EMPLOYMENT EFFECTS OF PRODUCTIVITY IMPROVEMENT AND MIGRATION OF SKILLED WORKERS

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Abstract

This paper studies the employment of unskilled labor effects on skilled labor productivity improvements, alternatively with and without skilled migration between two countries under the assumption that the level of unemployment benefits depends on the average income observed in each country. In models of bargaining wage, this level of benefits is generally considered exogenous. Endogeneity of the level of benefits leads to an interdependence between wages and benefits. As a consequence, the wage for unskilled labor is linked to the wage for skilled labor. Under this assumption, skilled productivity improvements, in absence of migration, decreases the employment of unskilled labor. With migration, the employment of unskilled labor in the host country is ambiguous, but in the send country, employment decreases.

ملخص

تدرس هذه الورقة تأثير تشغيل العمالة غير الماهرة على تحسين إنتاجية العمالة الماهرة، بالتبادل مع أو بدون هجرة العمالة الماهرة في دولتين بافتراض أن مستوى إعانات البطالة يعتمد على متوسط الدخل في كل دولة. وفي نماذج الأجور التفاوضية، عادة ما يعتبر هذا المستوى من الاعانات متغير خارجي. فافتراض أن الإعانات متغير داخلي سيؤدي إلى إعتماد تبادلي بين الأجور ةالاعانات. وعليه فإن ذلك سيؤدي إلى ربط أجور العمالة غير الماهرة بأجور العمالة الماهرة .وفي إطار هذا الأفتراض، تقلل تحسينات الإنتاجية الماهرة، في غياب الهجرة، من تشغيل العمالة غير الماهرة. وعند أخذ الهجرة في الاعتبار، فإن تشغيل العمالة غير الماهرة في الدولة المضيفة يصبح أمراً مبهماً، بينما يقل التشغيل في الدولة المصدرة للعمالة.

1. Introduction

The findings in the beginning of the 1990s that wage differences among workers had increased significantly in the U.S. since the early 1980s (Juhn, Murphy and Pierce (1993), Katz and Murphy (1992) and Gottschalk (1997) have triggered two interesting research topics. The first is identifying the cause of change for which the main reasons were: increased trade with less developed countries (Wood, 1995 and Leamer, 1996) and skill-biased technical progress (Krugman, 2000 and Krusell, Ohanian, Rios-Eull and Violante, 2000). Currently, skill-biased technical change is regarded as the quantitatively more important of the two accounts. The second research topic suggests that the trend increase in European unemployment is partly a result of technical change similar to that in the U.S. as well as rigid relative wages, which are presumed to be the result of institutional factors such as collective wage bargaining, trade union policies, unemployment benefits, and social security.

We assume that wages are determined by a generalised Nash bargain between each firm and its union. In the literature, there is no consensus about which union objective function to use. The most popular one is the utilitarian objective function, according to which, the union not only cares about wage but also about employment. However, some authors have argued that trade unions might rather be concerned with the well-being of their employed members (insiders), neglecting the interests of unemployed persons (Weizman, 1987; Oswald, 1993). Here, we follow the latter approach.

In standard models of wage bargaining, wage is determined in negotiations between the firm and the union (Mac Donald and Solow, 1981 and Layard, Nickel and Jackman, 1991). When a negotiation fails, the unemployed receives an income which is assumed to be exogenous; but in reality, this income is often linked to indices of standards of living.

In this paper, we extend the traditional wage bargaining model by the assumption that the level of unemployment benefits depends on the average income observed within the country. In developed industrial countries, these benefits are considered at the level that guarantees a "decent life" rather than just the amount of money that suffices survival. Thus, we study the employment of unskilled labour effects on the improvement of skilled labour productivity. This improvement may be due to technical progress. However, different studies highlight that skilled-biased technological change influences employment of unskilled labour if the two types of labour are substitutes or complements and if the level of unemployment benefits is exogenous.

We suppose that there are two symmetrical countries in which two kinds of productive labour can be differentiated: high wage-earning skilled labour and low wage-earning unskilled labour. We adopt the viewpoint of Davis (1998) and Krugman (1995) by supposing that the wage of skilled labour is flexible[1], and the wage of unskilled labour is determined by the bargaining between the firm and the union.

It is the unskilled workers who are by far more likely to be unemployed as evident from Table 1.

In the model, we assume that the two types of labour are neither substitutes nor complements, and the level of unemployment benefits depends on the average income observed in the country. Endogeneity of benefit levels allows for the existence of interdependence between the wage of the unskilled and the level of unemployment benefits *via* the average income. As

[1]In Davis (1998), the wage for unskilled labour is modelled as an exogenously given fixed minimum wage; while Krugman (1995) assumed it to be a fixed proportion of the skilled worker's wage.

a result of wage bargaining, if the average income decreases (increases) the wage of the unskilled decreases (increases) and their employment increases (decreases). While the average income depends positively on the total income and negatively on the total population, it is important to study variations following rise of productivity and emigration of skilled labour. The paper is organised as follows: Section 2 sets up the basic model; Section 3 studies the static comparative; and the last section concludes.

2. The Model

Two countries *j=1,2* compose our economy. In each country, there is a representative firm, a given number of skilled labor N_j^s , unskilled labour N_j^u , a government, and a union.

2.1 Firm

The firm produces a single good by using two input factors: skilled labour l_j^s and unskilled labour l_j^u . The production function is given by the expression

$$Y_j = A_j\left(l_j^s\right)^{\alpha} + A_j\left(l_j^u\right)^{\beta} \qquad (1)$$

where *Y* is the quantity of final good, A_j^s , A_j^u are productivity parameters and $\alpha,\beta \in]0,1[$ are supposed to be identical for the two countries.

At given wage levels, the firm chooses the level of employment so as to maximize its profits. The demand for skilled and unskilled labour is respectively

$$l_j^s = \left(\frac{\alpha A_j^s}{w_j^s}\right)^{\frac{1}{(1-\alpha)}} \qquad (1.1)$$

$$l_j^u = \left(\frac{\beta A_j^u}{w_j^u}\right)^{\frac{1}{(1-\beta)}} \qquad (1.2)$$

w_j^s and w_j^u are the wages for skilled and unskilled labour in country *j* respectively.

Lemma1: In absence of emigration, the wage of skilled labour is determined by exogenous parameters and variables:

$$l_j^s = N_j^s \quad \Leftrightarrow \quad w_j^s = \frac{\alpha A_j^s}{N_j^{s(1-\alpha)}}$$

The full employment hypothesis implies that all skilled workers are employed and that the wage adjusts to clear the market. Therefore, the wage is determined by the productivity and the supply of work. In reality, the wage of skilled workers is not flexible. The new theories of labour market explain this rigidity by efficiency wages (summers, 1988) and union bargaining (Mac Donald and Solow, 1981 and Layard, Nickel and Jackman, 1991).

2.2 Workers

Preference for each type of work *i=s,u* in the country *j=1,2* depends on the net tax wage. This function is given by the equation

$$U_j^i = \ln\left(w_j^i(1-t)\right) \qquad (2)$$

where *t* is the tax rate on the wage supposed to be identical for the two countries.

2.3 Government

The government finances unemployment benefits through a proportional income tax. The number of unemployed workers multiplied by the level benefits must be equal to the tax income.

$$tY_j = B_j(N_j^u - l_j^u) \tag{3}$$

where $B_j = \delta(1-t)\bar{Y}_j$ is the unemployment benefits, $\bar{Y}_j = \frac{Y_j}{N_j = N_j^s + N_j^u}$ is the average income and δ is the indexation rate of unemployment benefits in country *j*.

2.4 Wage bargaining

All unskilled workers are assumed to be members of the union. The union maximizes the expected utility V_j of its representative member conditional on the wage level.

$$V_j = q_j \ln(w_j^u(1-t)) + (1-q_j)\ln B_j \tag{4}$$

where $q_j = \frac{l_j^u}{N_j}$ is the probability for any union member to become (or remain) employed and $(1-q_j)$ is the probability for unemployed. The wage of unskilled labour is assumed to be determined in negotiations between the firm and the union. When negotiation fails, all unskilled workers are unemployed and the union's disagreement point in bargaining is identified with the value of its objective function

$$V_j^0 = \ln B_j \tag{5}$$

In this case, only skilled workers are employed and the firm's profit is $\Pi_j^0 = A_j^s l_j^{s\alpha} - w_j^s l_j^s$.

We assume that bargaining covers only the wage, whereas the level of employment is determined by the firm. Thus, the outcome of bargaining is modelled as the Nash bargaining solution

$$w_j^u = \arg\max_{w_j^u} \left(V_j - V_j^0\right)^{\lambda} \left(\Pi_j - \Pi_j^0\right)^{1-\lambda} \tag{6}$$

where $0<\lambda<1$ measures the union's relative bargaining power.

Substituting equations (1-2) and (3) in equation (4) and solving $V_j - V_j^0$ yields

$$V_j - V_j^0 = \frac{l_j^u}{N_j^u} \ln \frac{w_j^u(1-t)}{B_j} \tag{7}$$

The firm's profit is

$$\Pi_j = Y_j - w_j^s l_j^s - w_j^u l_j^u \tag{8}$$

The firm's disagreement point is identified with the level of profit (Π_j^0) when negotiation on the wage for unskilled labour fails and only skilled workers are employed.

Substituting equation (1-2) in equation (8) and solving for $\Pi_j - \Pi_j^0$ yields

$$\Pi_j - \Pi_j^0 = (1-\beta) A_j^u l_j^{u\beta} \tag{9}$$

Plugging expressions (1-2), (7) and (9) in equation (6) yields the Nash maxim as function of the wage for unskilled labour.

$$w_j^u = \arg\max_{w_j^u} \left(\frac{\beta A_j^u}{w_j^u} \right)^{\frac{\lambda+\beta(1-\lambda)}{1-\beta}} N_j^{-\lambda} \left[(1-\beta) A_j^u \right]^{1-\lambda} \ln \left[\frac{w_j^u (1-t)}{B_j} \right]^{\lambda} \quad (10)$$

Solving the maximization problem yields the following result

$$w_j^u = \exp\left(\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)} \right) \frac{B_j}{1-t} \quad (11)$$

The wage for unskilled labour is an increasing function of the level of benefits and the union's bargaining power.

2.5 Migration of Skilled Workers

We assume that only skilled workers migrate between these two countries and the existence of unemployment prevents unskilled workers to migrate. The cost of emigration is assumed to be null.[2] Skilled workers migrate from country *j'* to country *j* only if

$$U_j^s > U_{j'}^s \quad (12)$$

So that the difference in utility levels persists, emigration continues until utilities between these two countries equalize. In this case, the demand for skilled labour in country *j* is $l_j^s = N_j^s + M_{j'}^s$ and in country *j'* is $l_{j'}^s = N_{j'}^s - M_{j'}^s$. Substituting equation (1-1) for each country in equation (14) yields the number of migrants.

$$M_{j'}^s = \frac{g N_{j'}^s - N_j^s}{1+g} \quad (13)$$

where $g = \left(\frac{A_j^s}{A_{j'}^s} \right)^{\frac{1}{1-\alpha}}$ represents the gap of productivities (or the gap of wages) between these two countries.

Lemma 2: The number of migrants is an increasing function of the gap of productivities between these two countries.

$$\frac{\partial M_{j'}^s}{\partial A_j^s} = \frac{g^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_{j'}^s} \frac{N_{j'}^s + N_j^s}{(1+g)^2} > 0$$

This expression explains that the migration decision is faction of wage gaps between these tow countries. Full employment and migration is not associated with any costs.[3]

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[2] Bouzahzah and Saber (2002) have shown, in a dynamic framework model that psychological and monetary costs play an important role on the decision to immigrate.

[3] The so-called 'new economics of migration' takes a different view of the microeconomic determinants of migration. Their motivations were some observations in the LDCs which could not be explained by the extended neoclassical approach. Therefore, according to the 'new economics of migration', the individual is not the decision-maker, and the decision is the household's or the family's respectively. Although it is also possible to analyze migration as a family decision within the neoclassical framework (Mincer, 1978), the perspective of the New Economics is a fundamentally different one. Individual family members migrate because the dependence on the situation in single labour markets is reduced. Similar to a portfolio decision of an investor, there is risk reduction by diversification. This form of risk diversification is a particularly important aspect in LDCs where public social security is inadequate, and working private capital markets are rare. Remittances are often the biggest part of household budgets and are also highly important for many LDCs on the macro-economic level.

3. Comparative static

The comparative static properties of equilibrium allocation are presented in the following propositions:

Proposition 1: In the absence of migration, an increase in the unskilled workers' productivity in the country *j=1,2* leads to an increase in the unskilled labour demand.

$$\frac{\partial l_j^u}{\partial A_j^u}>0$$

Proof: See Appendix A.1.

An increase in the unskilled workers' productivity leads, via an increase in average income, to an increase in the unskilled workers' wage. But this increase is less than what would be justified by the productivity gain because the wage is linked to the average income level, which increases by less than the unskilled workers productivity; therefore, employment increases. This failure of wage to fully adjust to changes in productivity can be seen as rigidity of relative wage w_j^u/w_j^s. While the wage of skilled workers adjusts to clear the market, the wage of unskilled workers depends on the productivity of both, skilled and unskilled workers. This rigidity leads to a decrease in unemployment in response to an increase in the productivity of unskilled workers. The same results are obtained in traditional models of wage bargaining where the reservation wage of the workers is exogenous.

Proposition 2: In the absence of migration, the rise of skilled labourers' productivity in the country *j=1,2* diminishes demand for unskilled workers.

$$\frac{\partial l_j^u}{\partial A_j^s}<0$$

Proof: See Appendix A.1.

In absence of emigration, an increase in the skilled labourers' productivity in the country *j=1,2* reduces demand for unskilled workers. In fact, this rise of productivity leads to an increase in unskilled workers' wage *via* a rise in the average income of the country. Thus, the rise of wage increases the production costs and decreases demand for unskilled workers[4].

This result is consistent with the common view that skill-biased technical change leads to an increase in wage inequality in the U.S. and the U.K., and an increased unemployment in Europe[5].

Proposition 3: In the presence of migration between these countries, an increase in the skilled workers' productivity has an ambiguous effect on demand for unskilled workers in the host country *j*; but in the send country *j'*, demand for unskilled workers falls.

$$\frac{\partial l_j^u}{\partial A_j^s}><0 \text{ and } \frac{\partial l_{j'}^u}{\partial A_j^s}<0$$

Due to emigration, an increase in the skilled workers' productivity in country *j* leads to an increase in skilled workers' wage and the number of emigrants from country *j'*. As a consequence, the total population in country *j* increases. Income in the host country increases by the immigration process (but less proportionally to the increase in total population) and by the rise in productivity. However, the extent of the shock and the number of emigrants can

[4] The results do not rely on any complementarity or substitution relations between the different types of labour in the production process.

[5] See, eg, Freeman (1995), Krugman (1995), Siebert (1997) and Davis (1998).

influence the average income. In other words, if the average income decreases (respectively increases) unskilled workers' wage decreases (respectively increases) and demand for the unskilled workers increases (respectively decreases).

Concerning the send country *j'*, emigration decreases both total population and income; but income decreases less proportionally to total population. This leads to an increase in average income and wage for unskilled workers. Consequently, demand for unskilled workers in the send country decreases.

In order to verify these effects in the two countries, assuming emigration, we proceed to a numerical simulation. Thus, we suppose that the two countries are symmetrical and only the skilled productivity A_1^s in the country 1 is subjected to a positive shock.

Parameters α=0.15 and β=0.55 were calibrated so as to imply (w_u/w_S) to be around 1/2 with $A^u=A^s=1$. We assume that both firm and union have the same weight in the wage bargaining λ=1/2; and indexation rate of unemployment benefits is calculated so as to obtain an unemployment rate around 8% (δ=0.65).

Concerning the effect of an increase in skilled workers' productivity in country 1, table 2 shows that unskilled workers' demand l_1^s increases (respectively decreases). In fact, income increases due to the rise of the productivity of skilled workers and due to immigration following the rise of wages. When the income increases less than the increase of the total population, the average income (reservation wage) adjusts less to the level where it would have to be if there was no immigration, and demand for unskilled workers increases. However, when income increases relatively more than the increase in total population, the average income (reservation wage) increases, and demand for unskilled workers decreases. Therefore, the rise of skilled workers' productivity does not always affect the increase in unemployment of unskilled workers since immigration can increase the employment by preventing the increase of the reservation wage.

In the send country 2, the demand for unskilled workers l_2^s decreases with emigration of skilled workers because average income Y_2/N_2 increases. In fact, emigration decreases the income and the total population; but the diminution of income is less important than the population. So, the average income (or reservation wage) increases and demand for unskilled workers decreases. This last result is consistent with the common view that emigration of skilled workers has a negative effect on the send country by decreasing the income (Bhagwati and Hammada, 1974 and Miyagiwa, 1991).

However, an increase in the skilled workers' productivity in country *1* leads to an increase in skilled workers' wage and the number of emigrants from country *2*. As a consequence, the total skilled workers *2* increases and the total skilled workers in country 2 decreases as shown in table 2.

4. Conclusion

By developing the model of bargaining wage, we have shown that a rise of skilled workers' productivity influences demand for unskilled workers, assuming that: i) the two types of workers are neither substitutes nor complements; and ii) unemployment benefits depend on the average income. Even if this hypothesis remains to be verified empirically, it allows the same results as the skill-biased technological change.

These findings are consistent with unskilled workers' wages and employment evolution in Europe over the past decades. Wages for all skill levels have risen steadily over this period, and employment prospects of less skilled workers have been deteriorating. Thus, rise of

unemployment of unskilled workers can be attributed to the absence of emigration between European countries because migration can improve employment.

This paper can be extended in future work by taking into account the migration of unskilled workers and comparing results to these of standard models of wage bargaining.

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Table 1: European Union Unemployment Rates by Education Attainment for persons aged 25-64, Both Sexes, 2000

Low Education	Medium Education	High Education
5.20%	**4.52%**	**3.51%**

Source: OECD (2004).

Table 2: Skilled, Unskilled Workers' Demand in the Two Countries (all values are in %)

A_1^s	0*	10	20	30	40	50	60	70
Y_1/N_1	0	-0.22	-0.34	-0.36	-0.30	-0.18	0.002	0.23
l_1^u	0	0.5	0.74	0.80	0.62	0.40	-0.004	-0.5
Y_2/N_2	0	1.05	2.04	2.96	3.83	4.64	5.40	6.11
l_2^u	0	-2.30	-4.40	-6.29	-8.01	-9.59	-11.03	-12.36
N_1^s	0	5,6	10,7	15,3	19,5	23,4	26,9	30,2
N_2^s	0	-5,6	-10,7	-15,3	-19,5	-23,4	-26,9	-30,2

Notes : *: This column represents the reference's state before the shock.

Appendix

A.1 Proof of the Comparative Static Results for Equilibrium Level of Employment without Emigration

Proof: Divide both sides of the equilibrium condition (11) by βA_j^u and replace $\left(w_j^u/\beta A_j^u\right)$ by $l_j^{u-(1-\beta)}$ to get

$$l_j^{u-(1-\beta)}=\frac{\delta}{\beta A_j^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}\frac{Y_j}{N_j^s+N_j^u} \tag{14}$$

define the right hand side of (14) as $F(l_j^u,A_j^s,A_j^u)$

$$F(l_j^u,A_j^s,A_j^u)=\frac{\delta}{\beta A_j^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}\frac{Y_j}{N_j^s+N_j^u} \tag{15}$$

so that the equilibrium level of employment is given by

$$l_j^{*u-(1-\beta)}=F(l^{*u}_j,A_j^s,A_j^s)$$

Totally differentiating and solving for ∂l_j^u yields:

$$\partial l_j^u=-\frac{\partial F/\partial A_j^s}{\left[(1-\beta)l_j^{u-(1-\beta)}+\partial F/\partial l_j^u\right]}\partial A_j^s+-\frac{\partial F/\partial A_j^u}{\left[(1-\beta)l_j^{u-(1-\beta)}+\partial F/\partial l_j^u\right]}\partial A_j^u$$

From (15), it can easily be seen that $\frac{\partial F}{\partial l_j^u}>0$, $\frac{\partial lF}{\partial A_j^u}<0$ and $\frac{\partial F}{\partial A_j^s}>0$ and implying that $\frac{\partial l_j^u}{\partial A_j^u}>0$ (proof of proposition 1) and $\frac{\partial l_j^u}{\partial A_j^s}<0$ (proof of proposition 2).

A.2 Proof of the Comparative Static Results for Equilibrium Level of Employment in *j* and *j'* Countries with Emigration

In the host country *j*, demand for skilled workers is $l_j^s=N_j^s+M_{j'}^s$

The equation (14) can be written as:

$$l_j^{u-(1-\beta)}=\frac{\delta}{\beta A_j^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}\frac{Y_j}{l_j^s+N_j^u} \tag{16}$$

define the right hand side of(16) as $G(l_j^u,A_j^s)$

$$G(l_j^u,A_j^s)=\frac{\delta}{\beta A_j^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}\frac{Y_j}{l_j^s+N_j^u} \tag{17}$$

so that the equilibrium level of employment is given by

$$l^{*u-(1-\beta)}_j=G(l^{*u}_j,A_j^s)$$

Totally differentiating and solving for ∂l_j^u yields:

$$\partial l_j^u=-\frac{\partial G/\partial A_j^s}{\left[(1-\beta)l_j^{u-(1-\beta)}+\partial G/\partial l_j^u\right]}\partial A_j^s$$

We calculate $\frac{\partial G}{\partial A_j^s}$

$$\frac{\partial G}{\partial A_j^s}=k_j\left[l_j^{s\alpha}+\left(\alpha A_j^s l_j^{s(\alpha-1)}-\frac{Y_j}{N_j}\right)\frac{\partial M_j^s}{\partial A_j^s}\right]$$

where $k_j=\frac{\delta}{N_j\beta A_j^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}$ and $N_j=N_j^s+M_j^s+N_j^u$.

We distinguish two cases:

Case N°1:

If $\alpha A_j^s l_j^{s(\alpha-1)}>\frac{Y_j}{N_j} \Rightarrow \frac{\partial G}{\partial A_j^s}>0$ and $\frac{\partial l_j^u}{\partial A_j^s}<0$.

Case N°2:

if $\alpha A_j^s l_j^{s(\alpha-1)}<\frac{Y_j}{N_j}$ and $l_j^{s\alpha}<\left(\alpha A_j^s l_j^{s(\alpha-1)}-\frac{Y_j}{N_j}\right)\frac{\partial M_j^s}{\partial A_j^s} \Rightarrow \frac{\partial G}{\partial A_j^s}<0$ and $\frac{\partial l_j^u}{\partial A_j^s}>0$.

In send country *j'*, demand for skilled workers is $l_{j'}^s=N_{j'}^s-M_{j'}^s$

The equation (18) can be written for country *j'* as

$$l_{j'}^{u-(1-\beta)}=\frac{\delta}{\beta A_{j'}^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}\frac{Y_{j'}}{l_{j'}^s+N_{j'}^u} \qquad (18)$$

define the right hand side of (18) as $G(l_{j'}^u,A_j^s)$:

$$G(l_{j'}^u,A_j^s)=\frac{\delta}{\beta A_{j'}^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}\frac{Y_{j'}}{l_{j'}^s+N_{j'}^u}$$

so that the equilibrium level of employment is given by

$$l_{j'}^{*u-(1-\beta)}=G(l^*_{j'}^u,A_j^s)$$

Totally differentiating and solving for $\partial l_{j'}^u$ yields:

$$\partial l_{j'}^u=-\frac{\partial G/\partial A_j^s}{-\left[(1-\beta)l_{j'}^{u-(1-\beta)}+\partial G/\partial l_{j'}^u\right]}\partial A_j^s$$

We calculate $\frac{\partial G}{\partial A_j^s}$

$$\frac{\partial G}{\partial A_j^s}=k_{j'}\left[\left(\frac{Y_{j'}}{N_{j'}}-\alpha A_j^s l_j^{s(\alpha-1)}\right)\frac{\partial M_j^s}{\partial A_j^s}\right]$$

where $k_{j'}=\frac{\delta}{N_{j'}\beta A_{j'}^u}e^{\frac{\lambda(1-\beta)}{\lambda+\beta(1-\lambda)}}$ and $N_{j'}=N_j^s-M_j^s+N_j^u$

Since $N_{j'}$ decreases more than $l_j^{s(\alpha-1)}$, $\frac{Y_{j'}}{N_{j'}}$ increases more than $\alpha A_j^s l_j^{s(\alpha-1)}$. So $\frac{Y_{j'}}{N_{j'}}>\alpha A_j^s l_{j'}^{s(\alpha-1)}$ and $\frac{\partial G}{\partial A_j^s}>0$. Where $\frac{\partial l_{j'}^u}{\partial A_j^s}<0$ (proof of proposition 3).