

**Does The Specification of A New Class
of Poverty Measures Matter? Evidence
from Tunisia**

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POVERTY MEASURES MATTER? EVIDENCE FROM
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1. Introduction

The horizontal equity (HE) principle requires that like individuals be treated alike, while the vertical equality (VE) principle stipulates suitable differentiation among unlike individuals. Poverty alleviation programs based on imperfect targeting are often unable to align with the first principle. For example, when benefits awarded to the poor are based on regional targeting, all individuals within a region are treated identically as with a universal transfer scheme; but only some regions are targeted within this social program. Because poor people of untargeted regions are not served, regional targeting scheme leads to violations of HE.

There is no disagreement that HE, i.e. the extent to which equals are treated equally, is a worthy goal of any design. Yet, given the heterogeneity characterizing the individuals' preferences, all public intervention modifying the real income distribution will incur violations of HE, the extent of which needs to be evaluated. The absence of consensus around the treatment of this issue is behind the emergence of various indexes of horizontal inequity (HI) in order to be included as a component of the social well-being variations.¹ However, the cost of HI is rarely considered when evaluating the effects of poverty alleviation programs.²

Sen (1976) was the first to propose a poverty measure that was sensitive to this transfer axiom, which corresponds to VE requirement. He also expressed his poverty measures as a function of the normalized income-gap ratio and the Gini coefficient of the poor income distribution. Adopting the general approach to the measurement of inequality, developed by Atkinson (1970), we can derive a cost of inequality from a poverty measure respecting the transfer axiom of Sen, which can be broken into two components, corresponding to vertical and horizontal inequality respectively.

When the cost of inequality approach is restricted to poverty measures of the Foster, Greer and Thorbecke (1984) class (henceforth FGT class), it is no longer possible to have different preferences toward VE and HE. Nevertheless, according to Auerbach and Hasset (1999) and Duclos and Lambert (2000), it is useful to let attitudes differ about HI and vertical inequality (VI). We then extend the application of the cost of inequality approach to capture the extent of HI, only after specifying a new class of poverty measures. The measures of the new class are parameterized by two coefficients, allowing, therefore, different preferences toward these two principles. When these two coefficients are identical, the new poverty measures class reduces to that of the FGT, whose poverty measures imply the same aversion to VI and HI.

The poverty measures of the new class can be broken into two components, an income-gap measure and a cost of inequality. The cost of inequality can also be broken into a contribution of VI and HI. The starting point is a local measure of HI, capturing the dispersion of post-reform income gap among individuals having the same pre-reform income gap. When this local measure is aggregated, using an appropriate weighting system, a global cost of the HI results. While other approaches focus on the errors of exclusion to capture the extent of horizontal inefficiency following the reforms of poverty alleviation programs, our cost of HI provides a measure of the mean saving of the available budget that would come from eliminating HI poverty-neutrally. Further, with poverty measures allowing more sensitivity to HI than VI, it becomes possible to define a set of poverty measures such that policymakers are indifferent to a choice between a given reform and the status quo.

To resolve the identification problem of like individuals, we follow a non-parametric estimation procedure, described by Silverman (1986), to assess the extent of HI. Hence, using micro-data from the 1990 Tunisian Household Survey, we apply this procedure to assess the effects of a hypothetical reform on the leakages to non-poor, HI, and VI. This reform advises substituting a direct transfer program, based on proxy means tests, for the current universal food subsidies system.

The next section of the paper presents the necessary prerequisites for the analytical framework we shall adopt. The decomposition of the inequality cost into VI and HI contributions after specifying a

¹ See, for instance, Aronson et al. (1994), Auerbach and Hasset (1999), Berliant and Strauss (1985), Brennan (1971), Duclos and Lambert (2000), Feldsein (1976), King (1983a), Lambert and Ramos (1997), and Plotnick (1981, 1982).

² Duclos (1995) develops a theoretical and an empirical analysis of the impact of imperfect targeting, participation costs and incomplete take-up upon the level of progressivity, VE, HI, and redistribution exerted by state benefits. Nevertheless, he follows the "re-ranking" approach to measuring HI by decomposing the variation of Gini index into progressivity, average benefit, VE, and HI.

new class of poverty measures constitutes the aim of Section 3. Section 4 applies the new poverty measures to the Tunisian Household Survey for 1990. Finally, Section 5 concludes the paper.

2. The Basic Model

Let us consider a distribution of incomes (y^1, y^2, \dots, y^I) among a population of I individuals where, without loss of generality, $y^1 \leq y^2 \leq \dots \leq y^I$. Let $(y^1, y^2, \dots, y^{I_p})$ be the restriction of the previous distribution to the poor segment of the population. In other words, I_p is the population having an income, y^i , below the poverty line. So, we have $y^1 \leq \dots \leq y^{I_p} < z \leq y^{I_p+1} \leq \dots \leq y^I$ where z is the poverty line. We consider the general class of poverty measures which expression is given by:

$$P_\alpha = P_\alpha(g^1, \dots, g^i, \dots, g^{I_p}, 0, \dots, 0), \quad (1)$$

where g^i is the income-gap of any individual i given by:

$$\begin{aligned} g^i &= z - y^i && \text{if } y^i < z \\ &= 0 && \text{otherwise.} \end{aligned} \quad (2)$$

For any income distribution, it is possible to aggregate the income gap distribution so as to obtain the non-normalized FGT class of poverty measures, the form of which can alternatively be given by:³

$$P_\alpha(z, y) = \frac{1}{I} \sum_{i=1}^{I_p} (z - y^i)_+^\alpha, \quad (3)$$

$$P_\alpha(z, y) = \int_0^{+\infty} (z - y)_+^\alpha f(y) dy, \quad (3')$$

where $f(y)$ is the income density function, and α is a coefficient of poverty aversion. These poverty measures are non increasing in y^i , symmetric or anonymous with regard to individual well-being, and quasi-convex for $\alpha > 1$, which ensures that an equalizing transfer of income from a poor person to anyone who is poorer decreases the selected poverty measure.⁴ A monotonic transformation of $P_\alpha(z, y)$ enables us to define the “equally distributed equivalent (EDE) level of income-gap” as that level of income-gap which, if shared equally by all people, would produce the same level of poverty as that generated by the actual distribution of income gaps:

$$P_\alpha(\mathbf{g}_\alpha, \dots, \mathbf{g}_\alpha, \dots, \mathbf{g}_\alpha) = P_\alpha(g^1, \dots, g^i, \dots, g^{I_p}, 0, \dots, 0). \quad (4)$$

Given the ordinal characteristic of the FGT class of poverty measures, we may write the level of aggregate poverty as:

$$\begin{aligned} \mathbf{g}_\alpha(z, y) &= [P_\alpha(z, y)]^{\frac{1}{\alpha}} && \text{if } \alpha \geq 1 \\ &= P_0(z, y) && \text{if } \alpha = 0. \end{aligned} \quad (5)$$

Sen (1976) has attacked the choice of the headcount ratio, given by $\alpha = 0$, as a poverty measure since it is insensitive to an income reduction of a poor person. Also, according to his N axiom, $\mathbf{g}_1(z, y)$ is an adequate poverty measure only if all the poor have the same income. Otherwise, α should be greater than 1 in order to have poverty measures that are sensitive to inequality among poor segment of population. Indeed, the more important the difference between $\mathbf{g}_\alpha(z, y)$ and $\mathbf{g}_1(z, y)$, the more unequal is the distribution of income gaps. Hence, an appropriate measure of the cost of inequality can be given by:⁵

³ Chakravarty and Mukherjee (1998) have also used non-normalized poverty measures to study the problem of allocating an antipoverty budget among the poor.

⁴ This suggestion is in line with the transfer axiom of Sen (1976).

⁵ This average cost of inequality, $C_\alpha(z, y)$, can be considered as a transposition of the cost of inequality proposed by Atkinson (1970) when the income of non-poor people is just equal to the poverty line.

$$C_\alpha(z, y) = \mathbf{g}_\alpha(z, y) - \mathbf{g}_1(z, y) \quad \text{if } \alpha > 1 \\ = 0 \quad \text{otherwise.} \quad (6)$$

Then, $C_\alpha(z, y)$ is a monetary evaluation of the inequality cost when the income of the non-poor segment of population is set equal to z . Using equation (6), the FGT poverty measure respecting the transfer axiom can be expressed as:

$$\mathbf{g}_\alpha(z, y) = \mathbf{g}_1(z, y) + C_\alpha(z, y), \quad \forall \alpha > 1. \quad (7)$$

When the income gap is equally distributed among total population, the cost of inequality equalizes zero and $\mathbf{g}_1(z, y)$ will be, as revealed by equation (7), an appropriate poverty measure. Although it recalls the N axiom of Sen (1976), this result is slightly different since $C_\alpha(z, y)$ is not the cost of inequality within the poor segment of population, but within the total population, under the assumption that the income of the non-poor is just equal to z . This cost will be positive as soon as we have a segment of the population that is poor, even in the absence of inequality within this segment.⁶

If one assumes that policymakers seek to improve targeting of the poor population by reforming the current allocation of an anti-poverty budget, with perfect targeting, the income gap measure, $\mathbf{g}_1(z, y)$, could be decreased by an amount equal to the leakage average (i.e. the average cost of the inclusion error) of the current distribution of the anti-poverty budget. However, as perfect targeting is costly according to Besley and Kanbur (1993), the reduction of $\mathbf{g}_1(z, y)$ with imperfect targeting will be just equal to the cost reduction of awarding the non-targeted group:

$$\mathbf{g}_1(z, y^p) = \mathbf{g}_1(z, y^o) - [F(z, y^o) - F(z, y^p)], \quad (8)$$

where $y^o, y^p, F(z, y^o)$, and $F(z, y^p)$ are respectively the income distribution and the average of leakage in the pre- and post-reform situation.

Because the ability of a design to concentrate benefits on the poor is only one determinant of its impact on poverty, it is relevant to focus on the poverty outcome, which can be given by:

$$E_\alpha(z, y^o, y^p) = \mathbf{g}_\alpha(z, y^o) - \mathbf{g}_\alpha(z, y^p), \quad (9)$$

where $E_\alpha(\cdot)$ is a performance measure of the suggested reform that is supposed revenue-neutrally.

Furthermore, it is useful to present this outcome as a function of the reform ability to focus benefits on the poor. Hence, using equations (7) and (8), we can rewrite equation (9) as:

$$E_\alpha(z, y^o, y^p) = \mathbf{g}_0(z, y^o) - \mathbf{g}_0(z, y^p) \quad \text{if } \alpha = 0 \\ = [F(z, y^o) - F(z, y^p)] + [C_\alpha(z, y^o) - C_\alpha(z, y^p)] \quad \text{if } \alpha \geq 1. \quad (10)$$

The above equation shows that if $\alpha = 0$, the efficiency of the reform under consideration is captured by its ability to lift the richest of the poor out of poverty.⁷ For $\alpha = 1$, the effectiveness of the reform is simply appreciated by its targeting accuracy improvement, i.e. its ability to award benefits to poor people, regardless of their poverty level. Finally, for $\alpha > 1$, the impact on inequality of the reform is equally considered. This impact will be positive if the reform enables to award more benefits to the poorest. As α becomes very large, $E_\alpha(\cdot)$ approaches a ‘‘Rawlsian’’ measure which exclusively focuses on the welfare improvement of the poorest, even at the risk of increasing leakages.

⁶ We have a correspondence with the N axiom of Sen (1976) only if all people are poor and having the same income.

⁷ If we adopt the terminology of Bourguignon and Fields (1997), this is a p-type transfer.

3. The Model with Horizontal Inequity

The like individuals (or equals) that the HE principle advises to treat alike are, according to Feldstein (1976), those with the same utility. The need to assess to what extent a given reform generates HI can be satisfied when the equals are identified. We consider as equals those having the same pre-reform income gap, or, said differently, those with the same income under the assumption that the income of the non-poor equalizes z .

Let j denote the group of individuals with exactly the same pre-reform income gap, $g(z, y_j^o)$, and J denote the set of the equals groups. Locally, we seek to assess the extent of HI resulting from unequal treatment within a given group of equals. We can assume that the local HI is the inequality introduced by the reform within each group. Hence, the cost of inequality approach, developed in the previous section, can be used to assess locally the magnitude of HI.

Thus, let $T(\cdot)$ be the vector of the net transfers corresponding to the difference between the transfer awarded to each one after and before the reform, $\mathbf{g}_1(z, y_j^p)$ be the mean of the post-reform income gap in j , and $\mathbf{g}_\alpha(z, y_j^p)$ be the post-reform EDE income gap in j . A first measure of the local cost of HI at j can be given by:

$$H_\alpha(z, y_j^p) = \mathbf{g}_\alpha(z, y_j^p) - \mathbf{g}_1(z, y_j^p). \quad (11)$$

This approach assumes that differences among individuals within any equals group induce the same increase in poverty as differences among individuals in different groups. Nevertheless, following Auerbach and Hasset (1999) and Duclos and Lambert (2000), it is useful to allow attitudes to differ about these two types of inequality. Since the poverty measures of FGT class, $\mathbf{g}_\alpha(z, y)$, prevent different preferences toward HI and VI, we need a new class including poverty measures which satisfy the following axiom of HE: the *horizontal equality axiom*, which states: given other things, a pure transfer of income from one to another person within the same group of equals should increase the poverty measure, even if we are insensitive to inequality among different groups.

While $\mathbf{g}_\alpha(z, y)$ does not satisfy this axiom, it can be generalized to a class which contains poverty measures that do. Formally, this new class can be defined as:

$$\mathbf{g}_{\alpha, \beta}(z, y^p) = \begin{cases} \left[\sum_j \frac{I_j}{I} \left(\frac{1}{I_j} \sum_{i \in j} (z - y_j^{ip})_+^\beta \right)^{\frac{\alpha}{\beta}} \right]^{\frac{1}{\alpha}} & \text{if } \alpha \geq 1 \text{ and } \beta \geq 1 \\ = P_0(z, y^p) & \text{otherwise,} \end{cases} \quad (12)$$

where y_j^{ip} is the post-reform income level of an individual i in a group j , β is the inequity aversion within groups, and α is the inequality aversion between groups. If $\alpha = \beta$, this reduces to FGT class of poverty measures.

Hence, with $\mathbf{g}_{\alpha, \beta}(z, y)$, it is possible to be averse to HI even when we have not any preferences toward VE, i.e. when $\alpha = 1$. Indeed, while $\mathbf{g}_{1,1}(z, y)$ is not in line with the HE axiom, for $\beta > 1$, $\mathbf{g}_{1, \beta}(z, y)$ fulfils this axiom since it is sensitive to unequal treatment of equals. In addition, for $\beta > \alpha > 1$, preferences toward HE are more important than those toward VE.⁸

The amount that we would be prepared to sacrifice to remove locally HI without increasing poverty is henceforth given by:

$$H_\beta(z, y_j^p) = \mathbf{g}_{1, \beta}(z, y_j^p) - \mathbf{g}_{1,1}(z, y_j^p). \quad (13)$$

⁸ Since the poverty measures of FGT class which satisfy the transfer axiom of Sen (1976) imply the same preferences toward VE and HE, the case in which we are less averse to HE than to VE is excluded.

The next step is to aggregate the local measure of the HI cost, $H_\beta(z, y_j^p)$, into a global cost, $H_\beta(z, y^p)$, using an appropriate weighting system. Heeding Musgrave's (1990) dictum to avoid inappropriate comparisons between unequals, $H_\beta(z, y^p)$ should be unaffected by vertical considerations. It is then relevant to choose a weighting scheme so that the importance attributed to a local HI does not depend upon the income (or income gap) level at which it is occurred.⁹ This condition is fulfilled as soon as the share of each group of equals in overall population is chosen as its weight. Therefore, we have:

$$H_\beta(z, y^p) = \sum_j \frac{I_j}{I} H_\beta(z, y_j^p). \quad (14)$$

An attractive feature of $H_\beta(z, y^p)$ is that it sets a monetary value upon HI. $H_\beta(z, y^p)$ depends on the "local" inequity aversion parameter, β , but not on the "global" inequality aversion parameter, α . As β becomes large, the willing to pay of policymakers to eradicate horizontal equity violations rises, and so $H_\beta(z, y^p)$ increases.

Defining $T_\beta^{\text{Pn}}(\cdot)$ as an alternative design which would entail the same post-reform poverty level in j without violations of HE:

$$T_\beta^{\text{Pn}}(z, y_j^o, y_j^p) = \mathbf{g}_{1,1}(z, y_j^o) - \mathbf{g}_{1,\beta}(z, y_j^p) - F_j^o(z) \quad \forall \beta > 1. \quad (15)$$

Because we assume that the original poverty alleviation program, generating $F_j^o(z)$, treats all equals exactly identically,¹⁰ $T_\beta^{\text{Pn}}(\cdot)$ is thus a poverty-neutral reform.¹¹ It goes without saying that there would be losers and winners from this hypothetical process of HI removal. There is no question to command $T_\beta^{\text{Pn}}(\cdot)$ instead of $T(\cdot)$, but only to assess the cost of HI resulting from imperfect targeting, which precludes treating equals equally.

Let $T^{\text{Bn}}(\cdot)$ be a second hypothetical design with no HE violations defined by:

$$T^{\text{Bn}}(z, y_j^o, y_j^p) = \mathbf{g}_{1,1}(z, y_j^o) - \mathbf{g}_{1,1}(z, y_j^p) - F_j^o(z). \quad (16)$$

This alternative hypothetical process of HI elimination alleviates more poverty within j while requiring the same anti-poverty budget as $T(\cdot)$ at the most. Thus, comparing the two HE replacement designs $T_\beta^{\text{Pn}}(\cdot)$ and $T^{\text{Bn}}(\cdot)$, the first enables more budget saving and the same poverty level, while the second enables a better performance in alleviating poverty without calling for more public expenditures, even when overall population are poor. Our first result shows that $H_\beta(z, y_j^p)$ may be interpreted as the average budgetary saving which would come from substituting $T_\beta^{\text{Pn}}(\cdot)$ to $T^{\text{Bn}}(\cdot)$. Hence, the local HI cost can be written as:

$$H_\beta(z, y_j^p) = T^{\text{Bn}}(z, y_j^o, y_j^p) - T_\beta^{\text{Pn}}(z, y_j^o, y_j^p). \quad (17)$$

Theorem: $H_\beta(z, y^p)$ measures the average saving of the anti-poverty budget which would come from a better targeting allowing to substitute $T_\beta^{\text{Pn}}(\cdot)$ to $T^{\text{Bn}}(\cdot)$.

⁹ For example, because they adopt a weighting system that depends upon the income level, the global HI index of Berliant and Strauss (1985) does not satisfy the Musgrave's (1990) recommendation.

¹⁰ If the original scheme violates HE, the leakages will not be equally distributed within equals groups and so $T_\beta^{\text{Pn}}(\cdot)$ will not be poverty-neutrally. Also, it is obvious that $F_j^o(z) = 0$ if $\mathbf{g}_{1,1}(z, y_j^o) > 0$.

¹¹ There is another advantage from this hypothetical reform, that is, it removes leakages.

Proof: The average budget saving which would come from removing local HI with poverty indifference in j is $H_\beta(z, y_j^p)$. Therefore, the average budget saving overall is $\frac{1}{I} \sum_j I_j H_\beta(z, y_j^p)$, that is $H_\beta(z, y^p)$.

Let $V_{\alpha\beta}(z, y^p)$ be the cost of inequality which remains after the $T_\beta^{\text{Pn}}(\cdot)$ hypothetical process of HI elimination. This cost can be then expressed as:

$$V_{\alpha\beta}(z, y^p) = \mathbf{g}_{\alpha, \beta}(z, y^p) - \mathbf{g}_{1, \beta}(z, y^p) \quad (18)$$

where

$$\mathbf{g}_{1, \beta}(z, y^p) = \sum_j \frac{I_j}{I} \mathbf{g}_{1, \beta}(z, y_j^p).$$

Therefore, if all inequality resulting from $T(\cdot)$ is eliminated without increasing poverty, by moving from the post-reform distribution of income-gaps to the distribution in which the income-gap of everyone would be equal to $\mathbf{g}_{\alpha, \beta}(z, y^p)$, the average income saving would be equal to $C_{\alpha, \beta}(z, y^p)$, which is given by:

$$C_{\alpha, \beta}(z, y^p) = \mathbf{g}_{\alpha, \beta}(z, y^p) - \mathbf{g}_{1, 1}(z, y^p); \quad (19)$$

while if this movement to perfect equality is done after application of $T_\beta^{\text{Pn}}(\cdot)$, where the income-gap of everyone in j would be equal to $\mathbf{g}_{1, \beta}(z, y_j^p)$, the income saving would be $V_{\alpha\beta}(z, y^p)$. Adding the overall income saving that would result from the application of $T_\beta^{\text{Pn}}(\cdot)$ instead of $T^{\text{Bn}}(\cdot)$ and from moving to perfect equality, we have:¹²

$$C_{\alpha\beta}(z, y^p) = V_{\alpha\beta}(z, y^p) + H_\beta(z, y^p). \quad (20)$$

The global cost of inequality resulting from the application of the design $T(\cdot)$ is then the cost of VI after a horizontally equitable transfer scheme plus the cost of HI. Thus, heeding the possibility to be more averse to HI than to VI, we must rewrite expression (7) as:

$$\begin{aligned} \mathbf{g}_{\alpha, \beta}(z, y) &= \mathbf{g}_{1, 1}(z, y) + V_{\alpha\beta}(z, y) + H_\beta(z, y) & \text{if } \alpha, \beta \geq 1 \\ &= P_0(z, Y) & \text{otherwise.} \end{aligned} \quad (21)$$

Moreover, the decomposition of the global cost of inequality into between-and-within contributions enables us, using equations (10) and (21), to rewrite the performances of the reform as:

$$\begin{aligned} E_{\alpha, \beta}(\cdot) &= [F^o(\cdot) - F^p(\cdot)] + [V_{\alpha\beta}^o(\cdot) - V_{\alpha\beta}^p(\cdot)] - H_\beta^p(\cdot) & \text{if } \alpha, \beta \geq 1 \\ &= P_0(z, y^o) - P_0(z, y^p) & \text{otherwise.} \end{aligned} \quad (22)$$

¹² This between-and-within-groups decomposition is due to Blackorby et al. (1981). It is followed in particular by Duclos and Lambert (2000) and Lambert and Ramos (1997).

In this form, it is easy to see the trade-off between targeting accuracy, vertical inequality, and horizontal inequity. If the preference toward HE is very important, the post-reform EDE income gap in each group of equals will be given by the income gap of losers; the status quo will be preferred even if the new design is vertically more equitable. The specification (22) enables us so to define the policy indifference curve as the locus points in (α, β) space such that the policymakers are indifferent to the original and the new poverty alleviation program.

4. Implementation of the Methodology Using the Tunisian Household Survey

One of the principal uses of the new class is to examine the impact of poverty alleviation reforms using micro-data sets from which it is possible to compute the extent of HI for each equals group in the sample. Because there are typically few exact equals, HI will therefore be under-estimated if captured from sample micro-data. This identification problem is the main obstacle preventing measurement of HI and leading to the emergence of the re-ranking approach.¹³ To avoid this last one, Berliant and Strauss (1985) as well as Lambert and Ramos (1997) have banded into close-equals groups the pre-reform distribution to obtain what they call an index of pseudo-horizontal inequality. However, as an arbitrary choice of the band (or the bandwidth) is not wholly satisfactory, we follow a non-parametric estimation procedure to assess the magnitude of HI.¹⁴

Formally, given the technique of kernel density estimation, a natural way to estimate the mean of the post-reform income-gap in each equals group is first to calculate an estimate of the joint density $\hat{f}(y^o, y^p)$ of (y^o, y^p) , and then to aggregate it according to the formula:

$$\hat{\mathbf{g}}_{1,1}(z, y_j^p) = \int_j (z - y^p)_+ \hat{f}(y^p / y^o) dy^p. \quad (23)$$

Likewise, a natural estimator $\hat{\mathbf{g}}_{1,\beta}(z, y_j^p)$ for $\mathbf{g}_{1,\beta}(z, y_j^p)$ is given by:

$$\hat{\mathbf{g}}_{1,\beta}(z, y_j^p) = \left[\int_j (z - y^p)_+^\beta \hat{f}(y^p / y^o) dy^p \right]^{\frac{1}{\beta}}. \quad (24)$$

Using equation (13), we can compute an estimate of a local cost of horizontal inequity for equals group j :

$$\begin{aligned} \hat{H}_\beta(z, y_j^p) &= \hat{\mathbf{g}}_{1,\beta}(z, y_j^p) - \hat{\mathbf{g}}_{1,1}(z, y_j^p) && \text{if } \hat{\mathbf{g}}_{1,1}(z, y_j^p) > 0 \\ &= 0 && \text{if } \hat{\mathbf{g}}_{1,1}(z, y_j^p) = 0. \end{aligned} \quad (25)$$

The continuity of $\hat{f}(y^o, y^p)$ across y^o will ensure the continuity of these above estimators over y^o . Integrating $\hat{H}_\beta(z, y_j^p)$ over y^o will yield an estimator of the overall cost of HI:

$$\hat{H}_\beta(z, y^p) = \int_0^{+\infty} \hat{H}_\beta(z, y_j^p) \hat{f}(y^o) dy^o, \quad (26)$$

where $\hat{f}(y^o)$ is an estimator of the marginal density function for the pre-reform income distribution.

The estimation of the joint density $\hat{f}(y^o, y^p)$ is made following the non-parametric kernel estimation procedure, with Gaussian Kernel and bandwidth chosen to minimize the *MISE* (mean integrated

¹³ On the re-ranking approach, see Atkinson (1980), Duclos (1995), Feldstein (1976), King (1983a), and Plotnick (1981, 1982). This approach has been attacked, for instance, by Kaplow (1989) and Musgrave (1990).

¹⁴ On the non-parametric estimation approach, see Silverman (1986). To compute HI, this approach is also followed by Auerbach and Hasset (1999) and Duclos and Lambert (2000).

square error). The estimated distribution tends asymptotically to the true one if the later is continuous.¹⁵

The methodology presented in previous sections is applied to data from the 1990 Tunisian survey. This is a multipurpose household survey which provides information on expenditures and quantities for food items and expenditures for non-food items, as well as on many other dimensions of 7734 households' behavior including the consumption of own production, education, housing, region of residence, demographic information, and economic activities. Nevertheless, it does not include information on income distribution. Since our goal is not to discuss the choice of an individual's welfare indicator, we assume that total expenditures *per capita* of households is an appropriate *proxy* of income distribution.

As the main poverty alleviation program currently in force in Tunisia is based on targeting by commodities, it is instructive to compare its outcomes with those of an alternative one based, for instance, on targeting by indicators. This requires that poverty measures are sensitive to price system variations. Following the methodology of King (1983b), it is possible to compute the equivalent income distribution under targeting by commodities, that is y_e^o , and under targeting by indicators, y_e^p . Using proxy means tests through a model designed to minimize poverty, subject to a fixed fund available for transfers, Bibi (2001) has shown that this new revenue-neutral design is appealing in alleviating poverty relative to food subsidies scheme, when the effects on poverty are only computed by FGT measures.

To analyze the outcomes of this hypothetical design when preferences toward HE may be different to those toward VE, an equivalent poverty line (z_e) of 360 Tunisian Dinar (DT) per capita per year is used, which corresponds to 50% of the arithmetic mean of the expenditure distribution. Simulation shows that leakages would be reduced by 10.6 DT per capita. Table 1 summarizes the efficiency measures of this reform. It gives the cost of horizontal and vertical inequality, for different values of the two inequality aversion parameters, as well as the effects on poverty as computed by some poverty measures of the proposed class.

Simulation results show that for some combination of the inequality aversion parameters, the simulated design will be preferred to food subsidies scheme, and for other combinations targeting by commodities will be preferred. For instance, the above table reveals that as long as the HI aversion parameter is at most twice the VI aversion parameter, the reform seems to be unambiguously preferable. Yet, for some values of the HI aversion parameter, like $\beta = 3\alpha$, the loss of HE offsets the gain in VE and the targeting accuracy improvement, leading social policymakers to prefer the status quo.

5. Conclusion

The performance of a given reform on the poor's welfare is sometimes appreciated through its ability to reduce two common errors: that of exclusion—reform's failure to reach some members in the targeted group—and that of inclusion, where the new design reaches some non-poor people and so leads to leakages of program benefits. Following Atkinson (1995), the awarding of benefits to some members of the non-targeted group reduces the vertical efficiency of the new design; the errors of exclusion lead to horizontal inefficiency since the program becomes less effective in serving the targeted group. Ravallion and Datt (1995) argue that the ability of a design to concentrate benefits on the poor should not be confused with its impact on poverty; the former is only one determinant of the latter. Nevertheless, as policymakers are interested in the trade-off between targeting accuracy, vertical inequality, and horizontal inequity, it becomes instructive to deconstruct the impact of a design on poverty into these three components.

While it is easy to assess the cost of inclusion errors and let performance measures be a function of leakages-cost variation, reducing horizontal inefficiency to the percentage of the poor people incorrectly excluded from the program benefits prevents the sought deconstruction. A first solution would have been found in the re-ranking approach, so as to avoid the identification problem of equals.

¹⁵ See Silverman (1986).

But the use of the latter approach is not henceforth relevant since the emergence of the non-parametric estimation method, which allows the solving of the identification problem consistently. A best solution is then to use the general approach to the measurement of inequality in order to deduce a local measure of HI. When this local measure is aggregated, using an appropriate weighting system, an overall cost of the HI results.

A large preference toward HE could invert results of the reform's outcomes, leading policymakers to prefer the status quo. Formally, it is worthwhile to define the policy indifference curve bordering the set of poverty measures in which the reform is preferred from the set of measures in which it is better to preserve the original distribution. When we adopt poverty measures that allow only the same preferences toward HE and VE, like the measures belonging to the FGT class, it is no longer possible to achieve this task. Hence, the cost of inequality approach is followed, to make the required decomposition, only after specifying a new class of poverty measures, which enables attitudes to differ about HI and VI.

The method developed in this paper has been applied to assess the effects of a hypothetical reform on the leakages to non-poor, HI, and VI, using household data from Tunisia. This reform suggests substituting a direct transfer program, based on proxy means tests, for the current universal food subsidies system. Simulation reveals that expending the same anti-poverty budget to target direct transfers would be more effective in alleviating poverty as long as aversion parameter to HI is at most twice VI aversion parameter. Otherwise, the status quo would be preferred regardless of the poverty measure chosen.

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Table 1: Efficiency Measures of Imperfect Targeting

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
$V_{\alpha/\alpha}^o - V_{\alpha/\alpha}^p$	-	0	15.74
$H_{\alpha}(z_e, y_e^p)$	-	0	5.96
$E_{\alpha,\alpha}(z_e, y_e^o, y_e^p)$	0.044	10.6	20.38
$V_{\alpha/2\alpha}^o - V_{\alpha/2\alpha}^p$	-	0	12.49
$H_{2\alpha}(z_e, y_e^p)$	-	5.96	13.39
$E_{\alpha,2\alpha}(z_e, y_e^o, y_e^p)$	0.044	4.64	9.7
$V_{\alpha/3\alpha}^o - V_{\alpha/3\alpha}^p$	-	0	9.95
$H_{3\alpha}(z_e, y_e^p)$	-	10.17	18.07
$E_{\alpha,3\alpha}(z_e, y_e^o, y_e^p)$	0.044	-0.43	-2.48

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7 Boulou Hanna St. Dokki, Cairo, Egypt

Tel: (202) 3370810 – (202) 7485553 – (202) 7602882

Fax: (202) 7616042. Email: erf@erf.org.eg Website: <http://www.erf.org.eg>