Microeconomics for CGE Modeling

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Outline

- Introduction

- Production Functions
  - Generalities
  - Cobb-Douglas
  - Leontief
  - CES

- Utility Functions
  - Linear Demand
  - Cobb-Douglas
  - CES
  - Stone-Geary

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Utility and Production Function

- Utility and production functions can have the same mathematical formula with different variables.

- In both cases, we maximize them subject to a certain constraint.

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The Production Function

- Economists often use a production function to describe the relationship between the quantity of inputs used in production and the quantity of output from production.
- $Y = A\,F(L, K, H, N)$
  - Y = quantity of output
  - A = available production technology
  - L = quantity of labor
  - K = quantity of physical capital
  - H = quantity of human capital
  - N = quantity of natural resources
  - $F(\ )$ is a function that shows how the inputs are combined.

The Production Function

- A production function has constant returns to scale if, for any positive number x,

$$xY = A\ F(xL, xK, xH, xN)$$

- That is, a doubling of all inputs causes the amount of output to double as well.

The Production Function

- Production functions with constant returns to scale have an interesting implication.
  - Setting $x = 1/L$,
  - $Y/L = A\,F(1, K/L, H/L, N/L)$

    Where:

    Y/L = output per worker

    K/L = physical capital per worker

    H/L = human capital per worker

    N/L = natural resources per worker

The Production Function

- The preceding equation says that productivity (Y/L) depends on physical capital per worker (K/L), human capital per worker (H/L), and natural resources per worker (N/L), as well as the state of technology, (A).

Growth Accounting

$$Y = AL^{\alpha} K^{1-\alpha}$$

$$\text{Log } Y = \log(A) + \alpha \text{ Log}(L) + (1-\alpha) \text{ Log}(K)$$

Since the log is equal to growth rate

$$\mathbf{g_Y = g_A + \alpha\, g_L + (1-\alpha)\, g_K}$$

Therefore,

Growth rate of output =

growth rate of technology

+ share of labor * growth rate of labor

+ share of capital* growth rate of capital

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The General Form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Marginal productivities.
- Second derivatives.
- Interpretation of alpha.
- Increasing capital only.
- Increasing labor only.
- Increasing both.

Cobb-Douglas Production Function

- Suppose that the production function is

$$q = f(K,L) = AK^{a}L^{1-a} \quad A,a,1\text{-}a > 0$$

- This production function can exhibit any returns to scale

$$f(tK,tL) = A(tK)^{a}(tL)^{1-a} = At^{a+1-a}\,K^{a}L^{1-a} = t^{a+1-a}f(K,L)$$

  - if $a + b = 1 \Rightarrow$ constant returns to scale
  - if $a + b > 1 \Rightarrow$ increasing returns to scale
  - if $a + b < 1 \Rightarrow$ decreasing returns to scale

Cobb-Douglas Production Function

- The Cobb-Douglas production function is linear in logarithms

$$\ln q = \ln A + a \ln K + (1-a) \ln L$$

  - a is the elasticity of output with respect to K
  - $1-a$ is the elasticity of output with respect to L

Properties

(i) There are constant returns to scale.

(ii) Elasticity of substitution is equal to one.

(iii) α and β represent the labour and capital shares of output respectively.

(iv) α and β are also elasticities of output with respect to labour and capital respectively.

(v) If one of the inputs is zero, output will also be zero.

(vi) The expansion path generated by C-D function is linear and it passes through the origin.

Properties

(vii) The marginal product of labor is equal to the increase in output when the labor input is increased by one unit.

(viii) The average product of labor is equal to the ratio between output and labor input.

(ix) The ratio α / β measures factor intensity. The higher this ratio, the more labor intensive is the technique and the lower is this ratio and the more capital intensive is the technique of production.

Importance

(i) It suits to the nature of all industries.

(ii) It is convenient in international and inter-industry comparisons.

(iii) It is the most commonly used function in the field of econometrics.

(iv) It can be fitted to time series analysis and cross section analysis.

(v) The function can be generalised in the case of ‘n’ factors of production.

(vi) The unknown parameters α and β in the function can be easily computed.

(vii) It becomes linear function in logarithm.

(viii) It is more popular in empirical research.

Drawbacks

(i) The function includes only two factors and neglects other inputs.

(ii) The function assumes constant returns to scale.

(iii) There is the problem of measurement of capital which takes only the quantity of capital available for production.

(iv) The function assumes perfect competition in the factor market which is unrealistic.

(v) It does not fit to all industries.

(vi) It is based on the substitutability of factors and neglects complementarity of factors.

(vii) The parameters cannot give proper and correct economic implication.

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Fixed Proportions

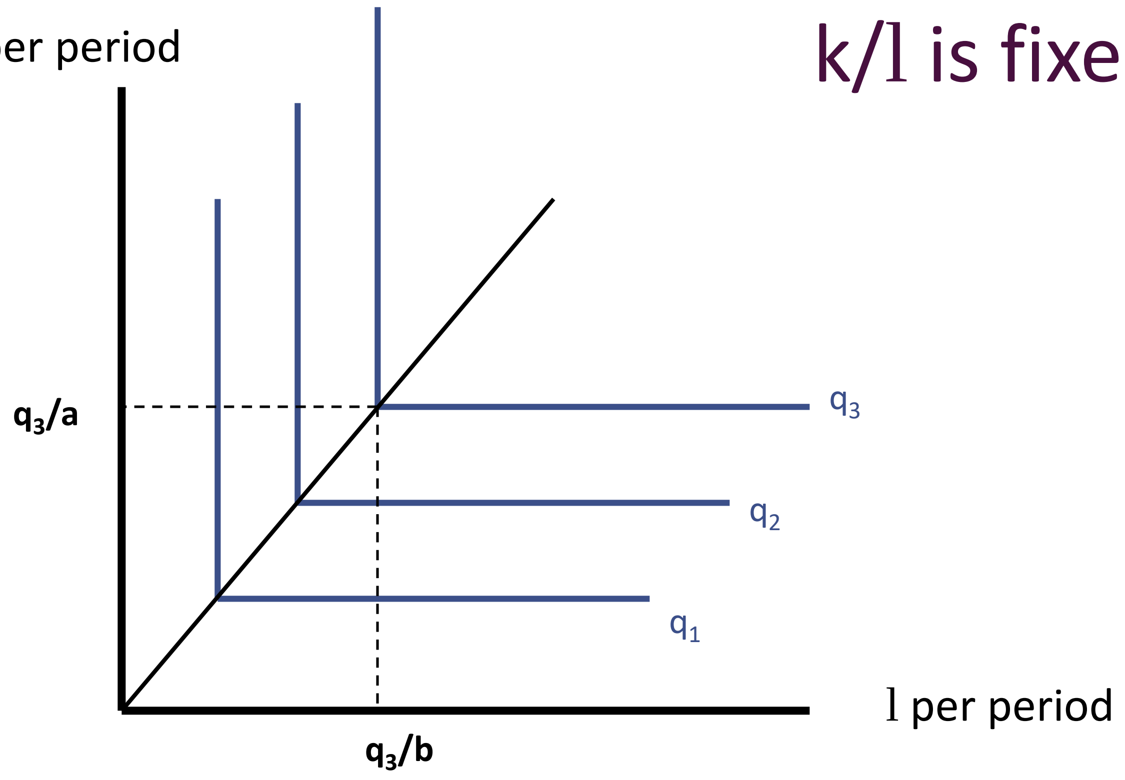
- Suppose that the production function is

$$q = \min(ak, bl) \quad a,b > 0$$

- Capital and labor must always be used in a fixed ratio
  - the firm will always operate along a ray where k/l is constant
- Because k/l is constant, $\sigma = 0$

Fixed Proportions

No substitution between labor and capital is possible



k/l is fixed at b/a

$\sigma = 0$

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The CES Production Functions

The constant elasticity of substitution production functions dominates in applied research. The parametric structure is

(1) $$Y = A\,[\,\theta(a_K K)^{\gamma} + (1-\theta)\,(a_N N)^{\gamma}\,]^{1/\gamma}.$$

Here $0 < \theta < 1$ is the share parameter and γ determines the degree of substitutability of the inputs. The parameters A, a_K, and a_N depend upon the units in which the output and inputs are measured and play no important role. The value of γ is less than or equal to 1 and can be $-\infty$. The two extreme cases are when $\gamma = 1$ or $\gamma = -\infty$.

The Case of Perfect Substitution ($\gamma = 1$): The function is

(2) $$Y = A\,[\,\theta\, a_K K + (1-\theta)\, a_N\, N\,].$$

The isoquants are straight lines for this production function.

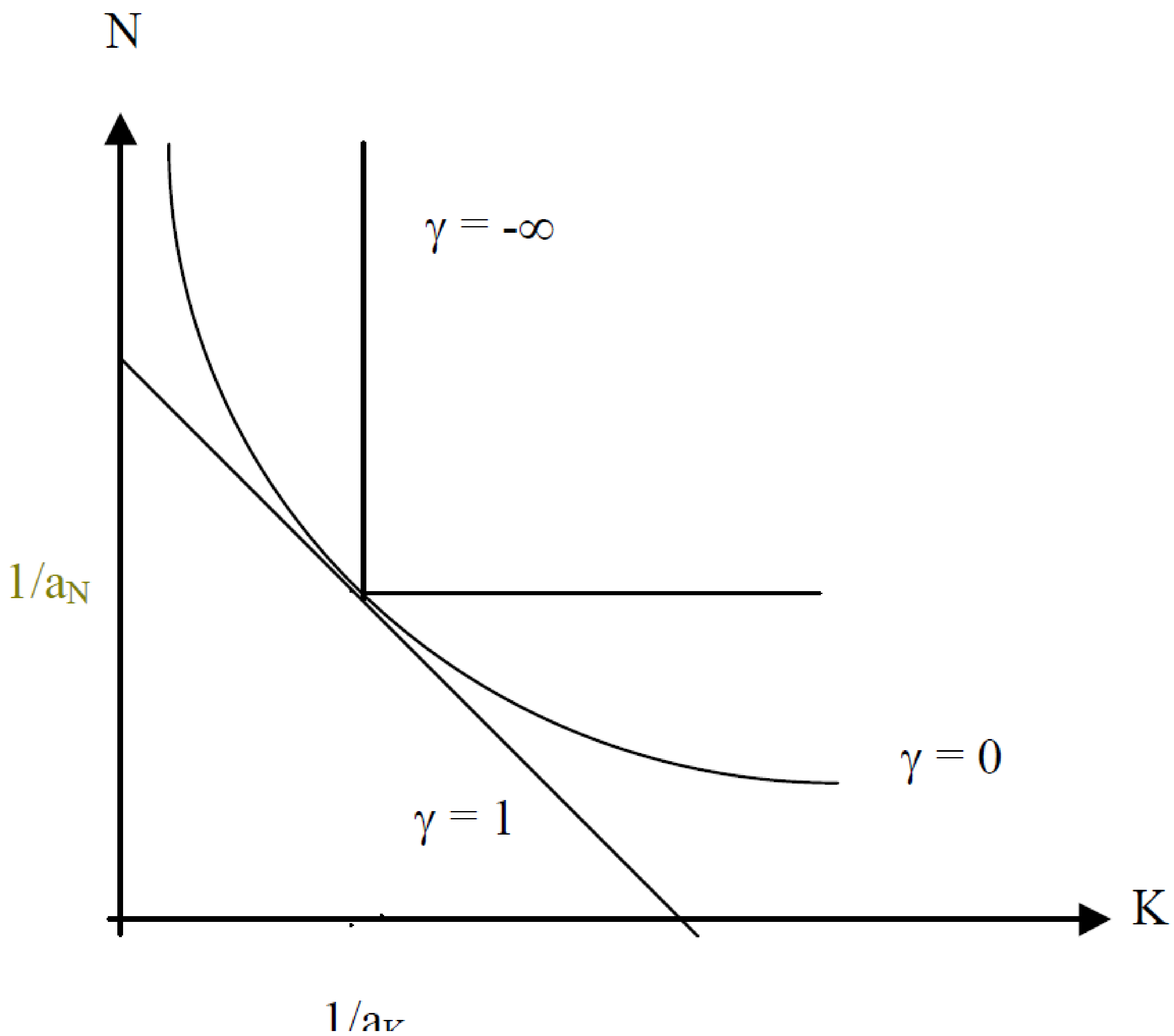
The Case of no Substitution ($\gamma = -\infty$): The function is

(3) $$Y = A \min\{ a_K K, a_N N \}.$$

The isoquants are at right angles. Factors are used in fixed proportions

The Case of Unit Elasticity of Substitution ($\gamma = 0$): The function is

(4) $$Y = A\, K^{\theta}\, N^{(1-\theta)}.$$


N
γ = -∞
1/a_N
γ = 0
γ = 1
K
1/a_K

Properties

(i) The value of elasticity of substitution depends upon the value of substitution parameter.

(ii) The marginal products of labour and capital are always positive if we assume constant returns to scale.

(iii) The marginal product of an input will increase when other factor inputs increase

(iv) When the elasticity substitution is less than unity the function does reach a finite maximum as one factor increases while other is held constant.

(v) The marginal product curves are sloping downward.

(vi) The estimation of the elasticity of substitution parameter requires the assumption of perfect competition.

Merits

(i) CES production function is more general.

(ii) CES covers all types of returns.

(iii) CES function takes account of a number of parameters.

(iv) CES function takes account of raw material among its inputs.

(v) CES function is very easy for estimation.

(vi) CES function is free from unrealistic assumptions.

Drawbacks

(i) The generated function suffers from the drawback that elasticity of substitution between any parts of inputs in the same which does not appear to be realistic.

(ii) In estimating parameters of CES production function, we may encounter a large number of problems like choice of exogenous variables, estimation procedure and the problem of multicollinearities.

(iii) Serious doubts have been raised about the possibility of identifying the production function under technological change.

In a nutshell

- Production function:
  - Leontieff between inputs and VA
- VA function:
  - CES between factors of production
  - Sometimes Cobb-Douglas (less)
- Intermediate inputs:
  - CS between different inputs

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Keynesian Consumption Function

- *Consumption Function*

$$C = A + mpc \times [GDP - TAX]$$

- *C* = Household Consumption Expenditure
- *A* = Autonomous Consumption { Consumption not dependent on current income}
- *mpc* = Marginal propensity to consume
  - {Fraction of extra income will be spent on consumption}
  - mpc will be smaller than consumption to GDP ratio if A is positive.

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Cobb-Douglas Demand Functions

- Cobb-Douglas utility function:

$$U(X,Y) = X^{\alpha}Y^{\beta}$$

- Setting up the Lagrangian:

$$\mathbf{L} = X^{\alpha}Y^{\beta} + \lambda(I - P_X X - P_Y Y)$$

- First-order conditions:

$$\partial\mathbf{L}/\partial X = \alpha X^{\alpha-1}Y^{\beta} - \lambda P_X = 0$$

$$\partial\mathbf{L}/\partial Y = \beta X^{\alpha}Y^{\beta-1} - \lambda P_Y = 0$$

$$\partial\mathbf{L}/\partial\lambda = I - P_X X - P_Y Y = 0$$

Cobb-Douglas Demand Functions

- First-order conditions imply:

$$\alpha Y/\beta X = P_X/P_Y$$

- Since $\alpha + \beta = 1$:

$$P_Y Y = (\beta/\alpha)P_X X = [(1- \alpha)/\alpha]P_X X$$

- Substituting into the budget constraint:

$$I = P_X X + [(1- \alpha)/\alpha]P_X X = (1/\alpha)P_X X$$

Cobb-Douglas Demand Functions

- Solving for *X* yields

$$X^* = \frac{\alpha I}{P_X}$$

- Solving for Y yields

$$Y^* = \frac{\beta I}{P_Y}$$

- The individual will allocate α percent of his income to good X and β percent of his income to good Y

Cobb-Douglas Demand Functions

- The Cobb-Douglas utility function is limited in its ability to explain actual consumption behavior
  - the share of income devoted to particular goods often changes in response to changing economic conditions
- A more general functional form might be more useful in explaining consumption decisions

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CES Demand

- Assume that $\delta = 0.5$

$$U(X,Y) = X^{0.5} + Y^{0.5}$$

- Setting up the Lagrangian:

$$\mathbf{L} = X^{0.5} + Y^{0.5} + \lambda(I - P_X X - P_Y Y)$$

- First-order conditions:

$$\partial \mathbf{L}/\partial X = 0.5X^{-0.5} - \lambda P_X = 0$$

$$\partial \mathbf{L}/\partial Y = 0.5Y^{-0.5} - \lambda P_Y = 0$$

$$\partial \mathbf{L}/\partial \lambda = I - P_X X - P_Y Y = 0$$

CES Demand

- This means that

$$(Y/X)^{0.5} = P_x/P_Y$$

- Substituting into the budget constraint, we can solve for the demand functions:

$$X^* = \frac{I}{P_X[1 + \frac{P_X}{P_Y}]}$$

$$Y^* = \frac{I}{P_Y[1 + \frac{P_Y}{P_X}]}$$

CES Demand

- In these demand functions, the share of income spent on either *X* or *Y* is not a constant
  - depends on the ratio of the two prices
- The higher is the relative price of *X* (or *Y*), the smaller will be the share of income spent on *X* (or *Y*)

CES Demand

- If δ = -1,

$$U(X,Y) = X^{-1} + Y^{-1}$$

- First-order conditions imply that

$$Y/X = (P_X/P_Y)^{0.5}$$

- The demand functions are

$$X^* = \frac{I}{P_X[1+\left(\frac{P_Y}{P_X}\right)^{0.5}]} \qquad Y^* = \frac{I}{P_Y[1+\left(\frac{P_X}{P_Y}\right)^{0.5}]}$$

CES Demand

- The elasticity of substitution (σ) is equal to $1/(1-\delta)$
  - when $\delta = 0.5$, $\sigma = 2$
  - when $\delta = -1$, $\sigma = 0.5$
- Because substitutability has declined, these demand functions are less responsive to changes in relative prices
- The CES allows us to illustrate a wide variety of possible relationships

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Stone-Geary (1951 and 1954) utility function

- There is a minimal level of each good that has to be consumed irrespective of its price or the consumer's income.
- Moreover, spending on a commodity consists of spending on the minimum required quantity for that commodity plus the proportion of the budget which is left over after paying for all minimum requirements.
- This proportion is the marginal budget share that determines the allocation of supernumerary income.

$$PC_{i,t}C_{i,h,t} = PC_{i,t}C_{i,h}^{min} + \gamma_{ih}(YDH_{h,t} - \sum_{i} PC_{i,t}C_{ih}^{min})$$

References

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Thanks for your attention!