UNCERTAINTY AND RISK AVERSION: IMPLICATION FOR TUNISIAN CEREALS CROPS

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Abstract

The main aim of this paper is to give the key features for the estimation of production structures using dual approach and to assess risk prevalence in agricultural activity. A supply/demand model, based on profit function in which land allocation is set endogenous and conditional on netput prices, is implemented within a dual approach. Aggregated data from Tunisian cereal crops sector are used. Results showed low price elasticities of supply. In order to modelize risk non-neutrality into the implemented dual approach, Mean-Variance utility function of profit is introduced thereafter as the objective function for the producer's optimization process (rather than profit). Uncertainty was set as a consequence of a weather variable variance; and randomness of output quantities were set endogenous and conditional on weather variable variance as suggested by Just and Pope (1978) and Coyle (1999). The coefficient of risk aversion was significant; and results from the Mean-Variance model show several salient results relative to both weather and yield variability. The average Risk premium during the sample period was equal to 13.58 percent.

ملخص

إن الهدف الرئيسي من هذه الورقة هو توضيح الملامح البارزة لتقييم هياكل الإنتاج باستخدام الطريقة المزدوجة. كما تهدف الورقة أيضا إلى تقييم غلبة عنصر المخاطرة في النشاط الزراعي. ويتنفذ في أثناء اتباع الاسلوب المزدوج نموذج العرض والطلب القائم على دالة الربح والتي يتم فيها تخصيص الأراضي داخليا وبناءا على صافي الأسعار. وبالاستناد إلى بيانات إجمالية من قطاع محاصيل الحبوب في تونس ظهرت نتائج مرونة العرض ذي السعر المنخفض. ولادخال عنصر عدم حيادية المخاطر في الأسلوب المزدوج المستخدم، سوف يتم فيما يلي إدخال دالة تنوع المنفعة المتوسطة الربح باعتبارها وظيفة موضوعية لعملية الاستفادة من المنتج (وليس الربح). واستحال الجزم على نحو قاطع في هذا الأمر نظرا للتفاوت في التقلبات الجوية كما قدرت كميات الانتاج بصورة عشوائية، حسب ظروف المكان والتفاوت في التقلبات الجوية. وهو ما أشار إليه جاست وبوب (1978) وسويل (1999). وكان معامل كراهية المخاطر ذا بال، وبينت نتائج النموذج تنوع المتوسط العديد من النتائج المهمة المتعلقة بتقلبات الطقس والإنتاج. وجاء معامل متوسط المخاطرة خلال فترة العينة 13.58%.

1. Introduction

Tunisia's agricultural policy has notably been designed to ensure food security for its population. Over the last forty years, the modernization of agriculture and the isolation from external competition has permitted in Tunisia during the 70s to substantially increase its outputs, yields, and self-sufficiency rates in products considered as being strategic, such as cereals, vegetables, oil, and livestock products. However, the most troubleshooting fact is that production enhancement was coupled with high volatility and stagnation of yields in a relatively inadequate level during the 90s. Agricultural productivity, especially in Arid and Semi-Arid Areas, remain persistently low and has not kept pace with effective demand. The high level of dependency of agriculture on exogenous conditions, such as hazardous and risky natural environment (drought, pests, flooding, insect infestations, disease, etc.), is one of the reasons that productivity growth is below a satisfying level (Mundlak, 1992). This was the main reason that led the Tunisian government to implement a large number of administrative regulation mechanisms to ensure adequate income levels for farmers and growers, national self-sufficiency for certain products, and low prices for principal commodities. These agricultural policy features were likely to entail several "reverse effects." Indeed, as argued by Ben Jemaa (2003), over subsidization of both input purchasing and output prices has led Tunisian large-holders to show unproductive behaviors and flat management during the 80s and early 90s.

The context is now changing. Like many MENA region countries, Tunisia has started to liberalize agricultural sector after signing the GATT agreement, and has taken part in the trade talks on agriculture held under the auspices of the WTO at the end of 1999. It has also engaged in a partnership with the European Union (EU). The agreement of association between Tunisia and the European Union aims at the progressive creation of a Euro-Mediterranean economic space which guarantees free circulation of goods, capital, and services. Three protocols, which have been re-examined in 2001, are governing agricultural shutter.

The need to design a policy mix that goes with Tunisia's multilateral and regional commitments and also affords sustainable increase in agricultural productivity is straightforward. Henceforth, any policy design should be based on a sound knowledge of the existing farming systems and decision-making behavior of the farm households. Unfortunately, major stresses of livelihoods and their local adaptive strategies for becoming the solid basis of sustainable livelihood have not been systematically studied and documented. Thus far, farmers' behavior under uncertainty and risk aversion is an important feature which requires a great deal of attention. No work has been undertaken to assess farmers' preferences in Tunisia or even MENA region except those based on mathematical programming, such as Bachta's (1991) and Hachicha's (1993) for Tunisia, and studies undertaken by ICARDA[1] for the Mashrek & Maghreb project. Among other characteristics of such work, the degree of risk aversion and behavior under uncertainty are corner stone features in modeling agricultural activity.

With respect to how risk aversion behavior hypothesis might be tested for, it was realized that any model strictly based on the neoclassical theory of the perfectly competitive firm would be inadequate. The assumption that producers are price taking profit maximizers and are operating exogenously, given the deterministic physical and market environments, rules out the possibility that supply response might support risk aversion.

Before the treatment of behavior under risk uncertainty and incorporation into supply response analysis, in the first section a deterministic empirical framework based on duality is

[1] International Center for Agricultural Research on Dry Areas.

estimated within an aggregated model of multi-output production using data from the Tunisian cereals crops sector. The problem of land allocation between the three main cereals crops in Tunisia: hard wheat, tender wheat and barely, is instigated by setting land allocation conditional on inputs and outputs (netputs) prices.

In section three, a model risk and uncertainty is stated. Firstly, it is shown how expected utility maximization framework is implemented into a dual approach in order to relax risk neutrality assumption and to make producers' decisions conditional to both weather and output variances. Secondly, an attempt to build up a multi-stochastic-output model for Tunisian cereals crops sector is made using a Mean-variance utility function. Application of the above methodology is illustrated here using the same data set used for the deterministic model. This permits a comparison with results reported under the assumption of non-stochastic outputs.

2. Production Technology and Land Allocation: A Profit Function Approach

Analytical Framework

The most comprehensive analytical approach in estimating output supply functions is to model them in terms of an equation system related to the underlying production technology as stated in expressions (9) and (10).

The total profit function is defined as the sum of all output-specific profit function, joint variable inputs, (quasi) fixed inputs, and a non-joint input,

$$\pi(P, W, L, Z) \tag{11}$$

where P stands for output price vector, W for variable input price vector (labor and fertilizers), L for land allocation vector, and Z for fixed inputs vector (capital).

First order conditions gives:

$$\frac{\partial \pi}{\partial p_i} = y_i^*(P, W, L, Z) \qquad i = 1,2,3 \tag{12}$$

$$\frac{\partial \pi}{\partial w_i} = x_i^*(P, W, L, Z) \qquad i = 1,2 \tag{13}$$

$$\frac{\partial \pi}{\partial l_i} = \frac{\partial \pi}{\partial l_j} = 0 \qquad i, j = 1,2,3 \tag{14}$$

(12) and (13) are the well known input demand and output supply equations derived using Hotelling Lemma. Equation (14) implies that optimal land allocation between crops is attended when marginal profit, or the shadow price, relative to land allocation is null; that is, there is no gain in changing land allocation. The allocation is done subject to the constraint of a fixed total land area in the long run.

Output supply, input demand, and land allocation functions can then be estimated jointly with or without the profit function[2]. Prominent among the functional forms commonly used to describe a profit or a revenue function is the normalized quadratic (NQ) specification (Villezca-Becerra and Shumway, 1994; Guyomard *et al.*, 1996) which allows linear equations for the examined products, except for the price used as *numéraire*.

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[2] Note that the dual approach is more adapted for multi-product production processes in the case of input joint technology.

The preceding analytical framework is used to estimate the supply/demand response for the three principal cereals crops produced in Tunisia; namely, hard wheat, tender wheat, and barely (henceforth denoted as crop 1, 2, and 3 respectively) over the last three decades.

The total production profit associated with these crops, given the acreage devoted in the cultivation of each crop, is assumed to take the quadratic form:

$$\pi = b_0 + \sum_{i=1}^{4} a_i \omega_i + \sum_{i=1}^{2} b_i l_i + \frac{1}{2}\left(\sum_{i=1}^{4}\sum_{j=1}^{4} a_{ij}\omega_i\omega_j \ + \ \sum_{i=1}^{2}\sum_{j=1}^{2} c_{ij} l_i l_j \right)$$
$$+\sum_{i=1}^{4}\sum_{j=1}^{2} b_{ij}\omega_i l_j + a_k k + \tfrac{1}{2} a_{5k} k^2 + \sum_{i=1}^{4} a_{i5}\omega_i k \sum_{j=1}^{2} b_{ik} l_i k + \sum_{i=1}^{4} a_{it}\omega_i t \tag{15}$$
$$+\sum_{j=1}^{2} b_{it} t + a_t t + \tfrac{1}{2} a_{tt} t^2$$

where π denotes total profit normalized by the price of barely (crop 3), ω_i ($i = 1,2,3,4$) denotes netput prices for two inputs: hard wheat and tender wheat and to outputs: labor and fertilizer divided by the price of barely, l_i ($i = 1,2$) denotes acreage cultivated for crops 1 and 2. The price of barely is used as *numéraire*. By Hoteling's lemma the supply/demand equations for netputs are given by:

$$\frac{\partial \pi}{\partial p_i} \equiv y_i = b_i + \sum_{j=1}^{4} a_{ij}\omega_j + \sum_{j=1}^{2} b_{ij} l_j + a_{ik} k + a_{it} t \quad i, j = 1,2,3,4 \tag{16}$$

where y_i denotes the quantity of netput i.

To the equation described in 16, are added the equations of land allocation for crops 1 and 2 (hard wheat and tender wheat). Land allocation equations take the following form:

$$\frac{\partial \pi}{\partial l_i} = b_i + \sum_{j=1}^{2} c_{ij} l_j + \sum_{j=1}^{4} b_{ij}\omega_j + b_{ik} k + b_{it} t \equiv 0 \quad i = 1,2 \ ; \ j = 1,2,3,4 \tag{17}$$

Due to the fact that π is quadratic in its argument, all first derivatives are linear. This specificity allows the following reformulation of land allocation equations:

$$l_i = -\alpha_i \left(b_i + \underset{i \neq j}{c_{ij} l_j} + \sum_{j=1}^{4} b_{ij}\omega_j + b_{ik} k + b_{it} t \right) \tag{18}$$

where $\alpha_i = 1/c_{ii}$; that is, the opposite of the inverse of the parameter associated to l_i^2 in the profit function.

Main Results

The linear supply/demand and land allocation equations described by equations (17) and (18) were jointly considered to comprehensively estimate parameters involved in the system. To implement the above specified model, aggregated annual data covering the period 1963-1999 for Tunisian cereals crops sector were used. Time series data on netput prices and quantities were compiled from the "Office des Céréales" and the National Statistic Institute (I.N.S). Three broad categories of cereal production factors are compiled using Törnqvist divisia index: capital, labor, and intermediate consumption. Capital included the flows of material, insurance costs, energy, and transport. Labor included hired and family labor. Intermediate consumption included fertilizers, pesticides, and herbicides.

To facilitate estimation, all explanatory variables were normalized to unit for 1963. The equations system described in equations (17) and (18) was estimated jointly using iterative three-stage least squares[3]. The estimation results are presented in the appendix.

Table 1 provides estimates of the crops' supply and input demand responsiveness to own and cross price changes. The own and cross price estimated parameters, which are central here, are statistically significant at least at the 10 percent level except for the labor own price elasticity of demand4. Own price parameters are positive, thus complying with economic observation. In addition, the cross price parameters between the three crops have a negative sign indicating a substitutability relationship between them. It should be noted that with respect to the numéraire equation (barely supply), all price elasticities can be derived from the estimated parameters because of shared parameters with the rest of the model equations. These are reported in Table 1.

The sensitivity of supply for the three examined crops to own and cross price changes is further analyzed using elasticities evaluated at the sample mean (Table 1). Since actual cultivated acreage and capital are included among the explanatory variables as (quasi) fixed factors, the computed elasticities represent short-run relationships. Own-price elasticities for all crop production are inelastic except for hard wheat own price elasticity. Inelastic supply must be due to technical limitation and to risk aversion. It is known that cereals production increases according to its variability; hence, the productivity effect must vanish under the allocation effect (Just and Zilberman, 1986 and Ben Jemaa, 2003). It is significant to sign the discrepancy between the input price elasticities. Demand for labor seems to be in the inelastic range. However, the demand for fertilizers and treatment chemicals are near the unity.

Let us examine the effects of netputs' prices on the land allocation. Allocation elasticities with respect to netpouts' prices are reported in Table 2. These elasticities are calculated at the sample mean and exhibit a positive relationship with respect to output prices. For tender wheat, own price land allocation elasticity is the smallest. This result corroborates the contention that shift in output price, when producer is risk averse, can lead to null or even 'perverse' land allocation as argued by Just and Zilberman (1986), especially when this cereal crop's yield is highly volatile. Those effects are endemic for risk adverse producers who proceed to allocate endowments into production activities based on a judgment between the outcome and the risky nature of the latter.

The purpose of this section was applying what section two described in addition to the introduction of a problem of land allocation. Even risk neutrality and full certainty are implicitly assumed. Contention in our results seems to assess for a risk aversion attitude observed in the behavior of Tunisian cereal crops sector producers. These findings call for considering risk aversion and uncertainty to deepen the analysis and to assess the real magnitude of risk effect on the production process. This will be the purpose of the next section where dealing with risk features is taken into account in the model, leading to more theoretical specifications since some properties of dual form do not hold under such assumptions.

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3 The three-stage least squares estimation technique was required because of the presence of l_i both as an endogenous and exogenous variable, and also to allow for cross correlation between the system equations and hetero-scedasticity of the errors.

4 This is generally due to the problems inherent to compilation and aggregation. Data used in this framework are the number of actives in the cereals crops sector and the number of owners. It is clear that this approximation brings some criticism, but these were the only available data on labor.

3. Yield Uncertainty and Risk Aversion in Duality Models: A Mean-Variance Approach

Neoclassical production theory assumes that production relationships are known with certainty; and, when this assumption is relaxed the predictions of the standard model can break down. In particular under uncertainty, the Law of Supply need not hold for risk adverse producers. For example, in a simple one output model of pure competition, Baron (1970) showed that when output price is stochastic and the entrepreneur is risk averse it is possible for a firm's short run supply function to have a negative slope. Similarly, Just and Zilberman (1986) used an allocation model in which production as well as output prices may be stochastic indicating that "*any price increase which inherently results in increased variability of returns can cause negative supply response*."

Both of these results were obtained using extensions of the neoclassical theory of the firm based on von Neumann-Morgenstern (vNM hereafter) utility theory in which the firm is assumed to maximize expected utility of profits, revenue, or any other measure of wealth rather than simply maximize profits. This approach allows any of the variables concerned, whether endogenous or exogenous to be treated as random, thereby allowing concepts of uncertainty and attitude toward risk in the theoretical framework.

Although the underlying theory is well developed, little empirical work has been done using the vNM approach (Hallam, Just and Pope, 1982; Behrmann, 1989; Saha, 1994, Coyle, 1999, and others). This is partly because the duality relationships in deterministic theory, which are so convenient for empirical work, break down under uncertainty. In particular, Pope (1980) has shown that output supply and input demand equations cannot be derived from a profit function by Hotelling's lemma.

As a result of these difficulties, studies attempting to accommodate risk tended to adopt *ad hoc* methodologies. Such methodologies often involved single equation estimation of output supply or input demand in which the conventional explanatory variables are supplemented by "risk variables." These variables are usually measures of the variability of prices or yields, such as a moving average squared deviation or simple range (Brennan, 1982), although the issue of which type of measure is most appropriate is still unresolved, reflecting the problem that much of the uncertainty facing individual farmers and their attitudes to such uncertainty is not directly observable.

An important theoretical point that should be stressed is the relationship between risk aversion and degree of efficiency. Attempts to assess both risk aversion and technical and alloted efficiency are made by several authors (see, for example, Horrace and Schmidt, 2000; Fraser and Horrace, 2003; and Kumbhakar, 2004). The consideration of inefficiency can be implemented by a stochastic production function rather than deterministic frontier. However, it is well accepted that risk aversion holds even under inefficiency. Moreover, the purpose here is basically to assess risk in the production process; in other words, to estimate the role of the risk in deterministic production shifts caused by decision making by farmers in terms of their endowment allocation and therefore output supply.

In the next part of this paper, those aspects of vNM utility theory which will be used in this analysis including the Arrow-Pratt measure of risk aversion, the certainty equivalent concept, and the constant risk aversion utility function, are summarized. How the dual approach to applied production analysis can be extended to cover production under uncertainty is then shown. A method for modeling a multi output technology, where uncertainty enters via random yields, is outlined. A model of Tunisian cereal crops production using this approach is implemented, thus extending neoclassical production theory and duality to include uncertainty and risk aversion. Thereafter, some concluding comments are made.

A linear Mean-Variance Model under Just-Pope Technology

The duality model is developed here under the following assumptions: linear mean-variance risk preferences or constant absolute risk aversion (CARA), Just-Pope technology, and price certainty. These assumptions regarding preferences and technology, albeit restrictive, have often been employed in empirical research in agriculture (Chavas and Pope 1982; Just and Pope 1978). Moreover they imply that the farm's objective function is almost linear in parameters, which simplifies exposition of the dual approach and, to some extent, simplifies empirical application.

The farmers risk preferences are specified in terms of a utility function that is linear in expected profits $\bar{\pi}$ and profit variance σ_π^2, viz.

$$E[U(\pi)] = \bar{\pi} - \frac{1}{2}\alpha\sigma_\pi^2 \tag{19}$$

where $\alpha > 0$ is the coefficient of absolute risk aversion. Profits are $\pi = py - wx$ where p, y denote the price and level of a single output respectively, and w, x denote vectors of farm input prices and levels respectively. The Just-Pope production function is:

$$y = f(x) + g(x)\varepsilon$$

where farm inputs' quantities x are deterministic and ε denotes a stochastic weather variable with mean and variance, mean ($\bar{\varepsilon}$) and variance(σ^2), of output conditional on x:

$$\bar{y} = f(x) + g(x)\bar{\varepsilon}$$

$$\sigma_y^2 = g(x)^2\sigma^2$$

The mean and variance of profit conditional on farm input levels and prices:

$$\bar{\pi} = p\bar{y} - wx = pf(x) + pg(x)\bar{\varepsilon} - wx \tag{20}$$

$$\sigma_\pi^2 = p^2\sigma_y^2 = p^2 g(x)^2\sigma^2 \tag{21}$$

Substituting (20) and (21) into (19), the producer's optimization problem is:

$$V(p, w, \bar{\varepsilon}, \sigma^2) = \max_{x \geq 0} pf(x) + pg(x)\bar{\varepsilon} - wx - \frac{1}{2}\alpha\, p^2 g(x)^2\sigma^2 \tag{22}$$

where $V(p, w, \bar{\varepsilon}, \sigma^2)$ is the producers dual indirect (expected) utility function, that is, the relation between maximum feasible (expected) utility and exogenous variables $(p, w, \bar{\varepsilon}, \sigma^2)$. Properties of this utility function are summarized in the following proposition (Coyle, 1999):

Assume existence of the dual utility function $V(.)$ (22) and twice differentiability. Then

Proposition 1 (Coyle, 1999),

(a) $V(\lambda p, \lambda w, \lambda\bar{\varepsilon}, \sigma^2/\lambda) = \lambda V(p, w, \bar{\varepsilon}, \sigma^2) \quad for \;\; \lambda > 0$

(b) (i) $\bar{y} = \dfrac{\partial V}{\partial p} + \alpha p \sigma_y^2$

(ii) $x_i = -\dfrac{\partial V}{\partial p} \quad i = 1, ..., n$

(iii) $\sigma_y^2 = -\dfrac{\partial V}{\partial \sigma^2}\dfrac{2\sigma^2}{\alpha p^2}$

(c) $V(.)$ is convex in w, and more generally $\left[V(.)\right]_{zz} + \left[p^2\right]_{zz}\sigma_y^2(\alpha/2)$ is symmetric positive semi-definite, where $\left[V(.)\right]_{zz}$ and $\left[p^2\right]_{zz}$ are Hessian matrices of second derivatives of $V(.)$ and p^2 with respect to $z=(p,w)$.[5]

This proposition generalizes the homogeneity, Hotelling's lemma, and convexity properties of standard risk-neutral models. Under risk neutrality, a dual profit function $\pi(p,w)$ is linear homogeneous and convex in (p,w) and satisfies Hotelling's lemma. Since the objective function for the producer's maximization problem (22) is nonlinear in parameters $(p,w,\overline{\varepsilon},\sigma^2)$ due to the term $p^2\sigma_y^2 = p^2 \;\; g(x)^2\sigma^2$, the standard homogeneity and convexity properties are modified. Proposition la indicates that the dual $V(.)$ is linear homogeneous in $(p,w,\overline{\varepsilon},1/\sigma^2)$ rather than in (p,w); that is, decisions $(x,\overline{y})$ are homogeneous of degree zero in $(p,w,1/\sigma^2)$ rather than in (p,w). Proposition 1a indicates that the dual $V(.)$ is convex in input prices w but not in (p,w).

The most important result for empirical applications is the generalization of Hotelling's lemma indicated in proposition lb. Equation lb (ii), relating input demands to derivatives of the dual with respect to input prices, is analogous to the standard Hotelling's lemma under risk neutrality and envelope results under price risk and CARA. Equation 1b (i) provides a specification of expected output supply equation in terms of the dual and relate output supply $\overline{y}$ to the dual's derivative of $V(.)$ relative to p and the endogenous output variance σ_y^2.

Equation 1b (iii) relates output variance σ_y^2 to a derivative of the dual relative to weather variance (σ_ε^2). Risk aversion and output uncertainty imply that the producer considers the impact of input choices x on output uncertainty as well as on expected output; that is, $\overline{y}$ and σ_y^2 are selected by x, as indicated in equation (22). Consequently output variance σ_y^2 is a decision variable for the firm along with input levels x and expected output $\overline{y}$. The generalized Hotelling's lemma equation b (iii) specifies the firm's output variance decision σ_y^2 in terms of the derivative of the dual with respect to weather variance σ_ε^2. This equation can be estimated jointly with factor demand and output supply equations.

According to this duality model, a firm's decisions regarding output uncertainty is endogenous. Standard risk-neutral models and models with price risk ignore output uncertainty (Coyle, 1992). Output uncertainty has been incorporated into recent cost function models (Pope and Chavas, 1996; Pope and Just, 1998); but these models treat output uncertainty as exogenous to the model.

Proposition 1 indicates that equations for decision variables $(x,\overline{y},\sigma_y^2)$ can be derived from a flexible functional form for the dual, essentially as in static risk-neutral models. For example, models based on a normalized quadratic function for the dual can be constructed as follows:

Selecting an input price w^0 as *numéraire*, in addition to the netputs normalization, we define the following normalization:

[5] Just and Pope (1978) suggest that model 4 implies that V is decreasing in w. Non-decreasing in $\overline{\varepsilon}$, and non-increasing in σ^2 (assuming $g(x) \geq 0$ and $\alpha \geq 0$).

$V^* = \frac{V}{w^0}$; $\sigma_\varepsilon^{2*} = \sigma_\varepsilon^2 w^0$

Applying the homogeneity property in proposition 1a (for $\lambda = 1/w^0$) to $V(p, w, \bar{\varepsilon}, \sigma^2)$ yields $V^*(p^*, w^*, \bar{\varepsilon}, \sigma^{2*})$. Homogeneity condition implies the following form of the derivative of V^* relative to σ_ε^2;

$$\frac{\partial V}{\partial \sigma_\varepsilon^2} = w^0 \frac{\partial V^*}{\partial \sigma_\varepsilon^{2*}}$$

Thus, assuming a quadratic function for $V^*(.)$, proposition 1b yields:

$$\bar{y} = a_1 + \sum_i a_{1i} z_i + \alpha p \sigma_y^2 \tag{23}$$

$$x_i = -a_i - \sum_j a_{ij} z_j \quad for \quad i \neq 0 \tag{24}$$

$$\sigma_y^2 = -\left(a_m + \sum_i a_{mi} z_i\right) \frac{2\sigma_\varepsilon^2}{\alpha p^{*2}} \tag{25}$$

where terms z_i (z_j) stand for normalized variables $\left\{p^*, w^*, \bar{\varepsilon}, \sigma^{2*}\right\}$. Estimates of the coefficient of risk aversion α can be obtained from equation (23) or (25) by substituting σ_y^2 in (23) by its expression in (25). Risk neutrality can be tested in terms of the joint restrictions $\alpha = 0$ and σ^2 excluded from the dual in the specification of expected output supply and factor demands.

Consistent estimates of expected output supply and input demand equations lb(i) and (ii) can be obtained by standard methods (similar to risk-neutral dual models), assuming adaptive (or rational) expectations for output. In contrast, direct estimation of a Just-Pope production function often requires more complex methods (Just and Pope 1978; Saha, Havenner, and Talpaz, 1997). Thus, the standard advantages of a dual approach over a primal approach for specification of policy relations and estimation in the deterministic case (Fuss and McFadden, 1979) may be amplified for this model. On the other hand, the dual approach does require estimates of mean and variance for a weather variable ε.

A Multi-Stochastic Output Model with Land Allocation

We now develop a duality model assuming linear mean-variance risk preferences (constant Absolute Risk Aversion), a general technology with multiple stochastic outputs (rather than a Just-Pope technology), and uncertainty regarding output levels. Risk preferences are specified in terms of a mean-variance utility function $U = U(\bar{\pi}, \sigma_\pi^2)$ where $(\bar{\pi}, \sigma_\pi^2)$ are the mean and variance of profit, which is a random variable due to the stochastic aspect of the bundle of produced quantities y_i. In other words, randomness of profit is imputable entirely to the revenue component rather than cost component since both input prices and quantities are supposed deterministic.

Revenue for the multi-output firm are:

$$R = p^T y \tag{26}$$

where T denotes transposition, p and y are price and quantity vectors, respectively, for m outputs. Output quantities y are uncertain, $\bar{y}$ is the vector of expected output quantities, and Ω_y is the ($m \times m$) quantities covariance matrix. The joint multi-output stochastic technology is designated as $F(y,x,\varepsilon)=0$, where stochastic output levels y are jointly determined by non-stochastic farm input levels x and stochastic weather variable (ε) with mean and variance ($\bar{\varepsilon},\sigma^2$). The mean and variance of the probability distribution for revenues R are designated as:

$\bar{R} = p^T\bar{y},\ \sigma_R^2 = p^T\Omega_y p$

Making use of a linear Mean-Variance utility form (19), the producer's maximization problem is:

$$\begin{aligned} \mathrm{V}(\mathrm{p},\mathrm{w},\Omega_y,\bar{\varepsilon},\sigma^2) &= \max_{x\geq 0} U \\ &= \max_{x\geq 0} \bar{R}(x,p,\bar{\varepsilon},M^e) - wx - \tfrac{1}{2}\alpha\sigma_R^2(x,p,\Omega_y,\bar{\varepsilon},\sigma^2,M^v) \end{aligned} \tag{27}$$

where (M^e, M^v) are all other moments determining R and σ_R^2 respectively.

Application of the above methodology is illustrated here using the same data set for Tunisia's cereal crops as in section two, permitting a comparison with results reported under the assumption of deterministic yields. This annual time-series data set (1963-99) of prices and quantities is aggregated national level; however, such aggregate data generally understate substantially the variation or uncertainty of yields at the farm level. The model assumes three outputs (hard wheat, tender wheat, and barely), two variable inputs (labor and fertilizers), one quasi-fixed input (capital services). Land is introduced into the model as acreage allowed to each crop as in section two in order to account for land allocation under fixed total acreage, at least in the short term. We assume that crops output quantities are uncertain.

Although data on production levels and acreage allocation are available only at the national level, data on monthly rainfall level are available for eighteen weather stations within Tunisia. Using data on rainfall level and data on regional shares allocated to cereal crops, it was possible to construct a weighted mean cereal-specific weather variable based on weighting both monthly and regional rainfall levels using biological water value and land allocation. Mean and variance of weather at time t were calculated as a weighted past realization:

$$\bar{\varepsilon}_t = 0.5\varepsilon_{t-1} + 0.33\varepsilon_{t-2} + 0.17\varepsilon_{t-3} \tag{28}$$

$$\begin{aligned} \sigma_{\varepsilon_t}^2 &= 0.5(\varepsilon_{t-1} - \bar{\varepsilon}_{t-1})^2 + 0.33(\varepsilon_{t-2} - \bar{\varepsilon}_{t-2})^2 \\ &\quad + 0.17(\varepsilon_{t-3} - \bar{\varepsilon}_{t-3})^2 \end{aligned} \tag{29}$$

Mean expression fit adaptive expectations where at time t is a weighted average of past realizations. The current variance equals the sum of squares of prediction errors of the three previous years with declining weights similar to various other studies (Chavas and Holt, 1990 and Coyle, 1999).

Means ($\bar{y}$) and Variances (Ω_y) of crops quantities are defined similarly to equation (28) and (29) respectively. Crops quantities' covariance (Ω_{ij}) is defined as follows:

$$\begin{aligned} \Omega_{ij_t} &= 0.5(y_{it-1} - \bar{y}_{it-1})(y_{jt-1} - \bar{y}_{jt-1}) \\ &\quad + 0.33(y_{it-2} - \bar{y}_{it-2})(y_{jt-2} - \bar{y}_{jt-2}) \\ &\quad + 0.17(y_{it-3} - \bar{y}_{it-3})(y_{jt-3} - \bar{y}_{jt-3}) \end{aligned} \tag{30}$$

These proxies presumably provide poor measures of outputs' uncertainty at the farm level, but farm-level data on output was unavailable.

Using *proposition* 1, a system of netputs supply/demand, land allocation, and outputs' variance equations is derived from (27):

$$\overline{y}_i = \frac{\partial V(.)}{\partial p_i} + \alpha \left[\frac{\partial \left(p^T \Omega_y p \right)}{\partial p_i} \right] \qquad i = 1,2,3 \tag{31}$$

$$x_j = -\frac{\partial V(.)}{\partial w_j} \qquad j = 1,2 \tag{32}$$

$$\frac{\partial V(.)}{\partial l_i} = 0 \qquad i = 1,2,3 \tag{33}$$

$$\Omega_{ii} = -\left(\frac{2}{\alpha} \right) \frac{\partial V(.)}{\partial \sigma_\varepsilon^2} \frac{\sigma_\varepsilon^2}{p_i^2} \qquad i = 1,2,3 \tag{34}$$

where equations (33) stand for land allocation. Note that outputs variance Ω_{ii} can be eliminated from the outputs supply equations by substituting (34) into (31). Thus, expected outputs supply relations can, in principle, be estimated independently of errors in measuring Ω_{ii}.

Accommodating linear homogeneity within the framework of a normalized Quadratic model as suggested by Coyle (1999), variable inputs and expected outputs are normalized by the fixed input's quantity (k). Outputs variance is normalized by k^2 and k is inc1uded as a separate explanatory variable.

The model specified in section three is used assuming a generalization of a normalized Quadratic dual profit function using labor price as *numéraire*. These equations can be modified as follows to accommodate outputs uncertainty:

$$\begin{aligned} y_i/k &= a_i + \sum\nolimits_{i=1}^{3} a_{ij} p_j + \sum\nolimits_{i=1}^{3} c_{ij} l_j + a_{i4} w_4 + b_{i1} \sigma^2 w_5 + b_{i2} w_5 \\ &\quad + b_{i3} \overline{\varepsilon} + b_{i4} k + \alpha \sum\nolimits_{j=1}^{3} p_{j\Omega_{ij}} + a_{it} t \end{aligned}$$

$$\begin{aligned} x_4/k &= -[a_4 + \sum\nolimits_{i=1}^{3} a_{ij} p_j + \sum\nolimits_{i=1}^{3} c_{4j} l_j + a_{44} w_4 + b_{41} \sigma^2 w_5 \\ &\quad + b_{42} w_5 + b_{43} \overline{\varepsilon} + b_{44} k + a_{4t} t] \end{aligned}$$

$$\begin{aligned} & c_i + \sum\nolimits_{j=7}^{9} d_{ij} l_j + \sum\nolimits_{k=1}^{3} c_{ik} p_k + c_{i4} w_4 + b_{i1} \sigma^2 w_5 + b_{i2} w_5 + b_{i3} \overline{\varepsilon} \\ &\quad + b_{i4} k + c_{it} t = 0 \end{aligned}$$

$$\begin{aligned} \Omega_{ii}/k^2 &= -\left(\tfrac{2}{\alpha} \right) \tfrac{\sigma_\varepsilon^2}{p_i^2} [e_i + \sum\nolimits_{j=1}^{3} b_{ij} p_j + b_{i4} w_4 + e_{i1} \sigma^2 w_5 + e_{i2} w_5 \\ &\quad + e_{i3} \overline{\varepsilon} + e_{i4} k + e_{it} t] \end{aligned}$$

where t is a time trend. Outputs quantities' variances are set to be endogenous using (34). Existence of an indirect utility function $V(.)$ consistent with equations (31)-(33) implies symmetry restrictions across netputs' supply/demand and land allocation equations:

$$a_{ij} = a_{ji}$$

$$c_{ij} = c_{ji}$$

Symmetry restrictions regarding the equation for the variances of outputs are also implemented.

Main Results

A nonlinear three stage least square was used to estimate parameters of the above model. Hypotheses of unit roots, using standard unit root tests (Dickey-Fuller, Phillips-Perron), were rejected for dependent and independent variables except for normalized price of fertilizers; and accordingly, this variable was rendered stationary. Thus, variables exhibiting trend were assumed to be trend stationary; and this case was accommodated by including time trend in regression equations.

Nonlinear three stage least square estimates are reported in the Appendix. Except for the time trend, coefficients can be interpreted as elasticities in 1999 (variables were normalized to 1 for 1999 and all across-equation restrictions were transformed accordingly).

As anticipated under risk aversion and crops' yield uncertainty, the coefficients b_{i1} (for $i = 1.2.3$) of weather variance in the crops' output equation are significant and negative. The significant negative sign for the b_{i1} 's implies that expected crops' output supply is decreasing in weather variance under risk aversion and yield uncertainty.

Outputs are also increasing in own prices. Fertilizers' demand is decreasing in own price and increasing in outputs' prices. Fertilizers' demand is found to be decreasing in weather variance (but not to expectation) which confirm the contention implying that fertilizers increase yield variability. Farmers tend to decrease the use of fertilizers when rainfall variability is high. On the other hand, estimate of the coefficient of absolute risk aversion α was found to be significant at the 5 percent significance level. This is another assessment of risk aversion behavior. Indeed, the significance of the coefficient of absolute risk aversion deals with the direct effect of yield volatility on risk preference. Coefficients in the equation for crops' output variances (Ω_{ii}) are insignificant, implying that decisions about endowments' allocation made by producers does not take into account yields' variances as target variable. This result can be the consequence for choosing variability proxies.

The *numéraire* price w_5 is significant in the model. Thus, the homogeneity restrictions under the standard risk-neutral model and under CARA are rejected. Weather uncertainty effects, substituted by weather variance calculated using equation (29), are also highly significant. The significance of weather variance suggests that this model deals both with direct and indirect effect of random environment on outputs supply.

the b_{i1}/d_{ii} 's ($i = 7.8.9$) that depict weather variance effect on land allocation are all highly significant which implies that weather variability seems to have an effect on land allocation between crops within a multi-output activity. While weather variability is land reducing for hard wheat and neutral for tender wheat, it increases land allocated to barely, a well known cereal that resists climate variations. Results concerning price elasticities for both supply and land allocation were found to be very similar to results from the model's deterministic components implemented in the framework of section two.

Price elasticities presented in Table 3 are partial price elasticities, and represent the intensity effect of a price change. All of the own price elasticities of the three cereals and the variable inputs have the correct sign theoretically; that is, positive definiteness of the Hessian implies that all own output (input) price elasticities are positive (negative). The exception is the positive sign on the hard wheat labor application in response to an increase in the price of labor. A 1 percent increase in fertilizers price has an elastic effect for the three crops.

The inclusion of the area allocation helps to capture the full complexity of the supply response (Table 4). For instance, an increase in the output prices for hard wheat and barely results in an increase in their land shares. Besides, an increase in the tender wheat's price results in a decrease in its acreage allocation. This salient finding is a considerable improvement of the model considered in section two assuming risk neutrality.

Making use of expression (19) of the mean-variance (expected) utility function for the crops' returns (rather than the profit), we were able to calculate the risk premium relative to total returns at every sample point. The average Relative Risk Premium (RRP) during the period 1967-1999 was equal to 13.58 percent.

As shown in Figure 1, risk premium has significantly increased since the early 80's. This date corresponds to the beginning of the implementation of the Agricultural Structural Adjustment Program (PASA) in Tunisia which aimed, among other things, at the elimination of agricultural inputs' subsidization.

3. Concluding Remarks

In this paper, the issue was to implement, within a dual approach, a supply/demand model based on profit function in which land allocation is set homogenous and conditional on netputs' prices. Aggregated data from Tunisia's cereal crops sector were used and results were characterized by low price elasticities of supply. In order to model risk non-neutrality into the implemented dual approach, mean-variance utility function of profit was introduced in section four as the objective function for the producer's optimization process (rather than profit). Uncertainty was set as consequence of a weather variable variance; and randomness of outputs' quantities was set endogenous and conditional on weather variable variance as suggested by Just and Pope (1978) and Coyle (1999). The main innovation in this model was the implementation of a multi-stochastic-output function which implies the consideration of co-variances between random output quantities.

Weather variable variance was found to have a negative impact on expected supplied quantity for all three crops. Its impact was found to be non-significant on input demand and land allocation. Estimate of Absolute Risk Aversion coefficient was highly significant and had the right theoretical sign, which is the main strong point of the model compared to previous work that failed to prove significant Absolute Risk Aversion coefficient. Nevertheless, insignificance of the coefficients in the output variance Ω_{ii} equation suggests that the use of aggregated measures of variance of crop output is poor proxy for yield uncertainty at the farm level.

Risk Premium for the Tunisian cereal crops sector has considerably increased since the 80's. This increase has several logical roots such as the great rainfall weather variability during this period and the increase in utilization of fertilizers. The liberalization process undertaken by the Tunisian government since the mid 80's to progressively eliminate agricultural inputs' subsidization is partially responsible for this increase.

In sum and although the model represents a set of good results, it is not free of shortcomings. Using aggregated data rather than farm-level data is not the best information for risk assessment and leads to some criticism. In future studies, farm-level data from farm-household producers in low rainfall locations in Tunisia should be used in order to assess production technology under uncertainty. Productivity and efficiency issues under uncertainty should be implemented.

References

Beattie, B.R. and C.R. Taylor, 1985, "The Economics of Production," J. Wiley.

Behrman, J.R., 1989, "Agricultural Supply," in Eatwell, J., Milgate, M., and Newman, P. (Editors), The New Palgrave: Economic Development, Macmillan, 35-42.

Ben Jemaa, M.M., 2003, "Local Productive Efficiency and Economic Integration: Assessment from the Tunisian Cereals Sector", Paper presented at the 5th conference on the Economics and Finance in The MENA, L.A.U, Lebanon and the 4th FEMISE conference, Marseille 4-6 December, 2003.

Brennan J.P., 1982, "The Representation or Risk in Econometric Models of Supply: Some Observations," *Australian Journal of Agricultural Economics*, Vol.26, 151-156.

Chambers, R.G., 1988, "Applied Production Analysis: A Dual Approach," Cambridge University Press.

Chavas J.P. and M.T. Holt, 1990, "Acreage Decisions under Risk: The Case of Corn and Soybeans," *American Journal of Agricultural Economics*, Vol.72, 529-38.

Chavas J.P. and R.D. Pope, 1982, "Hedging and Production Decisions under a Linear Mean-Variance Preference Function," *Western Journal of Agricultural Economics* Vol.7, 99-110.

Coelli T.J, 1996, "Measurement of Total Factor Productivity and Biases in Technological Change in Western Australian Agriculture," *Journal of Applied Econometrics*, Vol. 11, 77-91.

Colman D., 1983, "A Review of the Arts of Supply Response Analysis," *Review of Marketing and Agricultural Economics* 51(3), 201-230.

Coyle B.T., 1992, "Risk Aversion and Price Risk in Duality Models of Production: A linear Mean-Variance Approach," *American Journal of Agricultural Economics*, Vol., 74, 849-59.

Coyle B.T., 1999, "Risk Aversion and Yield Uncertainty in Duality Models of Production: A Mean-Variance Approach," *American Journal of Agricultural Economics,* Vol.81, 553-67.

Diewert, -VIE., 1974, "Applications of Duality Theory," in M.D. Intriligator and D.A. Kendrick (Editors), Frontiers of Quantitative Economics, Vol.11, North-Holland, Amsterdam, 106-171

Fraser, I. M. and W. C. Horrace, 2003, "Technical Efficiency of Australian Wool Production: Point and Confidence Interval Estimates*," Journal of Productivity Analysis*, Vol. 20, 169-190.

Fuss, M., McFadden, D. and Mundlak, Y., 1978, "A Survey of Functional Forms in the Economic Analysis of Production", in M. Fuss and D. McFadden (Editors), Production Economics: A Dual Approach - Theory and Applications, Vol. 1, Amsterdam: North-Holland, 219-68.

Gravelle, H. and Rees, R., 1981, Mico-economics, Longman.

Guyomard H. Baudry M. and Carpentier A., 1996, “Estimating Crop Supply Response in the Presence of Farm Programs: Application to the CAP,” *European Review of Agricultural Economics*, Vol.23, 401-20.

Hallam, J.A., RE. Just and R.D. Pope, 1982, “Positive Economic Analysis and Risk Considerations in Agricultural Production,” in G.C. Rausser (Editor), New Directions in Econometric Modelling and Forecasting in US. Agriculture, Elsevier North-Holland, New York, 179-202.

Horrace,W.C. and P. Schmidt, 2000, “Multiple Comparisons with the Best Economic Applications,” *Journal of Applied Econometrics*, Vol.15, 1-26.

Just R.E., Pope R.D., 1978, “Stochastic Specification of Production Functions and Economic Implications,” *Journal of Econometrics,* Vol., 7 67-86.

Just, R.E. and D. Zilberman, 1986, “Does the low of Supply Hold under uncertainty” *The Economic Journal,* Vol.96, 514-524, 57.

Kumbhakar S., 2004, “Risk Preferences, Price Uncertainties, and Production Risk: A Joint Analysis with Technical Inefficiency,” Paper presented at the Asia-Pacific Productivity Conference, 2004.

Marshak, J. and W. Andrews, 1944, “Random Simultaneous Equations and the Theory of Production,” Econometrica, Vol.12, 143-205.

Mundlak Y., 1992 “Agricultural productivity and Economic Policies: Concepts and Measurements,” Working Paper Series, Washington DC, *OCDE/GD.*

Mundlak, Y., 1988, “Endogenous Technology and the Measurement of Productivity,” In *Agricultural Productivity: Measurement and Explanation,* S.M. Capalbo and J.M. Antle, eds., Washington, D.C.: Resources for the Future.

Pope R.D and R.E. Just, 1998, “Cost Function Estimation under Risk Aversion,” *American Journal of Agricultural Economics*, Vol.80, 296-302.

Pope R.D. and J.P. Chavas, 1996, “On Defining Cost Functions under Uncertainty,” *American Journal of Agricultural Economics*, Vol. 76, 196-204.

Pope, R.D., 1980, The Generalized Envelope Theorem and Price Uncertainty, *International Economic Review,* Vol.21, 75-86.

Pratt, J.W., 1964, “Risk Aversion in the Small and in the Large,” *Econornetrica*, 30, 122-136.

Roumasset, J.A., M.W. Rosegrant, U.N. Chakravorty and J.R. Anderson, 1989, “Fertilizer and Crop Yield Variability: A Review,” in: J.R. Anderson and P.B.R. Hazell, eds., Variability in Grain Yields, Johns Hopkins University Press, Baltimore, 223-33.

Saha A., A. Havenner, and H. Talpaz, 1997, “Stochastic Production Function Estimation: Small Sample Proprieties of ML versus FGLS,” *Applied Econometrics*, Vol.29, 459-69.

Saha, A., C.R. Shumway and H. Talpaz, 1994, "Joint Estimation of Risk Preference Structure and Technology using Expo-power Utility," *American Journal of Agricultural Economics,* Vol.76, 173-84.

Shepard, R. W., 1953, Cost and Production Functions, Princeton University Press.

Varian, H. R. (1984, Second Ed.), "Microeconomic Analysis," Norton.

Villezca-Becerra P.A. and Shumway C.R., 1994, "State-level Output Supply and Input Demand Elasticities for the Agricultural Commodities," *Journal of Agricultural Economics and Research*, Vol.44, 22-34.

Wall, C.A. and Fisher, B.S., 1987, "Modelling a Multiple Output Production Systems: Supply Response in the Australian Sheep Industry," Research Report No. 11, Dept. of Agricultural Economics, University of Sydney, Australia.

Figure 1: Total Returns and Relative Risk Premium (in M 1990 TND)

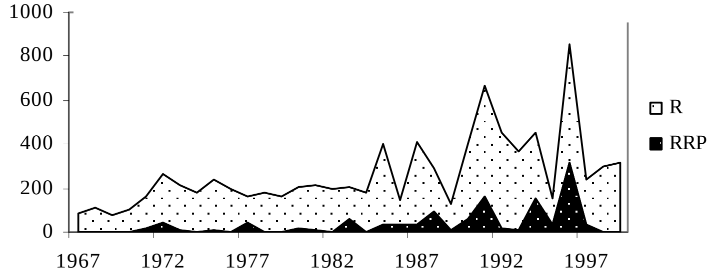

Table 1: Own and Cross Price Elasticities (Risk Neutrality Model)

	H. wheat	T. wheat	Barely	Fertilizers	Labor
H. wheat	1,119	-0,004	-0,357	-0,450	-0,001
T. wheat	-0,012	0,152	-0,917	-1,684	-0,026
Barely	-1,019	-0,917	0,400	-5,544	-0,142
Fertilizers	0,112	0,840	2,594	-0,889	0,334
Labor	-0,001	0,065	0,404	1,385	0,076

Elasticities are calculated at sample mean

Table 2: Effect of Netput Prices on Land Allocation (Risk Neutrality Model)

Change in land allocation relative to a 1% change in the price of :			
Relative to:	**Acreage 1**	**Acreage 2**	**Acreage 3**
H. wheat	0,401	0,290	-0,691
T. wheat	-0,083	0,033	0,050
Barely	-0,081	-0,169	0,249
Fertilizers	-0,181	-0,139	0,320
Labor	-0,135	0,730	-0,595

Elasticities are calculated at sample mean

Table 3: Own and Cross Price Elasticities (Mean-Variance Model)

Elasticity of netput (column) relative to the price of (row) :					
	H. wheat	**T. wheat**	**Barely**	**Fertilizers**	**Labor**
H. wheat	0,726	-0,724	0,142	-1,407	0,080
T. wheat	-0,023	0,146	-1,338	-3,675	-1,713
Barely	0,151	-1,331	0,433	-2,385	-0,519
Fertilizers	0,352	1,014	0,547	-1,224	-0,389
Labor	-0,047	0,780	0,291	-0,762	-0,346

Elasticities are calculated at sample mean

Table 4: Effect of Netput Prices on Land Allocation Table 4. Effect of Netput Prices on Land Allocation (Mean-Variance Model)

Change in land allocation relative to a 1% change in the price of:			
Relative to:	**Acreage 1**	**Acreage 2**	**Acreage 3**
H. wheat	0,186	0,004	0,358
T. wheat	-0,006	-0,067	0,681
Barely	-0,191	-0,711	0,483
Fertilizers	-0,501	0,349	-0,293
Labor	-0,160	-0,111	0,088

Elasticities are calculated at sample mean