## SELECTING A SOCIAL CHOICE RULE: AN

## EXPLORATORY PANEL STUDY

Murat R. Sertel and Ayca E.G. Kara*

Bogazici University
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## Abstract

This study is an attempt to empirically understand public preferences concerning social choice rules. We focus on four social choice rules (SCRs): Plurality, Plurality with a Run-off, the Majoritarian Compromise (MC) and Borda's SCR. We confront our subjects with hypothetical preference profiles of a hypothetical electorate over some abstract set of alternatives at which the four SCRs all disagree and we ask each subject which alternative should be chosen and which should be eliminated for the society whose preference profile is shown. The study was conducted with 288 subjects who were confronted with five preference profiles each. We found a very clear support for the MC and Borda's SCR, the subjects generally not favoring Plurality or Plurality with a Run-off. We noticed, however, that many of the subjects who chose the alternative which would be chosen by Borda's SCR were actually choosing it because it coincided with the Social Compromise winner. As a result, the net outcome of our study is the finding that our subjects strongly favor the MC, then a conglomerate of Borda's SCR and the SC, strongly disfavoring Plurality and even more so, Plurality with a Run-off.

## I. Introduction

This study is an attempt to empirically understand public preferences concerning social choice rules. We focus on four social choice rules (SCRs): Plurality, Plurality with a Run-off, the Majoritarian Compromise (MC) and Borda's SCR. We confront our subjects with hypothetical preference profiles of a hypothetical electorate over some abstract set of alternatives, and we ask each subject which alternative should be chosen for the society whose preference profile is shown. The least cluttered case that will serve our purposes is that of a set $\mathrm{A}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ of four alternatives and a 'society' of seven agents, each of which linearly orders these four alternatives, so that a typical preference profile here can be represented as a $4 \times 7$ table or matrix such as

| a | a | a | b | b | c | d |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| c | c | c | d | d | d | c |
| d | d | d | a | a | b | b |
| b | b | b | c | c | a | a |

Each column shows the ordering of A by a hypothetical agent in our hypothetical society, the agent's best alternative appearing at the top, the agent's second best alternative appearing in the second row, the agent's third best alternative appearing at the third row and the agent's worst alternative appearing at the bottom.
The preference profiles are chosen so that our four SCRs all disagree. Thus, one of them will choose ' $a$ ', another one choose ' $b$ ', another will choose ' $c$ ', and the remaining SCR will choose ' $d$ '. In the case of our above preference profile, for instance, plurality chooses ' $a$ ', plurality with a Run-off chooses ' $b$ ', MC chooses ' $c$ ' and Borda's SCR chooses ' d '.

If a subject tells us that, in such a society, 'a' (respectively b,c,d) should be chosen, we interpret this as the subject's preferring the implementation of the SCR of Plurality (respectively Plurality with a Run-off, the MC, Borda's SCR) to that of the other three SCRs

Our study was conducted with 288 subjects who were confronted with five preference profiles each, where the fifth was always the inverse of the first, showing the exact reverse orderings of A by the seven agents. Each subject was asked, looking at each preference profile, which alternative should be chosen and which should not. (The fifth profile being the inverse allowed as carrying out a consistency test on the subjects' responses.) We found a very clear support for the MC and Borda's SCR, the subjects generally not favoring Plurality or Plurality with a Run-off.

We noticed, however, that many of the subjects who chose the alternative which would be chosen by Borda's SCR were, according to their explanations, actually choosing it because it coincided with the SCR called Social Compromise. (Sertel, 1995 , 1998) As a result, when we deduct from the apparent Borda choosers, the subjects who were actually choosing the Social Compromise (SC), the net outcome of our studies is the finding that our subjects strongly favor the MC, then a conglomerate of Borda's SCR and the SC, strongly disfavoring Plurality and even more so, Plurality with a Run-off.

Now let us give a preview of the rest of the paper. Section II defines the SCRs we are focusing on and presents the design of the empirical study. In Section III, we present our empirical findings and discuss several consistency tests, which we applied in this study. Section IV gives our closing remarks.

## II. The Design

In this study, the subjects were presented hypothetical preference profiles concerning hypothetical alternatives (candidates) and were asked what they thought should be the electoral outcome. The profiles were designed so that each alternative coincides with a different social choice rule winner. The four social choice rules chosen for the experiments were Plurality, Plurality with a Run-off, Majoritarian Compromise and Borda's SCR.

## Definition:

a) The Plurality Rule is a typical "tops only" social choice rule. In any preference profile it chooses the alternatives receiving the greatest first-degree support of the electorate.
b) Plurality with a Run-off is a regular ${ }^{2}$ social choice rule as is Plurality. They differ in that if there is no alternative, which a strict majority ranks as best, Plurality with a Run-off declares as "first- round winners" the alternatives receiving the highest number of top-rank votes. For the second round of voting, the preference profile is restricted to only first-round winners and the Plurality Rule is then applied to the second round of voting.

[^0]c) The Majoritarian Compromise is a social choice rule introduced by Sertel (1986) and analyzed in detail by Sertel and Yilmaz $(1995,1998)$ and by Hurwicz and Sertel (1995). In contrast with that of the "tops only" category it takes into account more than the best alternatives of the voters. In the Majoritarian Compromise rule the first choice alternative in the preference profile wins only if that choice has also received the majority of the votes. If there is no such alternative, then the number of times each alternative is ranked either the first or the second is totaled. If any alternative receives majority of votes as the first or the second best choice, then the one gaining the maximal number of such approvals is picked as the winner of second degree of majority approval. ${ }^{3}$ In a case where there are several such candidates, a pre-agreed tiebreaking rule is applied. If no alternative has a majority, the same system is applied to the third level of preferences, and so on down through the preference profile levels until an alternative (alternatives) is (are) found with a majority of the votes. However, it is proved in Sertel and Yilmaz (1998) that the critical degree of majority approval $\mathrm{k}^{*}$ never exceeds the effective half of the number of available alternatives. (Thus, in our case of four alternatives, the critical degree of majority approval is two. Hence, the alternative that receives majority of votes either as the first or the second best choice is picked as the winner. It is guaranteed by the constraint imposed to the profiles to assure the SCRs to have distinct outcomes that each of the four SCRs picks a unique alternative as the winner without a need for a tie-breaking rule.)
d) Borda's SCR assigns a score to each of the available alternatives in a descending order. In a preference profile with four alternatives, for instance, if an alternative is ranked first, it takes a score of 4 points and if it is ranked the second then it would take 3 points, and so forth. The score of each alternative is summed across the electorate. The Borda winner is the alternatives with the most points (Borda, 1781).
A computer program, executable on a PC, was written to generate preference profiles at which each of the social choice rules defined above picks a distinct alternative as the electoral outcome. (Inal, 1999) The minimal cardinality of the set of individuals defining the society which enables each social choice rules in question to choose unique and distinct alternatives was found to be seven ${ }^{4}$. The very immediate reason for this is that, in the profiles with voters less than seven in number, the plurality and run-off rules pick the same alternative.

[^1]Let us now demonstrate through an example how our constraint enables the SCRs in question to pick distinct alternatives. How should the first row of the preference profiles look? First of all, can there be an alternative that appears more than 3 times in the first row? No, because that alternative, then, would be picked by Plurality, Plurality with a Run-off and Majoritarian Compromise as the winner. On the other hand, it is impossible not to have an alternative that appears more than 2 times in the first row since that, in a matrix of $4 \times 7$, would cause Plurality to pick more than one alternative as the electoral outcome. Or, consider the case where an alternative shows up 3 times and two other alternatives appear 2 times each in the first row. Then, Plurality with a Run-off would pick all three of them as the winners of the first round, but with only 7 voters, there would not be a second round that is different than the first. Thus, Run-off would choose the alternative that appears 3 times in the first row, as would Plurality. That is, the following is the way that the first row of the preference profiles must look like:
$\begin{array}{llllll}\text { a } & \text { a } & \text { a } & \text { b } & \text { b }\end{array}$
When the preference profiles with the dimension of 4(alternatives)x7(voters) fulfilling our constraint were refined so that the 'neutral versions's of each profile were ignored, 72 profiles were left. If the profiles are to be refined so that both anonymous ${ }^{6}$ and neutral versions are ignored, then we get only 3 profiles, which we call 'root profiles'.

A very important common characteristic of the 72 profiles is the nonexistence of the Condorcet winner. There are (4!) ${ }^{7}$ (around 4.5 billion) preference profiles with 7 voters and 4 alternatives. We also know through the celebrated work of Fishburn (1973) that approximately 15 percent of the profiles of this amount contain Condorcet cycle.
The reason for the nonexistence of the Condorcet winner in these profiles is our strong constraint imposed to the profiles guaranteeing the disagreement of the four social choice rules.

Each menu presented to the subjects contained four original profiles that were picked randomly without replacement out of 72 profiles. That is, 18 menus were created in order for all 72 profiles to be seen by the subjects. We decided to repeat this procedure 16 times so as to obtain a large enough sample to run the study on. Hence,

[^2]each of the 72 original profiles was seen 16 times and we had 288 subjects in total who were involved in the study.
The experiment sessions were run at Bogazici University. The subjects were students, mainly undergraduates, from 31 different departments (see Appendix I) and invited through ads placed in many localities on the campus. The subjects were paid $\$ 5$ each as the show-up price, but no additional payments took place. In order to avoid any kind of bias, the subjects were not informed about the team who was running the experiment and were not asked to put their names on the questionnaire.
On the cover of the each menu, the questionnaire was introduced, including a definition of the notion of ranking of alternatives. Each of the five profiles appeared on different sheets and for each of the profiles, the subject was asked "taking an impartial point of view," to indicate the alternative, namely ' $a$ ', ' $b$ ', ' $c$ ' or ' $d$ ', that they think should be adopted as the electoral outcome and the alternative he/she thinks should especially be avoided. The subjects were also required to give their reasoning along with their answers at each of the profile in their menus.(See Appendix II for a sample questionnaire) This was intended to help identity of the method followed by the subject in adopting or avoiding a particular alternative in each case, as well as to encourage the subjects to concentrate on their questionnaires. The average time spent by the subjects to answer the questionnaires was 18.3 minutes and the median timing was 16 minutes.

## III. Empirical Findings of the Study

## 1. The Tools used for the Analysis of the Data

The data collected during the experiment is analyzed on the basis of profiles. The number of profiles is counted for each of the social choice rule in question where the subjects state that rule as the top and the bottom choice. In addition, the recorded information about the subjects such as gender, department, school year and the time spent by each subject for the questionnaire, in order to make possible a further analysis of the data.
We used three main consistency measures to understand the level of consistency that each subject displays in the way that he/she answerd the questions:
Consistency Check \#1 (C1): The relation between the first and the fifth profiles in each menu is the essence of this consistency check. Since the fifth profile in each menu is a reversed version of the first profile that appears in that menu, a consistent subject is expected to state the representative of the winner of a social choice rule as the one to be avoided in the fifth profile while stating the representative of the same rule winner as the one to be adopted in the first profile. The subjects displaying such
kind of consistency are recorded as the consistent subjects . Similarly, the subjects who stated the representative of the winner of thesocial choice rule as the one to be avoided in the first profile while stating the representative of the same rule winner as the one to be adopted at the fifth profile are considered to be C1-10 consistent. If a subject displays a consistency in terms of both of the criteria described above, this subject is regard as C1-11 consistent. If the subject does not fulfill any of the consistency requirements of this type, then he/she is labeled as inconsistent.

Consistency Check \#2 (C2): This consistency measure counts the number of profiles at which a subject picks the representative of the same social choice rule winner as the one to be adopted (C21) and counts the number of the profiles at which a subject chooses the representative of the same rule winner as the one to be avoided (C22).

If a subject chooses the representative of the winner of each rule as the one to be adopted once in four of the original profiles he/she has in his/her menu, he/she is considered to be of the consistency level 1 in terms of C21. If he/she picks the representative of the winner of two different rules as the ones to be adopted two times each, then he/she is regarded to be of the level 2. Similarly, if he/ she states the representative of the same rule winner as the one to be adopted in three out of four profiles, then he/she is considered to be consistent of the level 3. When a subject chooses the representative of the same rule winner as the electoral outcome in all of the four profiles, this subject is regarded as a fully consistent agent (of level 4) according to consistency check C21. The same procedure works for the answers to the question concerning the alternative to be avoided (C22).
Consistency Check \#3 (C3): This measure takes the value of 0 if a subject chooses the representative of the same social choice rule winner as the one to be adopted in one profile and the one to be avoided in another profile. A subject is considered to be consistent (of level 1) in terms of C3 if and only if he/she does not pick the representative of the same rule winner as the one to be avoided in one profile while stating it as the one to be adopted in another preference profile.

## 2. Main Results of the Study

## (i) Overall results

1. The number of preference profiles presented to the subjects which are the anonymous versions of the 72 original profiles (the number of the profiles which appear as the first four profiles in each menu) is 1152 . The number of the valid profiles is 1090 out of 1152 .
2. The number of the profiles at which the alternative indicated as the one to be adopted represents

Plurality Rule: 112
Plurality with Run-off Rule: 22
Majoritarian Compromise: 468
Borda's SCR: 488

3. The percentage of the valid profiles at which the alternative indicated as the one to be adopted represents:
Plurality Rule: 10.27 percent percent
Plurality with Run-off Rule: 2 percent percent
Majoritarian Compromise: 42.93 percent percent
Borda's SCR: 44.77 percent percent

4. The number of the profiles at which the alternative indicated as the one to be avoided represents:
Plurality Rule: 425
Plurality with Run-off Rule: 585

Majoritarian Compromise: 43
Borda's SCR: 37

## Overall Results(Avoided Alternative)


5. The percentage of the valid profiles at which the alternative stated as the one to be avoided represents:
Plurality Rule: 39 percent percent
Plurality with Run-off winner: 53.67 percent percent
Majoritarian Compromise: 3.9 percent percent
Borda's SCR: 3.4 percent percent

## Overall Results (Avoided Alternative)


(ii) Distribution of top and bottom choices with respect to gender
6. The number of female subjects who gave (partially or completely) valid answers to the questions: 96
7. The number of male subjects who gave (partially or completely valid answers to the questions: 178
8. Distribution of Top Choices for the Profiles answered by the Female Subjects

Plurality: 52 ( 13.6 percent percent)
Run-off: 12 (3.1 percent percent)
MC: 152 (39.8 percent)
Borda: 166 ( 43.5 percent)

9. Distribution of Bottom Choices for the Profiles answered by the Female Subjects Plurality: 145 ( 37.8 percent)
Run-off: 203 (52.9 percent)
MC: 19 (4.9 percent)
Borda: 17 (4.4 percent)

10. Distribution of Top Choices for the Profiles answered by Male Subjects Plurality: 60 ( 9 percent)
Run-off: 10 ( 1.5 percent)
MC: 316 (44 percent)
Borda: 322 ( 45.5 percent)

## Distribution of TOP Choices (MALES)


11. Distribution of Bottom Choices for the Profiles answered by Male Subjects Plurality: 280 ( 39.6 percent)
Run-off: 382 (54.2 percent)
MC: 24 ( 3.4 percent)
Borda: 20 ( 2.8 percent)

(iii) Distribution of top and bottom choices under different types and levels of consistency tests
12. C1-01

Total number of profiles answered by the subjects who displayed consistency of C101: 100

TOP:
Plurality: 11 (11 percent)

13. C1-10

Total number of profiles answered by the subjects who displayed consistency of C110: 264

TOP:

Plurality: 28 (10.4 percent)
Run-off: 7 (2.6 percent)
MC: 124 (47 percent)
Borda: 105 (40 percent)


BOTTOM:
Plurality: 88 (33.4 percent)
Run-off: 165 ( 62.5 percent)
MC: 8 (3 percent)
Borda: 3 (1.1 percent)

14. C1-11

Total number of profiles answered by the subjects who displayed consistency of according C1-11: 312

TOP:
Plurality: 30 ( 9.7 percent)
Run-off: 1 ( 0.3 percent)
MC: 171 ( 55 percent)
Borda: 110 (35 percent)


BOTTOM:
Plurality: 71 (22.7 percent)
Run-off: 231 (74 percent)
MC: 6 (2 percent)
Borda: 4 (1.3 percent)


Distribution of TOP choices with respect to different kinds of C1 (\%)

| Plurality | Run-off | MC | Borda |
| ---: | :---: | :--- | :--- |


| C1-01 | 11 | 2 | 56 | 31 |
| :---: | :---: | :---: | :---: | :---: |
| C1-10 | 10.4 | 2.6 | 47 | 40 |
| C1-11 | 9.7 | 0.3 | 55 | 35 |




Distribution of BOTTOM Choices with Respect to Different Kinds and Levels of C1 (\%)

|  | Plurality | Run-off | MC | Borda |
| :---: | :---: | :---: | :---: | :---: |
| C1-01 | 45 | 42 | 5 | 8 |
| C1-10 | 33.4 | 62.5 | 3 | 1.1 |
| C1-11 | 22.7 | 74 | 2 | 1.3 |



Total number of profiles answered by the subjects with $\mathrm{C} 21=1: 123$
Plurality: 32 ( 26 percent)
Run-off: 11 (9 percent)
MC: 39 (31.7 percent)
Borda: 41 (33.3 percent)

$\mathrm{C} 21=2$
Total number of profiles answered by the subjects who displayed consistency of level 2 according to the consistency check C21: 176

TOP:
Plurality: 14 ( 9.5 percent)
Run-off: 6 ( 3.5 percent)
MC: 78 (44 percent)
Borda: 78 (44 percent)


Total number of profiles answered by the subjects who displayed consistency of leve 3 according to the consistency check C21:364

TOP:
Plurality: 29 (7.9 percent)
Run-off: 4 (1.1 percent)
MC: 179 (49 percent)
Borda: 152 (42 percent)

## Distribution of TOP Choices (C21=3)


$\mathrm{C} 21=4$
Total number of profiles answered by the subjects who displayed consistency of level 4 according to the consistency check C21: 416

TOP:
Plurality: 36 (9 percent)
Run-off: 0
MC: 168 (40 percent)
Borda: 212 (51 percent)


Distribution of TOP Choices with respect to Different Levels of C21 (\%)

|  | Plurality | Run-Off | MC | Borda |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{C} 21=1$ | 26 | 9 | 31.7 | 33.3 |
| $\mathrm{C} 21=2$ | 9.5 | 3.5 | 44 | 44 |
| $\mathrm{C} 21=3$ | 7.9 | 1.1 | 49 | 42 |
| $\mathrm{C} 21=4$ | 9 | 0 | 40 | 51 |

Distribution of TOP Choices w.r.t. Different Levels of C21


C22=1
Total number of profiles answered by the subjects with C22=1:112
Plurality: 38 ( 34 percent)
Run-off: 39 ( 34.8 percent)
MC: 23 (20.5 percent)
Borda: 12 (10.7 percent)

$\mathrm{C} 22=2$
Total number of profiles answered by the subjects who displayed consistency of level 2 according to the consistency check C22: 223

BOTTOM:
Plurality: 110 (49.3 percent)
Run-off: 111 (49.7 percent)
MC: 2 (1 percent)
Borda: 0

$\mathrm{C} 22=3$
Total number of the profiles answered by the subjects who displayed consistency of level 3 according to the consistency check C22: 480

## BOTTOM:

Plurality: 204 (42.5 percent)

Run-off: 244 (50.8 percent)
MC: 13 ( 2.8 percent)
Borda: 19 (3.9 percent)


$$
\mathrm{C} 22=4
$$

Total number of profiles answered by the subjects who displayed consistency of level 4 according to the consistency check C22: 264

## BOTTOM:

Plurality: 68 ( 25.7 percent)
Run-off: 188 (71.3 percent)
MC: 4 ( 1.5 percent)
Borda: 4 ( 1.5 percent)


Distribution of BOTTOM Choices with respect to Different Levels of C22 (\%)

|  | Plurality | Run-Off | MC | Borda |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{C} 22=1$ | 34 | 34.8 | 20.5 | 10.7 |
| $\mathrm{C} 22=2$ | 49.3 | 49.7 | 1 | 0 |
| $\mathrm{C} 22=3$ | 42.5 | 50.8 | 2.8 | 3.9 |
| $\mathrm{C} 22=4$ | 25.7 | 71.3 | 1.5 | 1.5 |


$\mathrm{C} 3=1$
Total number of profiles answered by the subjects who displayed consistency of level 1 according to the consistency check C3: 820

TOP:
Plurality: 43 ( 5.3 percent)
Run-off: 5 ( 0.5 percent)
MC: 371 ( 45.2 percent)
Borda: 400 (48.9 percent)


## BOTTOM

Plurality: 322 (39.4 percent)
Run-off: 460 ( 56 percent)
MC: 18 (2.2 percent)
Borda:19 (2.4 percent)

3. The Root Profiles and the Distribution of Top and Bottom Choices

The number of preference profiles with four alternatives and seven members fulfilling our constraint is declined to 72 after the anonymous versions are ignored. If the neutrality filter is imposed to the original 72 profiles, that is, if the versions that can be achieved through interchanging the names of the alternatives as well as the names of the members are eliminated, then only 3 profiles remain. We call these profiles 'root profiles'.

The structure of each root profile is presented below. Note that B, MC, P, R stand for Borda winner, Majoritarian Compromise winner, Plurality winner and Run-off winner respectively.

## Root Profile \#1:

| Member | Member | Member | Member | Member | Member | Member |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{2}$ |  | $\mathbf{3}$ |  | $\mathbf{4}$ |  |  |  |  |  |  | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ |
| P | P | P | R | R | MC | B |  |  |  |  |  |  |  |  |
| MC | MC | MC | B | B | B | MC |  |  |  |  |  |  |  |  |
| B | B | B | P | P | R | R |  |  |  |  |  |  |  |  |
| R | R | R | MC | MC | P | P |  |  |  |  |  |  |  |  |

Root Profile \#2:

| Member | Member | Member | Member | Member | Member | Member |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P | P | P | R | R | MC | B |
| MC | MC | B | B | MC | B | MC |
| B | B | R | P | B | R | R |
| R | R | MC | MC | P | P | P |

## Root Profile \#3:

| Member | Member | Member | Member | Member | Member | Member |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P | P | P | R | R | MC | B |
| MC | B | B | MC | MC | B | MC |
| B | R | R | B | B | R | R |
| R | MC | MC | P | P | P | P |

The distribution of top and bottom choices with respect to the type of root profiles is as follows:
Top Choice

| Root Profile | Plurality | Run-off | MC | Borda |
| :--- | :--- | ---: | ---: | ---: |
| $\# 1$ | $69(19 \%)$ | $1(0.3 \%)$ | $138(38.1 \%)$ | $154(42.6 \%)$ |
| $\# 2$ | $29(8.1 \%)$ | $1(0.3 \%)$ | $166(45.8 \%)$ | $166(45.8 \%)$ |
| $\# 3$ | $14(3.8 \%)$ | $20(5.5 \%)$ | $164(44.9 \%)$ | $167(45.8 \%)$ |



Bottom Choice

| Bottom Choice |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | ---: | :---: | :---: | :---: | :---: |
| Root Profile | Plurality | Run-off | MC | Borda |  |  |  |  |
| $\# 1$ | $17(4.7 \%)$ | $324(89.5 \%)$ | $9(2.5 \%)$ | $12(3.3 \%)$ |  |  |  |  |
| $\# 2$ | $143(39.6 \%)$ | $179(49.6 \%)$ | $23(6.4 \%)$ | $16(4.4 \%)$ |  |  |  |  |
| $\# 3$ | $265(72.4 \%)$ | $82(22.4 \%)$ | $11(3 \%)$ | $8(2.2 \%)$ |  |  |  |  |

15. Root profile \#1: The type of profile at which Plurality winner shows up two times at the bottom row. (That is Plurality winner happens to be the last choice of two out of seven members of the society.)
16. Root profile \#2: The type of profile at which Plurality winner shows up three times at the bottom row.
17. Root profile \#3: The type of profile at which Plurality winner shows up four times at the bottom row.

## Distribution of BOTTOM Choices

 w.r.t. Root Profiles

## 4. An Additional Consistency Check and the Distribution of Top and Bottom <br> Choices

The presence of four preferences profiles in each menu which are randomly picked out of 72 original profiles means that each subject is made sure to get at least two versions of the same root profile in his/her menu. This gives the opportunity to compare the answers given by the subject to the two versions of the same root profile. A consistent subject is expected to state the representative of the winner of the same social choice rule as the alternative to be adopted and to display the same consistent behavior for the alternative to be avoided. This new criterion of consistency is called the consistency check C4.

Let us now present the levels of consistency in terms of C4:
C4-1: A subject is C4-1 consistent if he/she picks the representative of the winner of the same SCR as the alternative to be adopted in both versions of the same root profile that appear in his/her menu of profiles, but fails to achieve the same consistency for the alternative to be avoided.

C4-2: A subject is C4-2 consistent if he/she picks the representative of the winner of the same SCR as the alternative to be avoided in both versions of the same root profile that appear in his/her menu of profiles, but fails to achieve the same consistency for the alternative to be adopted.
C4-3: A subject is C4-3 consistent if he/she picks the representative of the winner of the same SCR as the alternative to be adopted and another SCR winner's representative as the one to be avoided in both versions of the same root profile that appear in his/her menu of profiles.
The results of the experiment with respect to the consistency check C 4 is presented below:

## C4-1

The number of subjects with C4-1: 26
TOP:
Plurality: 3 (11.6 percent)
Run-off: 0
MC: 9 (34.6 percent)
Borda: 14 ( 53.8 percent)
Distribution of Top Choices

| Root Profile | \#1 |  | $\boldsymbol{\# 2}$ |
| :--- | :--- | :--- | :--- |
| Plurality | $2(33.3 \%)$ | $1(7.2 \%)$ | 03 |
| Ryn-off | 0 | 0 | 0 |
| MC | $1(16.7 \%)$ | $6(42.8 \%)$ | $2(33.4 \%)$ |
| Borda | $3(50 \%)$ | $7(50 \%)$ | $4(66.6 \%)$ |
| TOTAL | 6 | 14 | 6 |

C4-2
The number of subjects with C4-2: 50
BOTTOM:
Plurality: 23 ( 46 percent)
Run-off: 27 ( 54 percent)
MC: 0
Borda: 0

| Distribution of BOTTOM Choices <br> (C4-2) |  |  |  |
| :--- | :--- | :--- | :--- | :--- |

Distribution of Bottom Choices:

| Root Profile | \#1 | $\mathbf{\# 2}$ |  |
| :--- | :--- | :--- | :--- |
| Plurality | 0 | 0 | \#3 |
| Run-off | 0 | 0 | 0 |
| MC | 0 | $4(50 \%)$ | 0 |
| Borda | $21(100 \%)$ | $4(50 \%)$ | $19(90.4 \%)$ |
| TOTAL | 21 | 8 | $2(9.6 \%)$ |



C4-3
The number of subjects with C4-3: 177
TOP:
Plurality: 17 ( 9.6 percent)
Run-off: 3 ( 1.7 percent)
MC: 74 (41.8 percent)
Borda: 83 (46.9 percent)


BOTTOM:
Plurality: 64 ( 36 percent)
Run-off: 109 (61.74 percent)
MC: 1 ( 0.56 percent)
Borda: 3 (1.7 percent)

Distribution of Top Choices

| Root Profiile | \#1 | \#2 | \#3 |
| :--- | :---: | :--- | :---: |
| Plurality | $15(22.4 \%)$ | $2(3.7 \%)$ | 0 |
| Run-Off | 0 | 0 | $3(5.3 \%)$ |
| MC | $24(35.8 \%)$ | $27(51 \%)$ | $23(40.3 \%)$ |
| Borda | $28(41.8 \%)$ | $24(45.9 \%)$ | $31(54.4 \%)$ |
| TOTAL | 67 | 53 | 57 |

Distribution of TOP Choices w.r.t. Root Profiles (C4-3)

Distribution of Bottom Choices

| Root <br> Profiile | \#1 | \#2 | \#3 |
| :--- | :---: | :---: | :---: |
| Plurality | 0 | $16(30.7 \%)$ | $48(82.7 \%)$ |
| Run-Off | $64(95.5 \%)$ | $35(67.4 \%)$ | $10(17.3 \%)$ |
| MC | $1(1.5 \%)$ | 0 | 0 |
| Borda | $2(3 \%)$ | $1(1.9 \%)$ | 0 |
| TOTAL | 67 | 52 | 58 |



Distribution of TOP Choices with respect to Different Levels of C4

|  | Plurality | Run-off | MC | Borda |
| :---: | :---: | :---: | :---: | :---: |
| C4-1 | 11.6 | 0 | 34.6 | 53.8 |
| C4-3 | 9.6 | 1.7 | 41.8 | 46.9 |

Distribution of TOP Choices w.r.t. Different Levels of C4


Distribution of BOTTOM Choices with respect to Different Levels of C4

|  | Plurality | Run-off | MC | Borda |
| :--- | :---: | :---: | :---: | :---: |
| C4-2 | 46 | 54 | 0 | 0 |
| C4-3 | 36 | 61.74 | 0.56 | 1.7 |

## Distribution of BOTTOM Choices

w.r.t. Different Levels of C4


## 5. The Problem Associated with Borda Winner

It is through the construction of the preference profiles that there always exists an alternative, which is not bottom-ranked by any of the members of the hypothetical society. We can call this alternative the Social Compromise (SC) winner. This alternative happens to be the representative of the Borda winner in all of the profiles. Since the subjects were required to give the reasoning for their answers, we were able to detect the motive for the subjects to state the Borda winner as the alternative to be
adopted. There are, mainly three different kinds of Borda choosers: the subjects who adopted the scoring method, the subjects who picked the SC winner and the subjects who had some other unknown motive.
The distribution of the varying motives among Borda choosers for overall results and different consistency checks and levels are presented below:
(i) Overall results
18. The number of valid profiles at which Borda winner is stated as the alternative to be adopted: 488 .
19. The number of valid profiles at which Borda winner is picked through applying scoring method: 112
20. The number of valid profiles at which Borda winner is picked since it coincides with the SC winner: 297
21. The number of valid profiles at which Borda winner is chosen because of another reason: 79
Thus, 60 percent of the Borda choosers actually picked the SC winner, 23 percent of the Borda choosers adopted the scoring method, and 17 percent of them had some other motive in choosing the representative of the Borda winner.

(ii) The distribution of motives behind choosing borda winner as the alternative to be adopted with respect to different types and levels of consistency
The distribution of the methods used by the subjects with C1-01 who stated Borda winner as the alternative to be adopted:

Total number of profiles: 31
Method used:

Scoring: 7 (22.5 percent)
SC Winner: 17 ( 55 percent)
Other: 7 (22.5 percent)


The distribution of the methods used by the subjects with C1-10 who stated Borda winner as the alternative to be adopted:

Total number of profiles: 105
Method used:
SC winner: 60 ( 57 percent)
Scoring: 30 ( 28.5 percent)
Other: 15 (14.5 percent)


The distribution of the methods used by the subjects with C1-11 who stated Borda winner as the alternative to be adopted:

Total number of profiles: 110
Method used:

SC winner: 39 (35 percent)
Scoring: 55 (55 percent)
Other: 16 ( 15 percent)


The distribution of the methods used by the subjects with C21=2 who stated Borda winner as the alternative to be adopted:

Total number of profiles: 78
Method used:
SC winner: 54 (70 percent)
Scoring: 6 ( 7 percent)
Other: 18 (23 percent)


The distribution of the methods used by the subjects with $\mathrm{C} 21=3$ who stated Borda winner as the alternative to be adopted:

Total number of profiles: 152

Method used:
SC winner: 87 (57 percent)
Scoring: 33 (22 percent)
Other: 32 (21 percent)

## Motive Behind Choosing Borda

Winner (C21=3)


The distribution of the methods used by the subjects with $\mathrm{C} 21=4$ who stated Borda winner as the alternative to be adopted:

Total number of profiles: 212
Method used:
SC winner: 133 (63 percent)
Scoring: 63 ( 30 percent
Other: 16 (7 percent)


Distribution of Motives Behind Choosing Borda Winner as the Alternative to be adopted for different Levels of C21 (\%)

|  | SC | Scoring | Other |
| :--- | :--- | :--- | ---: |
| C21=2 | 70 | 7 | 23 |
| C21=3 | 57 | 22 | 21 |
| C21 $=4$ | 63 | 30 | 7 |

Motive Behind Choosing Borda Winner w.r.t. Different Levels of C21


The distribution of the methods used by the subjects with $\mathrm{C} 3=1$ who stated Borda winner as the alternative to be adopted:

Total number of profiles: 400
Method used:
SC winner: 253 (63 percent)
Scoring: 89 ( 22 percent)
Other: 58 ( 15 percent)

## Motive Behind Choosing Borda Winner (C3=1)



## IV. Concluding Remarks

This study, to our knowledge, is the first attempt to understand public preferences concerning social choice rules, and therefore it is the first empirical study aiming to capture people's notion and valuation of legitimacy and fairness. Unlike most experimental studies at which valuation and individual decision-making mechanisms of individuals are detected in a money-based context, the present study tries to capture these notions in a world where it is impossible to convert any kind of valuation into money terms.
The study claims no statistical finesse. It applies no statistical tests, mainly because it is not clear to the authors precisely what significance or other statistical tests are appropriate for the analysis of the data obtained in the three studies. Hence, only a summary of the rough results are presented below:
The overall results of the study point out the rivalry between Borda's SCR and MC as the SCRs that subjects favor. (Borda's SCR is ahead of MC only by 1.84 percentage points.) On the other hand, the reasoning of the subjects concerning the alternative they stated as the one to be adopted enables us to understand that only 23 percent of the Borda choosers actually applied the scoring method. The share of the Borda choosers who, in fact, meant to pick the alternative that does not appear at the bottom row in the preference profiles is as high as 60 percent. That is, 60 percent of the Borda choosers actually picked the Social Compromise winner. It is through the construction of the preference profile that there is always one alternative that is not bottom-ranked in each profile and this alternative happens to coincide with the Borda winner in every profile. Thus, in 293 profiles out of 488 at which the Borda winner was stated as the alternative to be adopted, the subjects meant to choose the SC winner rather than the Borda winner.
Another remarkable result is the very low support for Plurality with a Run-off Rule (2 percent). In addition, the winner of the Run-off Rule is stated as the alternative to be avoided in 53.67 percent of the profiles. Furthermore, the support for the Run-off Rule declines as the consistency level increases in terms of all consistency checks that we applied to the data. This should be noted as an important result since the Run-off Rule has been highly recommended by some politicians and political scientists in order to be applied in the nationwide parliamentary elections.
It is also remarkably important that the support for MC increases as the level of consistency increases in terms of almost all consistency checks. (Only in terms of C101 , the support for MC declines by one percentage point as the level of consistency increases, but still the most consistent subjects with C11-11 display only 35
percentage support for Borda's SCR, whereas the support for MC is 55 percent.) On the other hand, the percentage of the profiles at which Borda's SCR is favored declines as the level of consistency increases in terms of consistency checks C1-10 and C4-01. This is again a very interesting result since it might have been easier to achieve consistency once Borda's SCR is consciously adopted: For one, scoring is an arithmetical counting method and leaves, in fact, no room for inconsistency. Secondly, it is very easy to detect the alternative which does not appear at the bottom row.

The Plurality Rule which is currently used in mayoral elections in Turkey and in elections seen in many other countries is favored only in 10.27 percent of the profiles and the share of the profiles at which it was disfavored is as high as 39 percent. (The disfavor against Plurality declines as the level of consistency increases, but still the lowest rate does not fall below 22.7 percent.)
In the study, the difference between the results obtained for females and males does not appear to be significant.

Finally, it should be noted that a useful consistency check concerning elimination could have been applied if the four SCRs again disagreed in the inverses of the profiles, but the size of the matrix is insignificant for this. On the other hand, the enlargement of the matrix would be too demanding in our experimental context, since it becomes more difficult for the subjects to examine the profiles as the matrix expands.

## Appendix I

## Departments of the Subjects Used in the Study

| Biology | Politics and International Relations |
| :--- | :--- |
| Business Engineering | Psychology |
| Business Administration | Psychological Consultancy |
| Civil Engineering | Sociology |
| Chemistry Education | Tourism |
| Chemistry | Teaching in English |
| Chemistry Engineering | Turkish Language and Literature |
| Computer Engineering | Translation |

Computer Engineering
Economics
Electronic Engineering
English language and Literature
Enviromental Engineering
History
Industrial Engineering
Mathmatics
Mathmatics Education
Mechanical Engineering
Management of Information
Systems
Philosophy
Physics Education
Physics

## Appendix II

## A Sample Questionaire

University:
Department:
Year:
A group of seven members faces four alternatives. Only one of these alternatives is to be adopted.

Each member of the group rank these four alternatives according to his/her own preference. For example, a member ranking the alternatives as
a
b
c
d
has ranked 'a' as his/her top choice, 'b' as his/her second choice, 'c' as his/her third choice and 'd' as his/her last choice.

Below are presented five distinct groups whose members exhibit various rankings of the alternatives according to their personal preferences. For each group, taking an impartial view you are asked to indicate which alternative ('a' or 'b' or 'c' or 'd') should be adopted and which should especially be avoided.

### 1.1.1

Group: 1.3.1

| mbr 1 | mbr 2 | mbr 3 | mbr 4 | mbr 5 | mbr 6 | mbr 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a | c | d | c | a | b | a |
| b | d | b | d | b | d | d |
| c | b | c | b | c | c | b |
| d | a | a | a | d | a | c |

If the rankings of the alternatives by the group members are as above, taking an impartial view which alternative should be adopted for this group?

Which alternative should especially be avoided?
Can you briefly explain the thinking on which your above views rest?
1.1.2

| Group: 1.57 .1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| mbr 1 | mbr 2 | mbr 3 | mbr 4 | mbr 5 | mbr 6 | mbr 7 |
| a | c | c | a | c | b | d |
| d | b | b | d | d | d | b |
| b | a | a | b | b | a | a |
| c | d | d | c | a | c | c |

If the rankings of the alternatives by the group members are as above, taking an impartial view which alternative should be adopted for this group?

Which alternative should especially be avoided?
Can you briefly explain the thinking on which your above views rest?
1.1.3

Group: 1.43.1

| mbr 1 | mbr 2 | mbr 3 | mbr 4 | mbr 5 | mbr 6 | mbr 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| b | a | c | d | a | a | b |
| d | c | d | c | c | c | d |
| a | d | b | b | d | d | a |
| c | b | a | a | b | b | c |

If the rankings of the alternatives by the group members are as above, taking an impartial view which alternative should be adopted for this group?

Which alternative should especially be avoided?
Can you briefly explain the thinking on which your above views rest?
1.1.4

Group: 1.38 .1

| mbr 1 | mbr 2 | mbr 3 | mbr 4 | mbr 5 | mbr 6 | mbr 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| b | d | b | c | d | a | d |
| a | c | c | a | a | c | a |
| c | b | d | b | c | b | c |
| d | a | a | d | b | d | b |

If the rankings of the alternatives by the group members are as above, taking an impartial view which alternative should be adopted for this group?

Which alternative should especially be avoided?
Can you briefly explain the thinking on which your above views rest?

| 1.1.5 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group: 1.75 .1 |  |  |  |  |  |  |
| mbr 1 | mbr 2 | mbr 3 | mbr 4 | mbr 5 | mbr 6 | mbr 7 |
| d | a | a | a | d | c | a |
| c | b | b | c | c | b | c |
| b | d | d | d | b | d | b |
| a | c | c | b | a | a | d |

If the rankings of the alternatives by the group members are as above, taking an impartial view which alternative should be adopted for this group?

Which alternative should especially be avoided?
Can you briefly explain the thinking on which your above views rest?

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[^0]:    Many common SCRs do not always use all the information contained in a preference profile and take only the alternatives that are ranked first by the voters into account. Such electoral systems are often referred to by the tag of "tops only."
    Given any "regular" social choice rule, if an alternative is seen as the best by a strict majority of votes that is, more than half of the electorate - it is elected. (Sertel and Sanver, 1999)

[^1]:    ${ }^{3}$ At any preference profile a voter "approves an alternative in the $\mathbf{k}^{\text {th }}$ degree," or equivalently, that the candidate "gains the $\mathbf{k}^{\text {th }}$ degree approval" of this voter. This is the case, if and only if the voter regards this alternative as $\mathbf{k}^{\text {th }}$ best or better.
    ${ }^{4}$ In the preference profiles with 4 alternatives and 7 voters, the pre-imposed constraint concerning the disagreement of the four SCRs about the winner acts as a pre-agreed tie-breaking rule.

[^2]:    ${ }^{5}$ The neutral versions are the profiles that are obtained through interchanging the names of the alternatives. ${ }^{6}$ The anonymous versions are obtained through interchanging the names of the voters of the original profile.

