction interval). The nonlinear model produces a better catch in this metric, though the difference is not substantial. For the hour 00:00 A.M. for

[17] Proof can be found in the Appendix.
[18] Proof can be found in the Appendix.

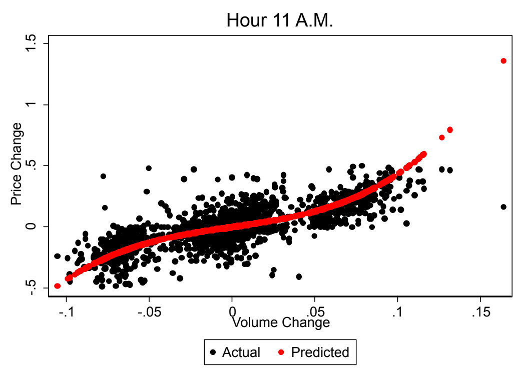
Figure 3: Estimated Reaction Functions



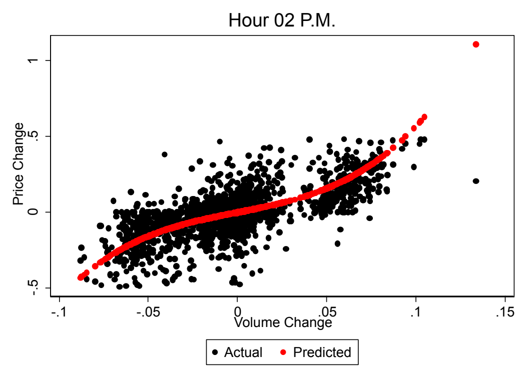
(a) Midnight



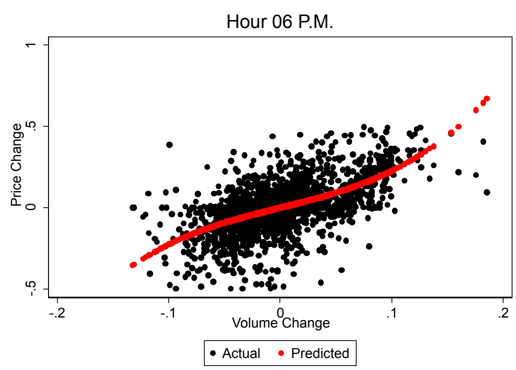
(b) Early morning



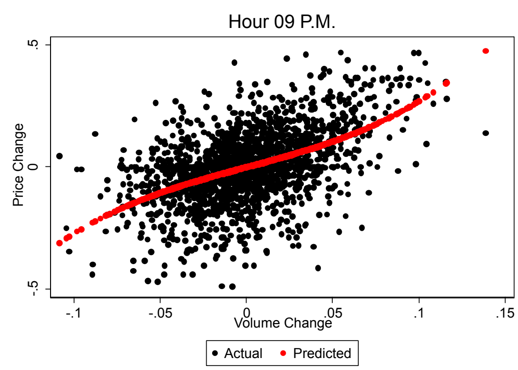
(c) Late morning



(d) Early afternoon



(e) Late afternoon



(f) Night

Note: Nonlinear fit is obtained from the latent excess demand using the conditional mean of the nonlinear model.

instance, the excess demand model predicts the price 93% of the time in the interval while the OLS estimation predicts 90% of the time. However, this difference in the percentages might undermine the performance of the excess demand model since both models perform well. Therefore next we look at the number of times the models miss the actual price change in the prediction interval. This is reported as the third statistics in the table. With this metric for instance, for the hour 00:00 A.M., the excess demand model misses 148 while OLS misses 201 cases in the whole period from January 2010 to January 2016. The same pattern is depicted for the more recent period of October 2015 to January 2016 where only 16 misses happen in the excess demand model compared to 22 from the OLS model.

Table 5: Prediction Results

	Whole Sample					
	Hour = 00 A.M.		Hour = 08 A.M.		Hour = 11 A.M.	
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.93	0.90	0.92	0.91	0.93	0.92
Expected Loss	0.0788	0.0740	0.0879	0.0758	0.0785	0.0748
Number of misses	148	201	144	162	148	172
N		2066		1712		2076
	Period 2015-10 to 2016-01					
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.84	0.78	0.73	0.71	0.88	0.91
Expected Loss	0.1009	0.0919	0.1041	0.0871	0.8750	0.9063
Number of misses	16	22	19	20	12	9
N		101		70		96
	Whole Sample					
	Hour = 02 P.M.		Hour = 06 P.M.		Hour = 09 P.M.	
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.93	0.91	0.93	0.91	0.93	0.90
Expected Loss	0.0828	0.0773	0.0835	0.0824	0.0644	0.0745
Number of misses	140	185	149	196	147	208
N		2076		2098		2141
	Period 2015-10 to 2016-01					
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.85	0.80	0.87	0.84	0.81	0.67
Expected Loss	0.8476	0.8000	0.8696	0.8435	0.1268	0.1222
Number of misses	16	21	15	18	17	29
N		105		115		88

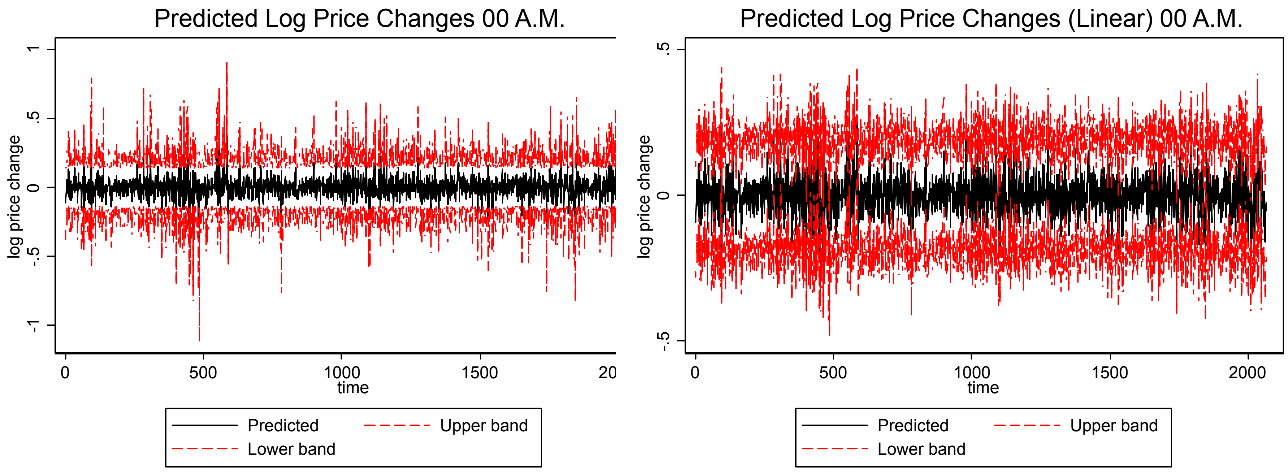
For the other hours (08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M.) similar results are obtained with the exception of the hour 11:00 A.M. only in the most recent period. For this hour in the period from October 2015 to January 2016, the linear model performs better, however the performance of the nonlinear model is superior when the whole sample period from October 2015 to January 2016 is considered. When the whole period is considered, for the 08:00 A.M.,11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M., the excess demand model predicts the price in the interval more often than the OLS model parallel with the hour 00:00 A.M. Specifically, excess demand model predicts 92%, 93%, 93%, 93% and 93% of the time in the interval while OLS model predicts 91%, 92%, 91%, 91% and 90%. Also, excess demand model misses 144, 148, 140, 149 and 147 cases while OLS model misses 162, 172, 185, 196 and 208 cases for the 08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M. respectively.

When the most recent period is considered, for the 08:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M., the excess demand model predicts better than the OLS model parallel with the results from hour 00:00 A.M.. Excess demand model predicts the price 73%, 85%, 87% and 81% of the time in the interval while OLS model predicts the price 71%, 80%, 84% and 67% of the time. Excess demand model misses 19, 16, 15 and 17 cases while OLS model misses 20, 21, 18 and 29 cases for the 08:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M. respectively. Also in Table 5, we report a version of an expected loss measure, which is calculated as the average of the absolute value of the deviation of the actual price from the predicted price in the instances where the actual price can not be covered in the 90% prediction interval. The expected loss results are comparable across the linear and nonlinear models.

In Figure 4, the prediction intervals (90%) are presented along with the mean predictions obtained from the excess demand and OLS estimates for the hour 00:00 A.M for illustration. The nonlinear model allows for wider prediction intervals, especially after days following a sudden large price change, the nonlinear reaction function translates that into a higher uncertainty with larger conditional standard deviations. This is obviously limited in the case linear model with a constant variance.

Figure 5 presents the estimated excess demand, price predictions and the conditional standard deviations from the nonlinear excess demand models for the hours 00:00 A.M., 08:00 A.M. respectively. The price changes in our formulation result from the changes in the excess demand which is a latent variable in the model. We use $\gamma' Z_t$ which is the expected value of the excess demand for time t to construct the first plot. Second graph illustrates the predicted price change. The expected value for the price change is obtained by the formula given in equation (29). Third graph presents the conditional standard deviation estimated from the excess demand model. Equation (30) is used to construct this series and it is plotted with the OLS standard error in Figure 4. Since OLS estimates a constant error variance, the standard deviation from OLS is plotted as the

Figure 4: Constructed Prediction Intervals



(a) Model (b) Linear

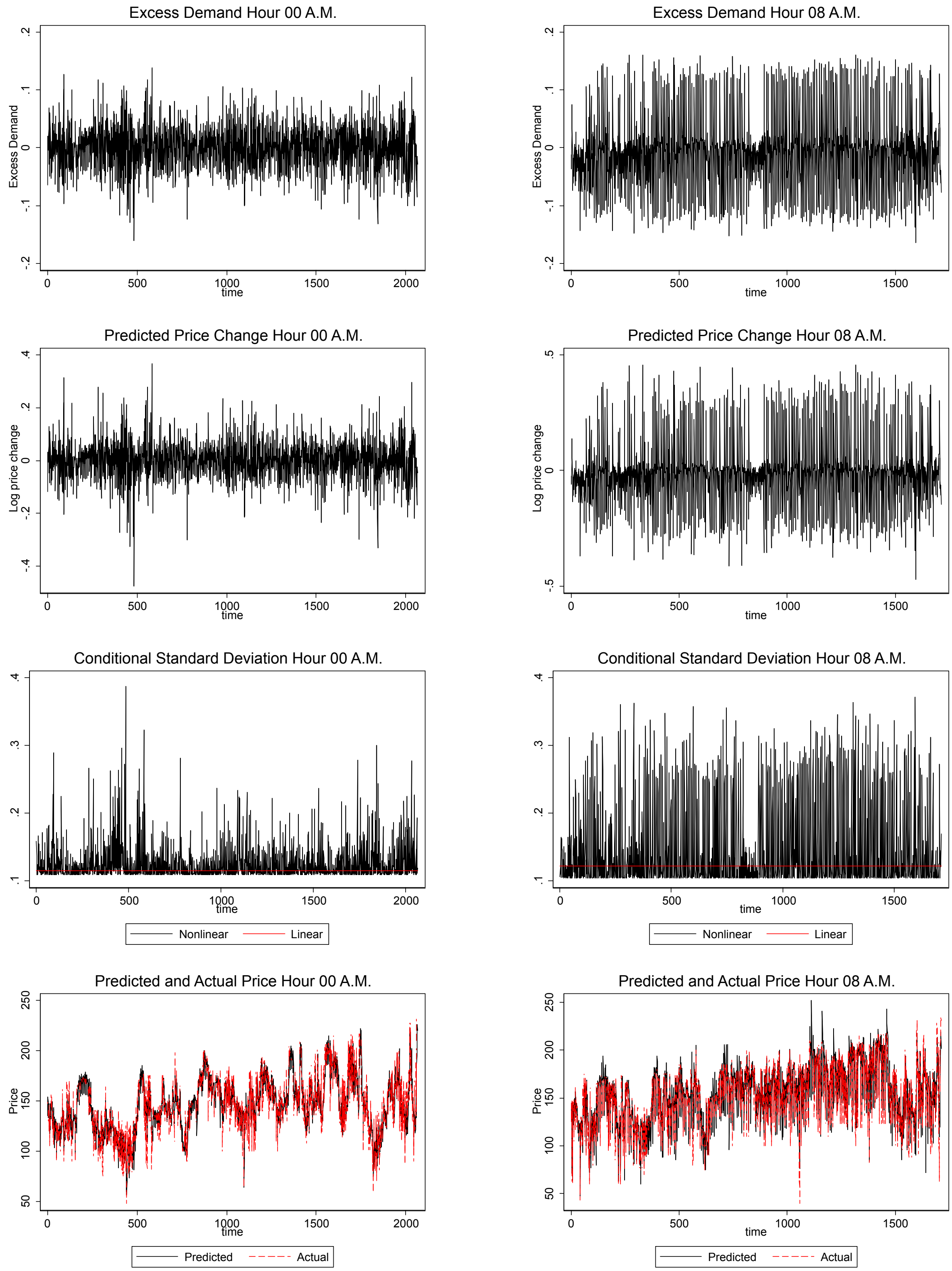
Note: The prediction intervals are for (90%). Mean predictions are obtained from the conditional mean of the nonlinear model in equation (29), and from the mean of the estimated regression equation ($\gamma' Z_t$) respectively.

horizontal line in the figure. The last graph presents the model implied price prediction for the price level. The estimated log pice changes are used to form one-day ahead price predictions in this graph. Similarly, Figure 6 presents the estimated excess demand, price predictions and the conditional standard deviations from the model for the hours 11:00 A.M., 02:00 P.M., and Figure 7 presents the estimated excess demand, price predictions and the conditional standard deviations from the model for the hours 06:00 P.M. and 09:00 P.M..

The results show that the estimated mean price changes are not quite different among the linear versus nonlinear model, however the nonlinear model specifies a time varying variance for the predictions which is essential in terms of the capturing the increase in the uncertainty in particular periods. In the figures both the excess demand and relatedly the price predictions exhibit substantial heteroskedasticity. The estimated conditional standard deviations confirm this observation and capture the time varying nature of the volatility process[19]. We can also identify differential characteristics of the excess demand and volatility predictions for the different hours we considered. For some hours the time varying volatility is more pronounced. Therefore understanding the price dynamics will inform us better to characterize the periods as more tranquil and more volatile. Restating again, the mean predictions from the linear and nonlinear models are not so different when the changes in the price are not large, however the prediction intervals are significantly larger under the nonlinear model in times of price instability. As stated before, the purpose of the prediction comparison is not a full forecast performance test. Obviously in that case a better comparison

[19]Linear models estimated via OLS or AR/RMA models will in general inherently suffer from this heteroskedasticity unless the errors are not specifically adjusted for this.

Figure 5: Model Fit, 00 and 08 A.M.



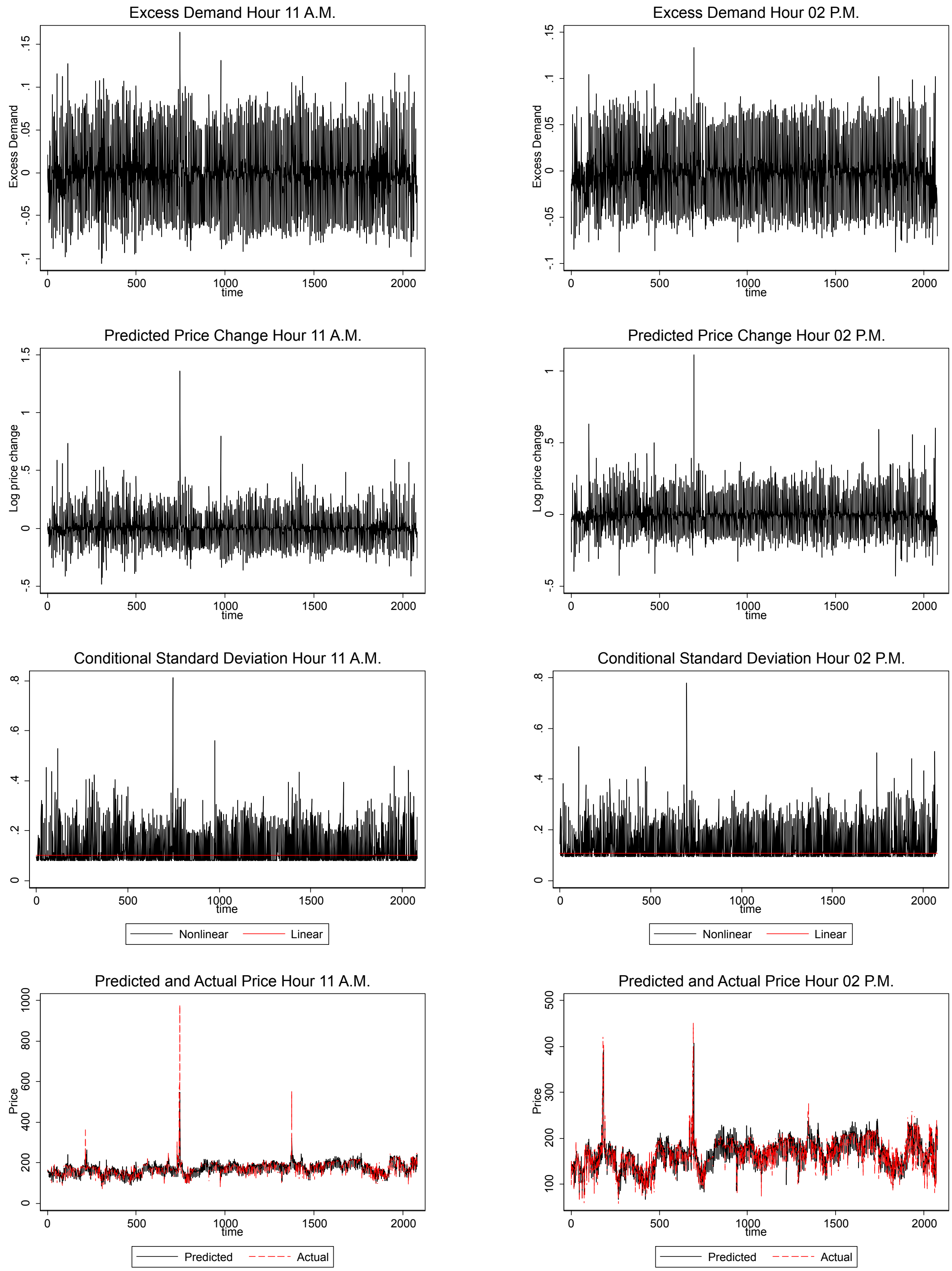
Note: $\gamma' Z_t$ - the expected value of the excess demand is used to construct the first series. Predicted price changes in the second graph are obtained from the changes in the latent excess demand using equation (29). Third graph draws the conditional standard deviation estimated from equation (30) and it is plotted with the OLS standard error (plotted as the horizontal line). The last graph plots the model implied price prediction for one-day ahead price.

should utilize a recursive specification test for the selection of explanatory variables for the nonlinear model. This paper aims to introduce the nonlinear excess demand model, its estimation and its capacity in producing mean predictions and time varying volatility, therefore this empirical exercise (with the same fixed set of explanatory variables) should be taken cautiously and should be interpreted accordingly. A full forecast performance comparison of the nonlinear model with linear as well, other widely used nonlinear models in time series modelling of electricity prices is left as future research.

5 Conclusion

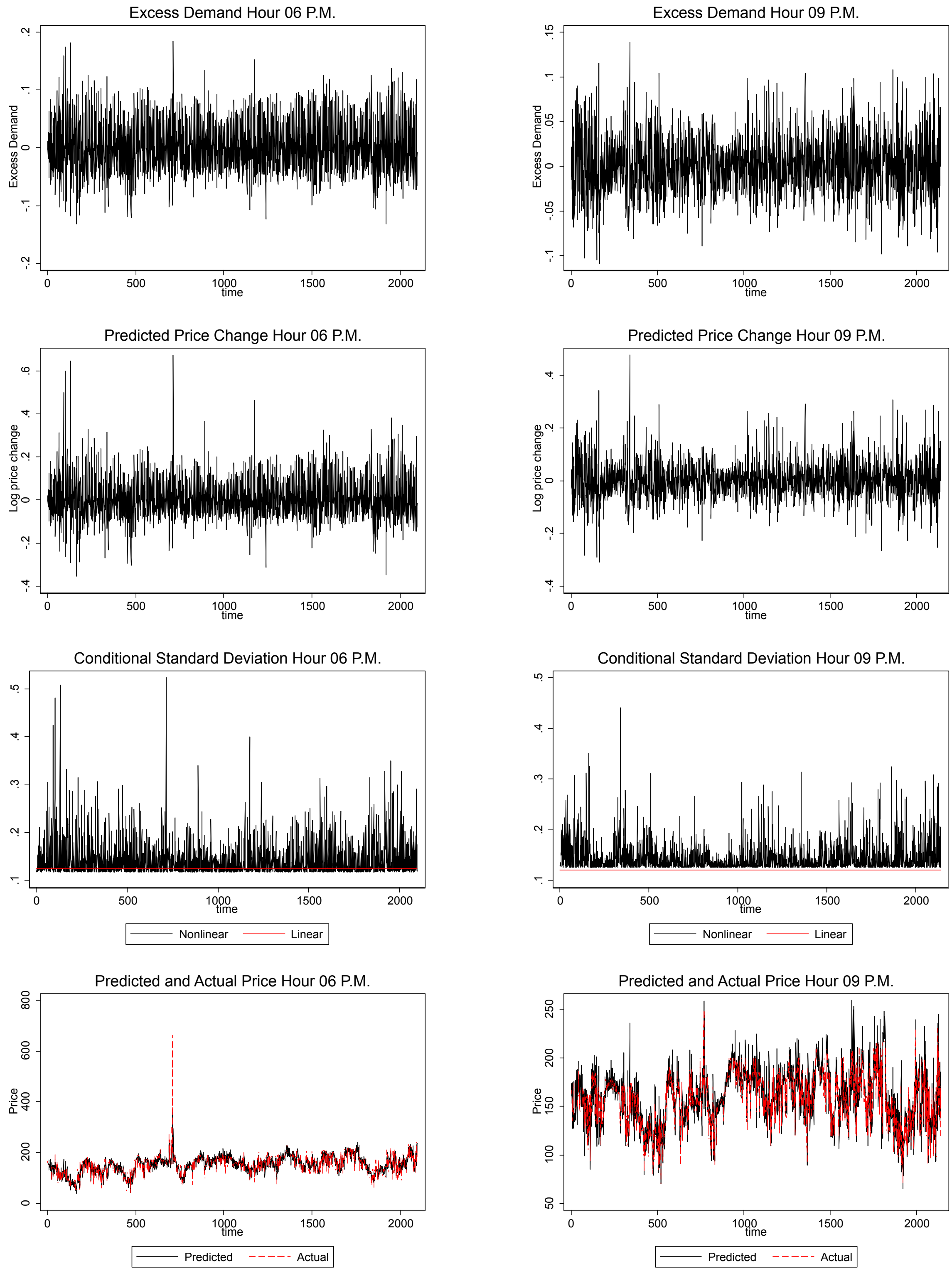
This paper develops a nonlinear stochastic excess demand specification for the formation of electricity prices. A maximum likelihood estimator is derived via a nonlinear reaction function of prices to the excess demand. The excess demand is defined as a normally distributed random variable conditional on variables derived from observable prices and volumes from the previous days. The conditional mean and variance of the price process are derived. Hourly prices in the Energy Exchange Istanbul (EXIST) are used to estimate the model parameters using the likelihood estimator. It is demonstrated that the nonlinear model fits the data better in terms of log-likelihood, AIC, BIC and pseudo-R^2 criteria compared to a naive linear model. Further, nonlinear model produces a time varying variance which is important in terms of capturing the increase in the uncertainty in volatile periods. However, apart from the model fit, two features of the excess demand model proves to be useful in understanding the short term price dynamics in electricity markets, namely (i): the model produces mean predictions for the excess demand (a latent variable in the model) which is economically the reason for the daily price variation; (ii) the implied volatility of the price changes are time varying in the model and they depend on the excess demand as well as the reaction function parameters. This basically brings two important features of time series modelling dynamics together, i.e. the nonlinear mean function, and time varying volatility in a parsimonious model. This paper focuses on demonstrating the derivation, estimation and capacitity for forecasting of the nonlinear excess demand model in the spot electricity market. Therefore a fixed set of explanatory variables are used for the demonstration, skipping the setep for the selection process for the explanatory variables in the nonlinear model. In nonlinear models, choosing the model variables might not be an easy task in general. For this reason, variable selection sometimes is bypassed by the researchers and an initial set of variables from a simplified model (mostly linear) is relied on. This basic idea however misses the opportunity of nonlinear dynamics that are not significant and as a result not captured for instance in the OLS estimation. Those considerations for the choice of set of explanatory variables in a recursive procedure, as well as the forecast performance comparision of the nonlinear model with linear and other widely used nonlinear models

Figure 6: Model Fit: 11 A.M. and 02 P.M.



Note: $\gamma' Z_t$ - the expected value of the excess demand is used to construct the first series. Predicted price changes in the second graph are obtained from the changes in the latent excess demand using equation (29). Third graph draws the conditional standard deviation estimated from equation (30) and it is plotted with the OLS standard error (plotted as the horizontal line). The last graph plots the model implied price prediction for one-day ahead price.

Figure 7: Model Fit, 06 and 09 P.M.



Note: $\gamma' Z_t$ - the expected value of the excess demand is used to construct the first series. Predicted price changes in the second graph are obtained from the changes in the latent excess demand using equation (29). Third graph draws the conditional standard deviation estimated from equation (30) and it is plotted with the OLS standard error (plotted as the horizontal line). The last graph plots the model implied price prediction for one-day ahead price.

in time series modelling of electricity prices is left as future research.

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A Appendix

Excess Demand and Mean Prediction:

The following expression is the expected price change from the model:

$$\begin{aligned} E(\Delta p_t|Z_t) &= E(E_t + \phi_2 E_t^2 + \phi_3 E_t^3|Z_t) \\ &= E(E_t|Z_t) + \phi_2 E(E_t^2|Z_t) + \phi_3 E(E_t^3|Z_t) \end{aligned}$$

replacing $E_t = \gamma' Z_t + v_t$

$$\begin{aligned} E(\Delta p_t|Z_t) &= E(\gamma' Z_t + v_t|Z_t) + \phi_2 E((\gamma' Z_t + v_t)^2|Z_t) + \phi_3 E((\gamma' Z_t + v_t)^3|Z_t) \\ &= E(\gamma' Z_t|Z_t) + E(v_t|Z_t) + \phi_2 E((\gamma' Z_t)^2|Z_t) + \phi_2 E(2\gamma' Z_t v_t|Z_t) + \phi_2 E(v_t^2|Z_t) \\ &\quad + \phi_3 E((\gamma' Z_t)^3|Z_t) + \phi_3 E(3(\gamma' Z_t)^2 v_t|Z_t) + \phi_3 E(3\gamma' Z_t v_t^2|Z_t) + \phi_3 E(v_t^3|Z_t) \end{aligned}$$

observe that $E(v_t \mid Z_t) = 0$ and $E(v_t^3 \mid Z_t) = 0$ given v_t distributed normal $(0, \sigma_v^2)$ conditional on Z_t. Also $E(v_t^2 \mid Z_t) = \sigma_v^2$. Replacing these into the above expression and taking the conditional expectations on Z_t:

$$\begin{aligned} E(\Delta p_t|Z_t) &= \gamma' Z_t + \phi_2(\gamma' Z_t)^2 + \phi_2 2\gamma' Z_t E(v_t|Z_t) + \phi_2 \sigma_v^2 \\ &\quad + \phi_3(\gamma' Z_t)^3 + 3\phi_3(\gamma' Z_t)^2 E(v_t|Z_t) + 3\phi_3 \gamma' Z_t E(v_t^2|Z_t) \end{aligned}$$

results in:

$$E(\Delta p_t|Z_t) = \gamma' Z_t + \phi_2(\gamma' Z_t)^2 + \phi_3(\gamma' Z_t)^3 + (\phi_2 + 3\phi_3 \gamma' Z_t)\sigma_v^2$$

.

Conditional Variance:

The conditional variance expression for the nonlinear model can be written as follows:

$$\begin{aligned} Var(\Delta p_t|Z_t) &= Var(E_t + \phi_2 E_t^2 + \phi_3 E_t^3|Z_t) \\ &= Var(E_t|Z_t) + \phi_2^2 Var(E_t^2|Z_t) + \phi_3^2 Var(E_t^3|Z_t) + \\ &\quad 2\phi_2 Cov(E_t, E_t^2|Z_t) + 2\phi_3 Cov(E_t, E_t^3|Z_t) + 2\phi_2\phi_3 Cov(E_t^2, E_t^3|Z_t) \end{aligned}$$

replacing $E_t = \gamma' Z_t + v_t$

$$\begin{aligned} Var(\Delta p_t|Z_t) &= Var(\gamma' Z_t + v_t|Z_t) + \phi_2^2 Var((\gamma' Z_t + v_t)^2|Z_t) + \phi_3^2 Var((\gamma' Z_t + v_t)^3|Z_t) + \\ &\quad 2\phi_2 Cov((\gamma' Z_t + v_t),(\gamma' Z_t + v_t)^2|Z_t) + 2\phi_3 Cov((\gamma' Z_t + v_t),(\gamma' Z_t + v_t)^3|Z_t) \\ &\quad +2\phi_2\phi_3 Cov((\gamma' Z_t + v_t)^2,(\gamma' Z_t + v_t)^3|Z_t) \end{aligned}$$

and deriving the expressions for $Var(.|Z_t)$ and $Cov(.|Z_t)$ for the joint items:

$$\begin{aligned} Var(\Delta p_t|Z_t) &= Var(v_t|Z_t) + 4\phi_2^2(\gamma' Z_t)^2 Var(v_t|Z_t) + \phi_2^2 Var(v_t^2|Z_t) + 2\phi_2^2(\gamma' Z_t) Cov(v_t, v_t^2|Z_t) \\ &\quad +\phi_3^2[9(\gamma' Z_t)^4 Var(v_t|Z_t) + 9(\gamma' Z_t)^2 Var(v_t^2|Z_t) + Var(v_t^3|Z_t) \\ &\quad +18(\gamma' Z_t)^3 Cov(v_t, v_t^2|Z_t) + 6(\gamma' Z_t)^2 Cov(v_t, v_t^3|Z_t) + 6(\gamma' Z_t) Cov(v_t^2, v_t^3|Z_t)] \\ &\quad +2\phi_2[2\gamma' Z_t Var(v_t|Z_t) + Cov(v_t, v_t^2|Z_t)] \\ &\quad +2\phi_3[3(\gamma' Z_t)^2 Var(v_t|Z_t) + 3(\gamma' Z_t) Cov(v_t, v_t^2|Z_t) + Cov(v_t, v_t^3|Z_t)] \\ &\quad +2\phi_2\phi_3[6(\gamma' Z_t)^3 Var(v_t|Z_t) + 6(\gamma' Z_t)^2 Cov(v_t, v_t^2|Z_t) + 2(\gamma' Z_t) Cov(v_t, v_t^3|Z_t) \\ &\quad +3(\gamma' Z_t)^2 Cov(v_t, v_t^2|Z_t) + 3(\gamma' Z_t) Var(v_t^2|Z_t) + Cov(v_t^2, v_t^3|Z_t)] \end{aligned}$$

and using the facts $Var(v_t|Z_t) = \sigma_v^2$, $Cov(v_t, v_t^2|Z_t) = 0$, $Var(v_t^2|Z_t) = 2\sigma_v^4$, $Cov(v_t, v_t^3|Z_t) = 3\sigma_v^4$ $Var(v_t^3|Z_t) = 15\sigma_v^6$, $Cov(v_t^2, v_t^3|Z_t) = 0$.

$$\begin{aligned} Var(\Delta p_t|Z_t) &= \sigma_v^2(1 + 4\phi_2^2(\gamma' Z_t)^2 + 9\phi_3^2(\gamma' Z_t)^4 + 4\phi_2\gamma' Z_t + 6\phi_3(\gamma' Z_t)^2 + 12\phi_2\phi_3(\gamma' Z_t)^3) + \\ &\quad 2\sigma_v^4(9\phi_3^2(\gamma' Z_t)^2 + 6\phi_2\phi_3(\gamma' Z_t) + \phi_2^2) + 3\sigma_v^4(6\phi_3^2(\gamma' Z_t)^2 + 2\phi_3 + 4\phi_2\phi_3(\gamma' Z_t)) \\ &\quad +15\phi_3^2\sigma_v^6 \end{aligned}$$

Rearranging and collecting similar terms yield:

$$\begin{aligned} Var(\Delta p_t|Z_t) &= (1 + 2\phi_2\gamma' Z_t + 3\phi_3(\gamma' Z_t)^2)^2\sigma_v^2 + (2\phi_2^2 + 6\phi_3 + 24\phi_2\phi_3\gamma' Z_t \\ &\quad +36\phi_3^2(\gamma' Z_t)^2)\sigma_v^4 + 15\phi_3^2\sigma_v^6 \end{aligned}$$