FINANCIAL GUARANTEE AS AN INNOVATION TOOL IN ISLAMIC PROJECT FINANCE

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Abstract

This paper proposes a model to study the arrangement of Islamic project finance with the participation of the government as a provider of loan guarantees. The owner-shareholders (musharaka certificate holders) initiate a project and raise funds by issuing Islamic profit-loss sharing mudaraba certificates. The government intervenes in providing financial guarantees in order to enhance the creditworthiness and increase the mudaraba capital capacity of the project. Our work raises several policy implications related to the structuring of Islamic project finance and the participation of both government and multilateral public agencies such as the Islamic Development Bank. It provides a unifying framework for the improvement of access to funds for Islamic projects and gives a rationale for government intervention in the arrangement of these projects.

ملخص

تقترح هذه الورقة نموذجًا لدراسة الإعداد لتمويل مشروع إسلامي بمشاركة الحكومة التي ستقوم بدور الضامن للقرض. وسوف يطلق المالك والمساهمون (حاملو شهادة المشاركة) مشروعًا ويجمعون التمويل المخصص له عن طريق إصدار شهادات مضاربة إسلامية يتم فيها اقتسام الربح والخسارة وسوف تتدخل الحكومة بتوفير الضمانات المالية من أجل تعزيز الملاءة المالية وزيادة القدرة الرأسمالية للمضاربة الخاصة بالمشروع. إن عملنا يثير العديد من مضامين السياسة المتعلقة بأسلوب تنظيم تمويل المشروعات الإسلامية ومشاركة كل من الحكومة وهيئات عامة متعددة الأطراف مثل بنك التنمية الإسلامي, كما إنه يعطي إطارًا موحدًا لكيفية تحسين إمكانية الحصول علي تمويل للمشروعات الإسلامية ومنطق تدخل الحكومة في الإعداد لمثل هذه المشروعات.

I. Introduction:

The foundations of Islamic finance are described in the Muslim Holy book and the traditions of the Prophet Muhammad (Peace be upon him). Under Islamic law (Sharia), making money from money, such as charging interest, is usury and therefore not permitted. Wealth should be generated only through legitimate trade and investment in assets. All forms of interests are forbidden. Moreover, investment in companies involved with illicit activities or goods such as alcohol, gambling, tobacco and pornography is strictly off limits. Islamic financing contracts should be designed to avoid risk-free return and money from money (riba), uncertainty (gharar) and gambling (maysir) (Ebrahim (1999), Esty (2004) and CFA Magazine (2005)). Islamic financial instruments have to be carefully structured so that the exchange involves goods for money or partnership shares for money over time. The crux of the Islamic financial system is based on risk-sharing not on risk-trading. Islamic banking is growing at a rate of 15% to 20% per year (Esty (2004), KPMG Tax Monitor (2005)). Some western countries are changing their tax codes to accommodate Islamic finance (KPMG Tax Monitor (2005), BBC News (2004)).[1] The singularities of Islamic finance require the conventional finance approach to be redesigned in order to satisfy the criteria of Islamic financing. These restrictions impose financing constraints on entrepreneurs seeking funds to undertake large scale investments based on Islamic principles.

Indeed, project finance is an arrangement in which a sponsor creates a new project company and looks to the project future cash flows as the main source of repayment to lenders. It allows better risk sharing because lenders have to evaluate and audit only the project assets without assessing both project and sponsor assets as would be the case with other financing vehicles. Project finance is an increasingly important method of financing large-scale capital-intensive projects, such as power plants, oil pipelines, automated steel mills, roads, ports, tunnels etc. The Government may want to provide financial guarantees for the promotion of projects for poverty alleviation, and increasing investment in high-return priority areas to accelerate growth and economic transformation.

Finally, the demand for financing often exceeds the supply capacity of the project sponsor and of local capital markets (Farrell, 2003). According to Esty (2004), project-financed investments have grown at a compound rate of almost 20 percent over the past 10 years and globally firms financed 234 billion dollar US of capital expenditures using project finance in 2004, up from 172 billion dollar US in 2003. Kleimeier and Megginson (2001) compare empirically portfolios of project finance loans to comparable samples of non-project finance loans. They find that project finance loans have longer maturities and are more likely to have third party guarantees. Moreover, projects funded with project finance loans are highly leveraged with an average loan to project value ratio of 67 percent. Ebrahim (1999) establishes a comparative study between Islamic and conventional project finance. Khan (2002) analyses cases involving Islamic instruments in financing Build Operate and Transfer (BOT) Projects.

This paper studies the dynamics between shareholders (musharaka certificate holders), entrepreneurs (mudaraba deposit holders) and the government in the arrangement of Islamic project financed investments. The shareholders (musharakah) being the entrepreneur initiates a project and seeks outside funds to finance it. The shareholders finance part of the project

[1] According to August 2005 KPMG Tax Monitor: "The last decade has seen tremendous growth in the Islamic financial system. More than 240 financial institutions – in more than 48 countries – now practice some form of Islamic finance. Moreover, Islamic banking assets worldwide are estimated at over US$200 billion with an average annual growth rate of 15 percent in recent years. The UK have published changes to tax rules in an effort to accommodate Islamic financial products. Should Ireland do the same?" For BBC News on Islamic banking go to http://news.bbc.co.uk/2/hi/business/3676138.stm.

investment with own capital and the remaining amount is provided by entrepreneurs in a profit-loss sharing mudarabah contract. In this financing agreement, entrepreneurs share the project after tax net-income with shareholders if the project is successful, but lose their investment in case of default by the project. To reduce the default risk and enhance the creditworthiness of the project, the government intervenes by providing partial financial mudaraba deposit guarantees. If the project turns out to be successful, the government gains tax revenues, entrepreneurs and shareholders share the after tax net-income using a preset sharing formula. We argue that by appropriate risk sharing and/or government financial mudaraba deposit guarantee, project sponsors in the Islamic world can enhance the creditworthiness and increase the attractiveness of their project and attract more mudaraba deposits.

Project finance involves huge amounts of financing and is highly levered, one way for lenders to hedge credit risk is to require financial guarantees for the loans they make. A financial guarantee is a promise from a third party to make good on payments to the fund provider when the borrower defaults. To have access to more funds and at lower costs, firms resort to financial guarantees to improve their credit rating and debt capacity (The World Bank; 1995, 2002). Government agencies and international organizations such as the World Bank, and Export Credit Agencies are some of the main providers of financial guarantees, especially to back large-scale projects financing (Dailami and Leipziger (1998), Ehrhardt and Irwin (2004)).[2]

We show that there is a trade-off between the profit sharing and the percentage of mudaraba to be guaranteed. For given levels of mudaraba financing, the shareholders have to decide ex-ante whether to give up more profit or seek government guarantee. On one hand, if there is no flexibility over the investment amount, increasing the share of the profit to entrepreneurs (mudaraba depositors) will result in decreasing shareholders (musharaka) equityholders' net-wealth. On the other hand, if the firm has flexibility over the amount it can invest, increasing either the share of the profit to entrepreneurs (mudaraba) or the guarantee percentage of the mudaraba deposits will increase the financing capacity of the firm. This increase in the financing capacity consequently induces more investment, thus more taxable income is available, which is beneficial for the government and entrepreneurs (mudaraba). Nonetheless, contrary to what we would expect, increasing the portion of the profit share to entrepreneurs (mudaraba) does not alter the net wealth to the shareholder as long as he has the flexibility over the amount to be invested. Both shareholders (musharaka) and the mudaraba certificate holders share risk and losses arising from genuine causes. The third party guarantee by the government, however, covers only the principle amount of the mudaraba capital from losses arising from both business failure, and negligence and improper practice of the owner-shareholders (musharaka certificate holders).

The remainder of the paper is as follows. Section II reviews the background of Islamic finance. Section III describes the structure of Islamic project financing. Section IV presents the model and analyses the payoffs of the participants. Section V presents and analyses the numerical simulations results. Section VI concludes.

[2] In the fiscal year 2005 (as of September 2005), Export-Import Bank the export credit agency of the United States supported by project finance $405 million dollar US in Qatar Liquified Gas Co. II, $230 million dollar US in Egypt Basic Industries Corp. (EBIC) and 263 million dollar US in Q-Chem II. (See Export-Import Bank of the United States website: http://www.exim.gov/products/guarantee/pfauth.html). On September 13, 2005, the Multilateral Investment Guarantee Agency, MIGA, a private sector branch of the World Bank Group, said it issued $1.2 billion in investment guarantees (insurance) for 33 new projects (62 contracts) in developing countries during the fiscal year ending June 30, 2005. (See MIGA website: http://www.miga.org/).

II. Background on Islamic Finance:

Islamic financing contracts should be designed to avoid risk-free return and money from money ((riba), uncertainty (gharar) and gambling (maysir). They have to be carefully structured so that the exchange involves goods for money or partnership shares for money over time. The Islamic financial model works on the basis of risk sharing. The main financial instruments used in Islamic finance are:

- Qardh Hasan is a risk-free asset with a nominal rate equal to the inflation rate. Therefore the real rate of return is zero.
- Murabahah (cost plus financing and deferred installment sale) is a form of credit facility which enables customers to purchase equipments/goods without having to take out an interest bearing loan. The bank buys an item and then sells it on to the customer on a deferred basis.
- Ijara (operating lease) is a leasing agreement whereby the bank buys an equipment or productive asset for a customer and then leases it back over a specific period. The client avoids initial capital outlay. In some cases, the customer is able to buy the item at the end of the contract.
- Mudarabah is a profit-and-loss sharing Islamic income or revenue bond contract. It offers specialist investment in which the project owner(s) and the investor share any profits. It does not guarantee any fixed rate of return (ribawi), instead, the investor receives a share of the profit or bears the losses generated by the business venture, and the principal is paid (in real terms) at the termination of the contract.
- Musharakah is an investment partnership in which profit sharing terms are agreed in advance, and losses are pegged to the amount invested. It is an equity participation or stock ownership contract.

Proponents of Islamic finance have repeatedly argued for its adoption chiefly because it can promote higher real investment and growth rates by encouraging risk and return sharing. However, Islamic financial institutions have so far focused mainly on debt financing rather than equity financing precisely because of concerns over risk. Equity financing may be represented by *Mudarabah* and *Musharakah*, the main profit-and-loss sharing instruments of Islamic finance. These instruments can be highly flexible as they allow projects to be partly or fully financed by lenders/shareholders, but they are considered as high risk instruments because of: (1) high uncertainty of return to financial institutions since returns depend on project performance; and (2) in addition to credit risk, they involve business risk.

Accordingly equity financing by Islamic financial institutions requires a high degree of trust in entrepreneurs and implies larger cost to financial institutions in terms of project evaluation, monitoring and supervision. To reduce the impact of risk on equity financing in Islamic financial systems, Islamic economists and *Shariah* boards have allowed use of collateral. But the use of collateral has its own limitations. First, Islamic *Shariah* Law permits the use of collateral as a guarantee against fraud, negligence or misconduct on the part of the entrepreneur; it does not permit compensation of shareholders or fund providers in the case of losses or business failure owing to genuine reasons (e.g. changes in market conditions, natural hazards, etc). Second, reliance on collateral imposes limitations on Islamic finance similar to those encountered by conventional finance, including limited access to finance for new and less established entrepreneurs who lack acceptable collateral as well as incentives for banks to prefer low return projects because they secured by adequate collateral.

In view of the above there is a need for policies and interventions to help both fund providers and entrepreneurs to reduce investment risk in *Mudarabah* and *Musharakah*. Thus the paper addresses an important issue and presents a model that justifies government guarantees of

projects from an economic and social viewpoint. As the paper explains, the government has an incentive to provide project guarantee since project success will mean increased tax revenue. Entrepreneurs will have greater access to finance with government guarantees and shareholders will be willing to finance more projects. Government project guarantee is justified as far as the return exceeds the cost of providing the guarantee. The paper presents mathematical illustrations, simulations and sensitivity analysis to describe interactions between the entrepreneur, lenders and the government.

We provide a model of Islamic project finance that can be used as a useful framework for governments to analyze potential policy options for encouraging and strengthening equity financing by Islamic financial institutions.

III. Structure of the Islamic Project Financing:

We consider an entrepreneur (or sponsor) who wants to undertake a new investment project. Several owners can co-exist in this new venture; however, we use the term entrepreneur to designate the 'representative shareholder'. The project will be built as a stand-alone firm, meaning that the project is an independent and separate entity. The project requires an initial investment I. The entrepreneur has limited liability; his only commitment in the project is its capital infusion S. Entrepreneurs provide the remaining amount $D = I - S$. However, the shareholder is facing the constraint that the project can be undertaken only if the investment level I is superior to the minimum required investment $\bar{I}$, $I \geq \bar{I}$. In this kind of Islamic financing, the shareholders (musharakah) have to share the profit with entrepreneurs (mudarabah deposits) if the project is successful. In stressful situations, the priority goes to entrepreneurs (mudaraba depositors). Entrepreneurs receive a fraction of the firm's after tax net profit plus their principal amount (the principal is paid in real terms not in nominal) if the project is successful. Hence, the shareholders keep the residual value of the project.

If the project is undertaken, the government benefits from it because of the corporate taxes. Therefore, it is in the interest of the government to encourage the shareholders and entrepreneurs to go ahead with the project. But, in the Islamic context, since the conventional capital market channel cannot always be used, it can be hard for the shareholders to raise enough money in order to finance the project. To overcome that, the government will intervene by providing partial mudaraba deposit guarantees for the project loan. There are many ways for the government to do so. One way will be through multilateral public agencies such as the Islamic Development Bank. The government will act so as far as the net social benefit outweighs the mudaraba certificate guarantee cost. The maximum government guarantee coverage should be obtained when the maximum benefit is equal to the marginal cost of guaranteeing the mudaraba certificate. We use as proxy for the social benefit, the amount of tax the government receives from the project.

Figure 1 describes the relationships between the shareholders, mudaraba certificate holders and the government. The project is financed by the entrepreneur and shareholders. The government enhances the creditworthiness of the project by providing mudaraba certificate guarantees. If the total financing exceeds the minimum capital requirement, the project goes ahead, otherwise the project is abandoned. When the project goes ahead, the investment is made and the project yields cash revenues. The cash revenues are then redistributed among the players, that is, the government collects taxes, mudaraba certificate holders receive their repayment which is composed of the principal and share of the profit, and shareholders are left with the residual value. Therefore, the success of the project benefits all parties involved which is consistent with the risk sharing philosophy behind Islamic finance.

Based on this Islamic project finance structure, next we present the model and analyze the payoffs to the participants.

IV. The Model:

As described in the previous section, we consider a shareholder (or sponsor) who wants to undertake a new investment project. The project will be built as a stand-alone firm, meaning that the project is an independent and separate entity. The project requires an initial investment I and its cash flows are characterized by the following technology $V = \widetilde{\theta} v(I)$ where $\widetilde{\theta}$ is the random output price which captures the stochastic nature of the cash flows and *v(I)* is a twice differentiable concave function of investment. We assume the project cash flows to have the following risk adjusted stochastic process:

$$dV_t(I) = (\mu - \lambda\sigma)V_t(I)dt + \sigma V_t(I)dW_t, \qquad (1)$$

where μ, σ and λ are respectively the instantaneous return, volatility and market price of risk of the project's asset returns. The random variable *W* is a standard Wiener process.

In Islamic finance, there is no risk-free rate, but instead any money deposit in a bank account and not used for investment will grow at the inflation rate π. Under the risk neutral valuation *à la* Harrison and Kreps (1979) and Merton (1974), the parameter λ is the market price of risk for the project. In the special case of a tradable security, $\mu - \lambda\sigma$ is equal to the inflation rate π in equation (1).[3]

As mentioned previously, the project requires an initial endogenous investment level *I*, but there is a minimum investment requirement in the sense that the shareholder (musharaka certificate holders) has to raise at least $\bar{I}$ before the project can be undertaken. The project sponsor (or shareholders) has limited financing capacity, therefore he can only finance an amount *S* of the total investment. He then needs outside financing in the form of profit and loss sharing mudaraba certificates. In other words, the entrepreneur can infuse a capital level *S* and has to borrow *I* - *S* to finance the new project. We assume a simple capital structure for the project, consisting of single mudaraba (profit-loss sharing certificates) and musharaka (equity) contracts. The maturity of the project is *T*. We assume the existence of corporate taxes. The project is owned by the entrepreneur and the project cash flows are used to pay the project mudaraba cerfiticate holders. In this financing framework, often referred to as non- or limited recourse financing, lenders depend on the performance of the project itself for repayment rather than the credit of the sponsor shareholder. The only commitment of the sponsor shareholder is its capital contribution.

In the following subsections, we present respectively, the cost/benefit to the government, the payoff from the participation constraint of mudaraba certificate holders, and the payoff to the owner-shareholders.

IV.I. Guarantee Cost and Tax Revenue to the Government:

The government acts as a stakeholder in the project by providing partial mudaraba certificate guarantees. The incentive for the government is to gain positive social benefits if the project is realized. The government insures a fraction ω of the mudaraba principal, with $0 \leq \omega \leq 1$.

[3] See Hull (2005) for a general discussion on the use of risk neutral valuation in valuing real options and the estimation of λ. Schwartz and Moon (2000) propose a risk neutral pricing model of internet companies.

From the structural approach of Merton (1974, 1977), the total loan guarantee cost to the government is equivalent to a put option and is given as follows

$$G = E\left[e^{-\pi T}\max(\omega D - V_T(I),0)\right] = \omega e^{-\pi T} DN(-z+\sigma\sqrt{T}) - V_0(I)e^{(\mu-\lambda\sigma-\pi)T}N(-z), \quad (2)$$

where $z = \dfrac{\ln(V_0(I)/\omega D) + (\mu - \lambda\sigma + 0.5\sigma^2)T}{\sigma\sqrt{T}}$ and $N(.)$ is the cumulative normal distribution function.

By guaranteeing the mudaraba certificate, the government expects to raise more taxes to fulfill its social agenda. If the project is realized, the expected tax revenue to the government will be

$$Taxes = E\left[e^{-\pi T}\tau_c \max(V_T(I) - I,0)\right] = \tau_c V_0(I)e^{(\mu-\lambda\sigma-\pi)T}N(x) - e^{-\pi T}\tau_c IN(x-\sigma\sqrt{T}), \quad (3)$$

where $x = \dfrac{\ln(V_0(I)/I) + (\mu - \lambda\sigma + 0.5\sigma^2)T}{\sigma\sqrt{T}}$ and τc is the corporate tax rate.

The net gain to the government is the difference between the tax gain and the cost of the guarantee, that is, *Taxes - G*. Therefore, the participation constraint of the government is such that the net gain to him is positive, *Taxes - G* ≥ 0.

IV.II. Mudaraba Certificate Holders' (Lenders') Participation Constraint and Project Mudaraba Capacity

As we mentioned above, the project is financed with musharaka (equity) and mudaraba (profit-loss sharing certificate). The entrepreneur contributes S and the rest I - S is financed by outside investors in the form of profit-sharing Islamic mudaraba certificates. Outside investors share the profit with the owner-shareholders. At the maturity T, if the project is successful, mudaraba certificate holders receive their principal otherwise they loose money since they won't be able to recover the full amount of the principal.

Optimal mudaraba without financial guarantee

In absence of guarantees, the payments to mudaraba certificate holders at date *T* are:

$$D_T = \begin{cases} D + q(1-\tau_c)(V_T(I) - I) & \text{if} \quad V_T(I) > I \\ D & \text{if } D < V_T(I) \leq I \\ V_T(I) & \text{if} \quad V_T(I) \leq D \end{cases}, \quad (4)$$

where *q* is the profit sharing allocation parameter.

Equation (4) states that if the cash flow generated by the project exceeds the initial investment amount, mudaraba certificate holders receive their principal amount *D* and share of the after tax net-income (percentage *q*). Otherwise, they receive only the principal amount when the project end value is greater than the principal amount and the project savage value in case the project market value is inferior to the principal amount. Therefore, the equilibrium

value of the mudaraba certificate holders today is the expected present value of the terminal payments to mudaraba cerfificate holders:

$$\begin{aligned} D &= E\left[e^{-\pi T} q(1-\tau_c)\max(V_T(I)-I,0)\right] + E\left[e^{-\pi T} D \times 1_{\{V_T(I)>D\}}\right] + E\left[e^{-\pi T} V_T(I) \times 1_{\{V_T(I)\le D\}}\right] \\ &= q(1-\tau_c)\left[V_0(I)e^{(\mu-\lambda\sigma-\pi)T}N(x) - e^{-\pi T} IN(x-\sigma\sqrt{T})\right] \\ &\quad + e^{-\pi T} DN(y-\sigma\sqrt{T}) + V_0(I)e^{(\mu-\lambda\sigma-\pi)T}N(-y) \end{aligned}, \quad (5a)$$

or

$$\begin{aligned} D &= V_0(I)e^{(\mu-\lambda\sigma-\pi)T}\left[q(1-\tau_c)N(x)+N(-y)\right] \\ &\quad - q(1-\tau_c)e^{-\pi T} IN(x-\sigma\sqrt{T}) + e^{-\pi T} DN(y-\sigma\sqrt{T}) \end{aligned}, \quad (5b)$$

where $x = \dfrac{\ln(V_0(I)/I) + (\mu - \lambda\sigma + 0.5\sigma^2)T}{\sigma\sqrt{T}}$ and $y = \dfrac{\ln(V_0(I)/D) + (\mu - \lambda\sigma + 0.5\sigma^2)T}{\sigma\sqrt{T}}$.

After few algebraic manipulations, we can show that the maximum total amount of mudaraba capital without guarantee will be

$$D = \frac{V_0(I)e^{(\mu-\lambda\sigma-\pi)T}\left[q(1-\tau_c)N(x)+N(-y)\right] - q(1-\tau_c)e^{-\pi T} IN(x-\sigma\sqrt{T})}{1-e^{-\pi T}N(y-\sigma\sqrt{T})}. \quad (6)$$

Given the level of sponsor contribution *S* or total investment *I* to be made, equation (6) allows us to solve for the maximum level of mudaraba capital outside investors will be willing to extend to the project. Equation (6) is a fixed point problem as *x* and *y* contains *D*, we solve for the level of maximum mudaraba capital using Matlab optimization toolbox. We denote by D_{NG} the optimal mudaraba capital level without financial guarantee. If the entrepreneur were to change the investment level, the mudraba capital amount will also change.

Optimal mudaraba capital with financial guarantee

Instead, in the presence of financial guarantee, the value of the capital-guaranteed mudaraba capital will be equal to the value of the mudaraba without guarantee plus the value of the guarantee, i.e., $D = D_{NG} + G$. Using equations (2) and (4), we obtain

$$\begin{aligned} D &= E[e^{-\pi T} D_T] + G \\ &= V_0(I)e^{(\mu-\lambda\sigma-\pi)T}\left[q(1-\tau_c)N(x)+N(-y)-N(-z)\right] - q(1-\tau_c)e^{-\pi T} IN(x-\sigma\sqrt{T}), \\ &\quad + e^{-\pi T} D\left[N(y-\sigma\sqrt{T}) + \omega N(-z+\sigma\sqrt{T})\right] \end{aligned} \quad (7)$$

which yields,

$$D = \frac{V_0(I)e^{(\mu-\lambda\sigma-\pi)T}\left[q(1-\tau_c)N(x)+N(-y)-N(-z)\right] - q(1-\tau_c)e^{-\pi T} IN(x-\sigma\sqrt{T})}{1-e^{-\pi T}N(y-\sigma\sqrt{T}) - e^{-\pi T}\omega N(-z+\sigma\sqrt{T})}. \quad (8)$$

Given the level of sponsor contribution *S* or investment *I*, equation (8) is a fixed point problem as *x, y* and *z* contains *D*. We use Matlab optimization toolbox to solve for *D*. We denote by D_G the optimal maximum amount of capital-guaranteed mudaraba. In conventional

modern finance, in the presence of credit insurance, debtholders will either decrease the credit premium they require to lend to the firm or they will increase their funding to the firm because the credit insurance enhances the project creditworthiness and increases its borrowing capacity. In Islamic banking, there is no return premium; the credit improvement will translate through changes in the profit-loss sharing parameter q and/or the mudaraba capital capacity D_G.

Figure 2 plots the payoffs to mudaraba certificate holders as a function of the value of the project. In the first graph (top graph), the payoff of the non-insured mudaraba is plotted. As illustrated, in bad performance states, mudaraba certificate holders loose their initial investment, but in good states, they recover the principal of their investment plus share of the profit. In the second graph (bottom graph), the payoff to the insured mudaraba certificate holders is plotted. As illustrated by the graph, the government mudaraba guarantee reduces the downside losses to mudaraba certificate holders. In good performance states, mudaraba certificate holders still receive their principal and share of the profit.

IV.III. Net-wealth to the Shareholders-Owners

The project receives D mudaraba capital at the beginning and promises to pay a fraction q of the after tax net-income plus the principal mudaraba amount D at date T. Hence, the owner-shareholder keeps the residual value of the project. Therefore, the residual to the owner-shareholder in the new venture at time T is as follows:

$$R_T = \begin{cases} V_T(I) - D - q(1-\tau_c)(V_T(I) - I) - \tau_c(V_T(I) - I) & \text{if} \quad V_T(I) > I \\ V_T(I) - D & \text{if } D < V_T(I) \le I \\ 0 & \text{if} \quad V_T(I) \le D \end{cases}, \tag{9}$$

Equation (9) states that the residual value to the owner-shareholders is the difference between the project value and the payments to mudaraba capital providers and government when the project value is greater than the initial investment amount. Otherwise, it is either the difference between the firm value and the principal payment or zero whichever is greater. The net-wealth of the owner-shareholder in the new venture is the expected present value of its terminal payoff given in equation (9) minus his initial capital contribution:

$$R = E\left[e^{-\pi T} R_T\right] - S, \tag{10a}$$

or

$$\begin{aligned} R = {} & V_0(I) e^{(\mu - \lambda\sigma - \pi)T} N(y) - e^{-\pi T} D N(y - \sigma\sqrt{T}) \\ & - (q(1-\tau_c) + \tau_c)\left[V_0(I) e^{(\mu - \lambda\sigma - \pi)T} N(x) - e^{-\pi T} I N(x - \sigma\sqrt{T})\right] - (I - D) \end{aligned}. \tag{10b}$$

From equation (10), the owner-shareholder is the residual claimant. The residual claim represents the value of the firm minus the expected value to mudaraba capital providers and tax payment to the government. The guarantee cost is a deadweight cost for the economy; however, at the government level, it is compensated by the tax gain.

Figure 3 plots the payoff to the owner-entrepreneur as function of the project value. As illustrated by the graph, if the project turns out to be unsuccessful, the owner-entrepreneur receives nothing and looses only his initial capital investment since he has limited liabilities. But if the project succeeds the owner-entrepreneur receives the residual value net of his initial capital investment.

As stated previously, the owner-shareholder has the flexibility over the choice of the level of investment I up to the maximum available funds. He can decide whether to go ahead with the project or not. Therefore, he will undertake the project only if the expected net gains to him are positive. All else being equal, the optimal level of I from the viewpoint of the shareholder-owner will be the one maximizing his net wealth R.

Next we perform several numerical analyses with the optimization toolbox of Matlab using defined sets of parameters value to gauge the impact of the sharing rule and the government guarantee on the optimal policies.

V. Simulations Results and Policy Implications

Several numerical analyses will be conducted to gauge for the effects of the changes in key policy parameter values by keeping others constant. Several parameters values such as the volatility level, the maturity of the project, etc. are set based on empirical evidence from Ebrahim (1999), Kleimeier and Megginson (2001), Khan (2002) and Esty (2004).

V.1. The Role of Financial Guarantee in the Absence of Investment Flexibility

We define the absence of investment flexibility by the fact that the project requires an exact investment amount and there is no room for the entrepreneur to change the level of the investment, in other words, either he raises the required amount or the project is not undertaken. The numerical simulation results are presented in Figure 4.

In Figure 4, without loss of generality, the level of investment I is normalized to 100. We analyze the sensibilities of the optimal policies (the government guarantee percentage, the cost of the guarantee to the government, the shareholders's equity capital and mudaraba debt capital) to the profit sharing parameter q for different levels of capital contribution from the shareholders measured as the percentage of sponsor own capital in total investment. From the percentage capital level of the sponsor-owner, we compute the amount of the capital contribution by the owner-shareholder S. The mudaraba amount to be raised is therefore the difference between the investment and the entrepreneur capital, that is, $I - S$. To raise the required mudaraba capital level, the entrepreneur needs financial guarantees as illustrated in Figure 4.

We observe that the government mudaraba guarantee enhances the creditworthiness of the project. However, the firm needs less government guarantees when the profit sharing parameter is high and the sponsor's own capital is big. Therefore, with limited capital, the owner-shareholder (musharaka) trades-off between the fraction of the profit to extend to mudaraba certificate holders and the partial guarantee portion. When the profit sharing parameter increases, the partial guarantee portion decreases. In this non-flexible investment environment, increasing the profit sharing parameter q decreases the net-wealth to the owner-shareholder. Intuitively, since the investment level does not change, the cash flow level is the same, and so is the before tax net-income. As a consequence, the government net-wealth is decreasing with the cost of the guarantee. We should keep in mind that we have assumed no investment flexibility; in the presence of investment flexibility, the outcome could be different.

V.II. The Role of Financial Guarantees with Investment Flexibility

In contrast to the absence of investment flexibility, here we assume that the owner-shareholder (musharaka) can increase or decrease the level of the investment as long as the investment level is above the minimum required investment. For the numerical simulations, we assume the capital contribution from the entrepreneur to be fixed, but the investment level is varying with the mudaraba capacity which is a function of the percentage of guarantee

provided and the sharing rule parameter. The numerical simulations results are plotted in Figure 5.

The graphs show the simultaneous impacts of the government guarantee and the profit-loss sharing parameter on the optimal policies (the maximum mudaraba capacity, the investment level, the shareholder's net-wealth, the guarantee cost, the total tax revenue and the government net-wealth). The owner-shareholder capital contribution S is fixed (without loss of generality, we normalized it to 100) and the total investment is equal to the shareholder's contributed capital S plus the total mudaraba capital D, $I = S + D$. The flexibility over the investment level comes from the amount of mudaraba raised. If mudaraba deposit holders are willing to extend more funds to the project, then the project investment will be high otherwise it will be low.

As illustrated by the graphs, the mudaraba capacity (D) of the project increases when either the government percentage guarantee increases or the profit sharing parameter increases. In this model, the investment level is endogenous since any changes on the mudaraba capital level affect the total investment level. On one hand, we observe that, increasing the profit sharing parameter q does increase the shareholder's net-wealth. Indeed, since the capital contribution of the entrepreneur is fixed, increasing the profit-loss sharing parameter increases the mudaraba capacity, which in turn increases the total investment level inducing more available profit to be shared. Therefore, the increasing size effect dominates the decreasing sharing fraction.

On the other hand, increasing the loan guarantee percentage improves the entrepreneur and government net-wealth. The explanation is as follows. With more guarantees, the firm is able to attract more mudaraba capital, yielding more investment, which increases the taxable income and the residual value to shareholders. For the government, more taxable income generates more tax revenues, and with our set of parameters values used to generate the graphs, the tax gain outweighs the guarantee cost for the government.

V.III. Financial Guarantees and Analysis of the Costs/benefits to the Government

As illustrated by the government net wealth graph in Figure 5, providing more guarantees always improves the net wealth to the shareholder. But for the government, it is not always beneficial. The guarantee portion or the profit-loss sharing parameter increases the mudaraba capacity of the project which increases the total investment since $I = S + D$. Using the government guarantee to enhance the creditworthiness and increase the mudaraba capacity of the project induces a cost for the government but at the same time, if the project is successful, the government gains from tax revenues collected. Therefore, there is an optimal guarantee level to be provided by the government. The optimal guarantee portion is obtained when the marginal gain (marginal tax revenue) is equal to the marginal cost (marginal guarantee cost).

V.IIII. Sensitivity Analyses to the Parameters Values

Sensitivity to the output price

Figure 6 presents the impact of the simultaneous changes in the guarantee percentage and the profit-loss sharing parameter on the optimal policies (the mudaraba capacity, the net-wealth to the entrepreneur and the government net-wealth) for two levels of output price. For lower output prices, it takes more guarantees to insure a positive net-wealth to the shareholder. Or put in other words, it takes more guarantees for the project to go ahead. But for the government, with lower levels of output price, it is more beneficial to lower the percentage of guarantee it provides if the project goes ahead. Unfortunately, that will not be the case if the owner-shareholder does not expect positive residual value. The government gets nothing

when the project is abandoned; therefore, it is in its interest to provide some level of financial guarantee which will create incentive for the owner-shareholder (positive net-wealth) to go ahead with the project. Hence, even though the zero guarantee contribution seems more plausible for the government at first look; it does not guarantee that the project will go ahead.

Sensitivity to the project risk level

Figure 7 illustrates the sensitivity results with respect to the project's asset returns volatility. When the project is too risky, mudaraba certificate holders require more part of the profit for compensation for their investment which decreases the net-wealth of the owner-shareholder. Therefore, more needs for guarantees, which affects negatively the net-wealth of the government. As we have argued previously, it is in the interest of the government to provide financial guarantee support even if it results in destruction of part of its net-wealth, otherwise the project will not go ahead. The government will act so as long as its net-wealth remains positive.

Sensitivity to the Project's Asset Growth Rate, the Market Price of Risk and the Inflation rate

Figures 8, 9 and 10 show the sensitivity results respectively for changes in the project asset returns growth, the market price of risk and the inflation rate. Lower levels of project asset growth rate (or higher levels of either the market price of risk or the inflation rate), all else being equal, imply less future expect value for sharing between the three players (mudaraba, musharaka and government).

Sensitivity to the Project Maturity

Figure 11 shows the sensitivity results with respect to the project maturity. When the maturity of the project increases, the net-wealth to the government and the shareholder (musharaka) decrease, and that because the amount of mudaraba capital extended to the project is lower. The intuition is as follows. Recall, from equations (4) and (7), the face value of the mudaraba capital is the initial amount of mudaraba since no money is made from nothing in Islamic financing, therefore the discounting factor has more impact with longer maturities than shorter maturities. As a consequence, mudaraba certificate holders require more share of the profit. However, increasing the profit sharing parameter will create less incentive for the shareholders to invest.

Sensitivity to the Tax Rate

Figure 12 exhibits the sensitivity results with respect to changes in the tax rate. When the tax rate increases, it creates less incentive for investment, therefore the total net taxable income is lower, which brings less revenue to the government. This phenomenon is more accentuated for higher levels of the profit-loss sharing parameter. Hence, for the government, increasing the tax rate will create the inverse effect, which is the decrease in the government revenue since it does not create incentive for investment.

Sensitivity to the entrepreneur's capital contribution

Figure 13 plots the sensitivity results with respect to level of the owner-shareholder's capital contribution. For these graphs, we take two values for the contribution of the shareholder, $S = 50$ and $S = 100$. We observe that when the value of S doubles, all the optimal policy values are doubled. Therefore, the optimal policies are proportional to the value of S. This is true

because the revenue function used in our simulation is linear to the investment level. With non linear revenue function specification, this proportionality will not hold. However, all the qualitative results obtained above will remain valid.

X Conclusion

This paper proposes a model to study the interactions between the project's owner-sponsor (musharaka certificate holders), its mudaraba certificate holders and the government. The owner-shareholder has an investment opportunity with positive expected net gain. To finance the project, the owner-shareholder contributes with their own capital and seeks outside funds by issuing Islamic profit-loss sharing mudaraba certificates. He also requires government financial guarantees to enhance the creditworthiness and increase the mudaraba capacity of the project. The payoffs to all the participants are derived from their participation and incentive constraints in equilibrium. The participation constraint of mudaraba certificate holders determines the optimal profit-loss sharing parameter and the maximum mudaraba capital. The percentage of guarantee is determined such that the government gains positive net-wealth from the project. The investment decision lies on the owner-shareholder (musharaka certificate holders) and he gets the residual value of the project.

Our work raises several policy implications related to the arrangement of Islamic project finance and the participation of both government and multilateral public agencies such as the Islamic Development Bank. It provides a unifying framework for the improvement of access to funds in the Islamic financing context and gives a rationale for the government intervention in the arrangement of Islamic project finance.

We, however, did not discuss in this paper the possibility of government failure or inefficiency of government project guarantee schemes. While we argue in this paper that it is justifiable from a social welfare viewpoint that the government should provide guarantees from big infra-structure projects, experiences indicate that government interventions in the financial market seldom work; they often create moral hazard problems. We argue that investors will be willing to take the risk if they believe that their capital is at least guaranteed by the government, which is a third part to the contract between an owner-shareholder (musharaka certificate holders) and mudaraba deposit holders, and this third-party guarantee is permissible in Islamic law.

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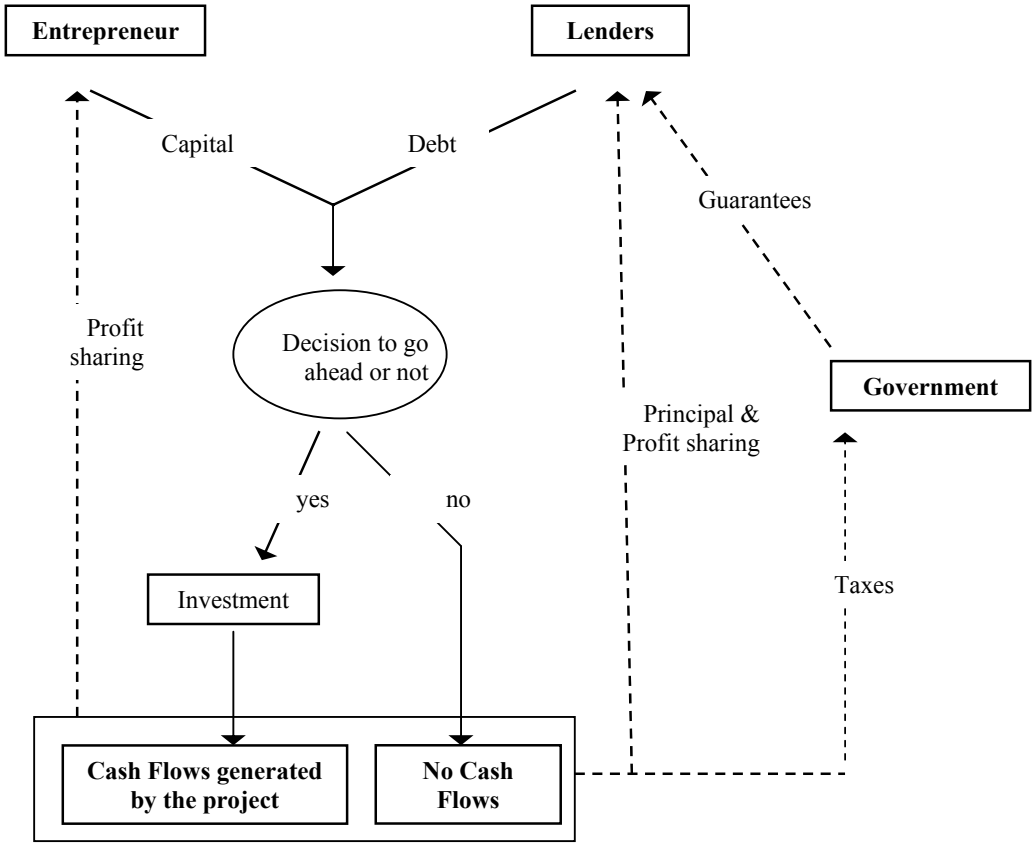
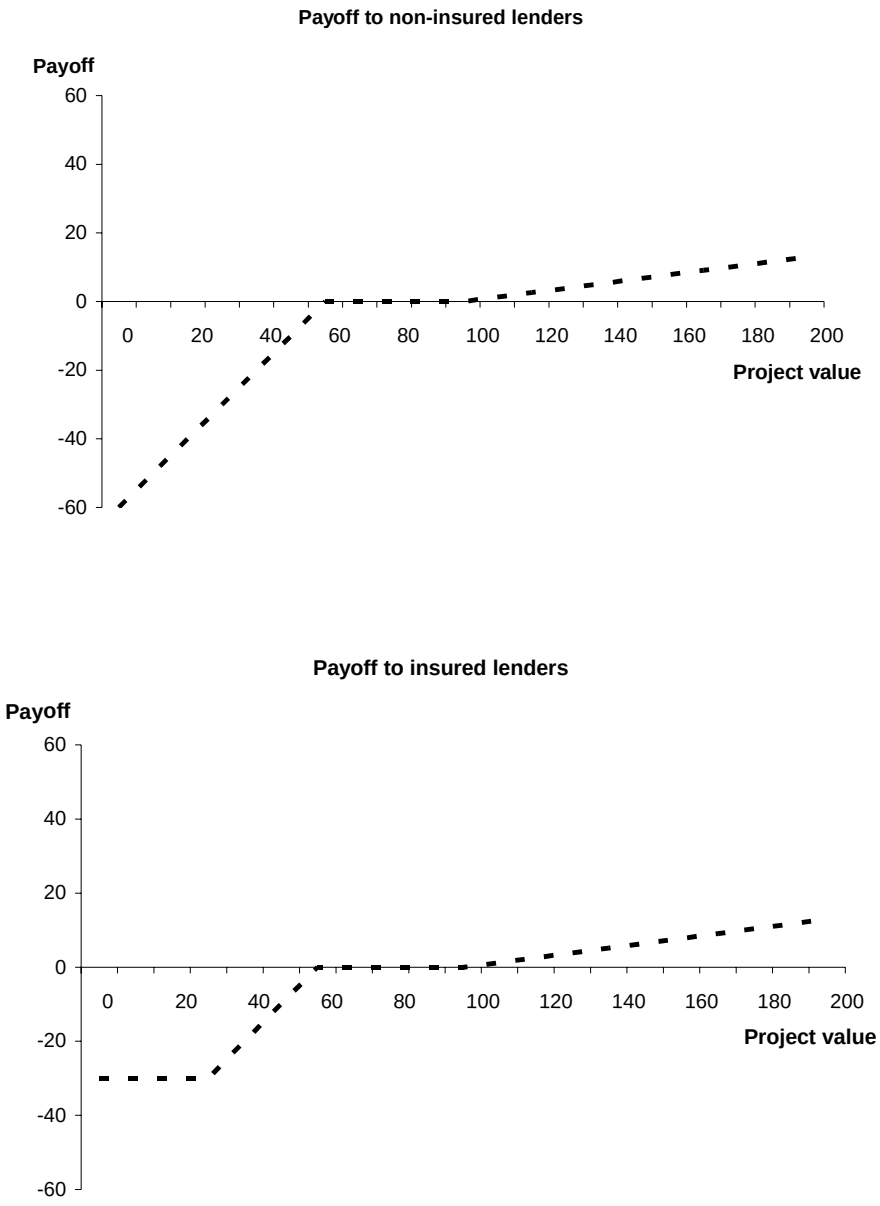
Figure 1: The Players Chart Flow



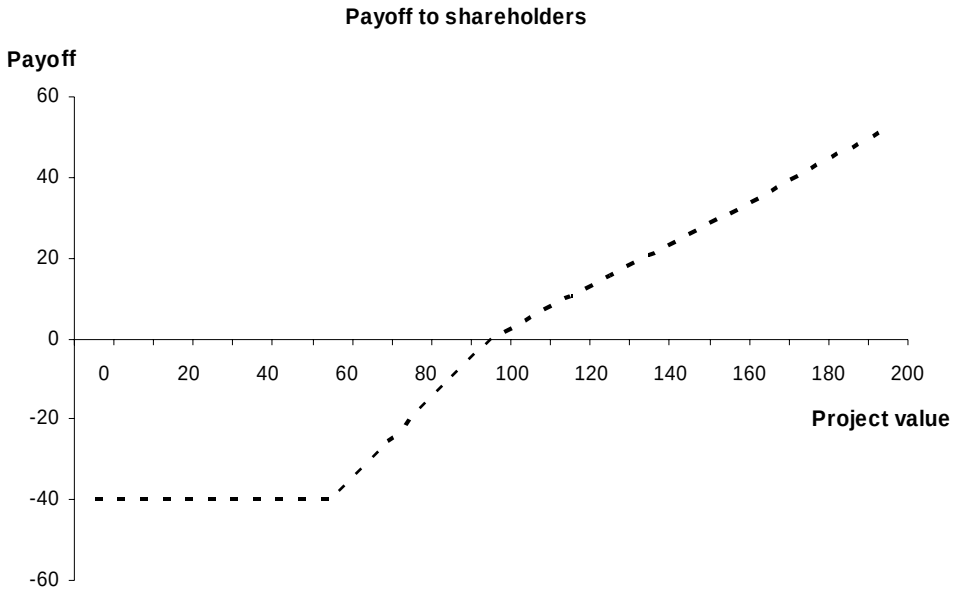
This chart plots the interactions between the entrepreneur, lenders and the government in the arrangement of Islamic project finance. The entrepreneur initiates a project idea and requires outside financing in the form of Islamic profit sharing debt. The government intervene by providing a financial guarantee in order to improve the project creditworthiness. If the investment is made, each stakeholder receives part of the cash flows generated by the project. The chart illustrates the cash inflows to and outflows from the project to the different stakeholders.

Figure 2: Payoff to Lenders (madarabah)



This graph represents the payoff to a holder of an Islamic profit sharing revenue bond. This bond consists of a security which pays a fraction of the project profit and the principal to the holder if the project is successful, otherwise the security holder receive the salvage value of the project in case of default. The top graph represents the payoff to non-insured lenders, while the bottom graph represents the payoff to insured lenders. Comparing the two graphs, we observe that, the financial guarantee limits the downside losses, while the security holder still maintains the option to profit any potential gain. The graphs have been constructed using the following parameter values: $q = 0.20$, $\omega = 0.50$, $\tau_c = 0.35$, $D = 60$, $I = 100$. The payoffs are obtained by subtracting from the final payment to lenders (D_T) the principal D: $D_T - D$.

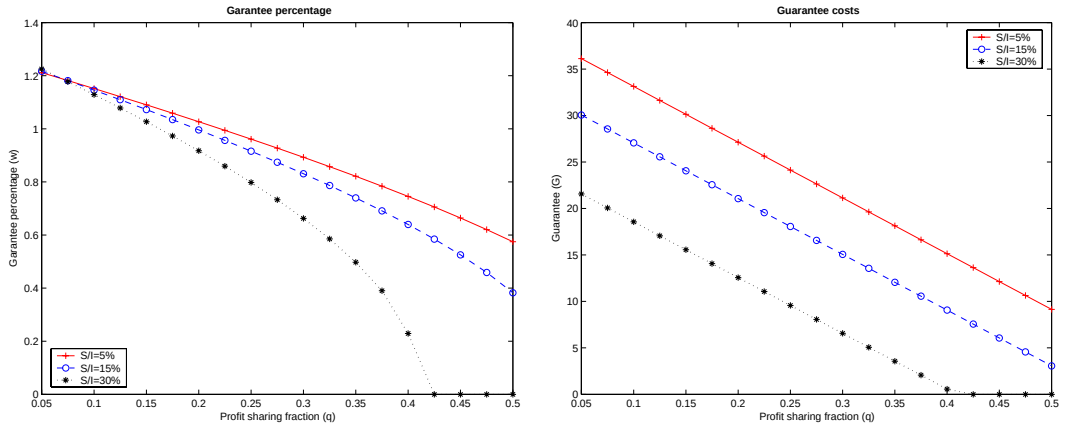
Figure 3: Payoff to the Entrepreneur or Shareholders (musharakah)



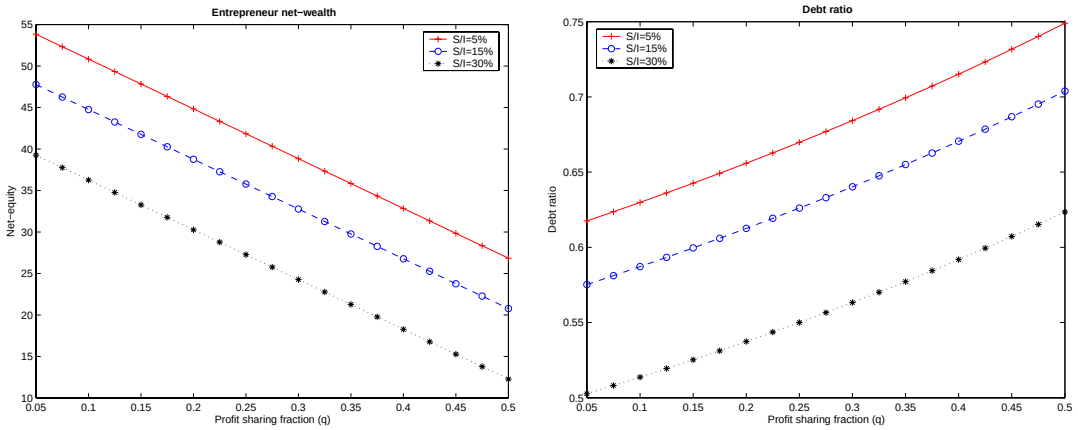
This graph represents the payoff to the entrepreneur or shareholders. The entrepreneur receives the residual value of the project. If the project is successful, the entrepreneur shares the after tax profit with lenders, otherwise he looses his initial investment. Comparing this payoff to the payoff of standard call option, commonly used to model equity, here there is a kink when the payoff crosses the horizontal line. The curve above the horizontal line is the profit sharing part presented in Islamic financing. The graphs have been constructed using the following parameters values: $q = 0.20$, $\omega = 0.50$, $\tau_c = 0.35$, $D = 60$, $I = 100$, $S = I - D = 40$. The payoffs are obtained by subtracting from the final payment to the entrepreneur (R_T) his initial capital investment S: $R_T - S$.

Figure 4: Optimal Policies with Fixed Level of Investment

(a) Percentage of loan guaranteed (ω) **(b) Guarantee costs (G)**

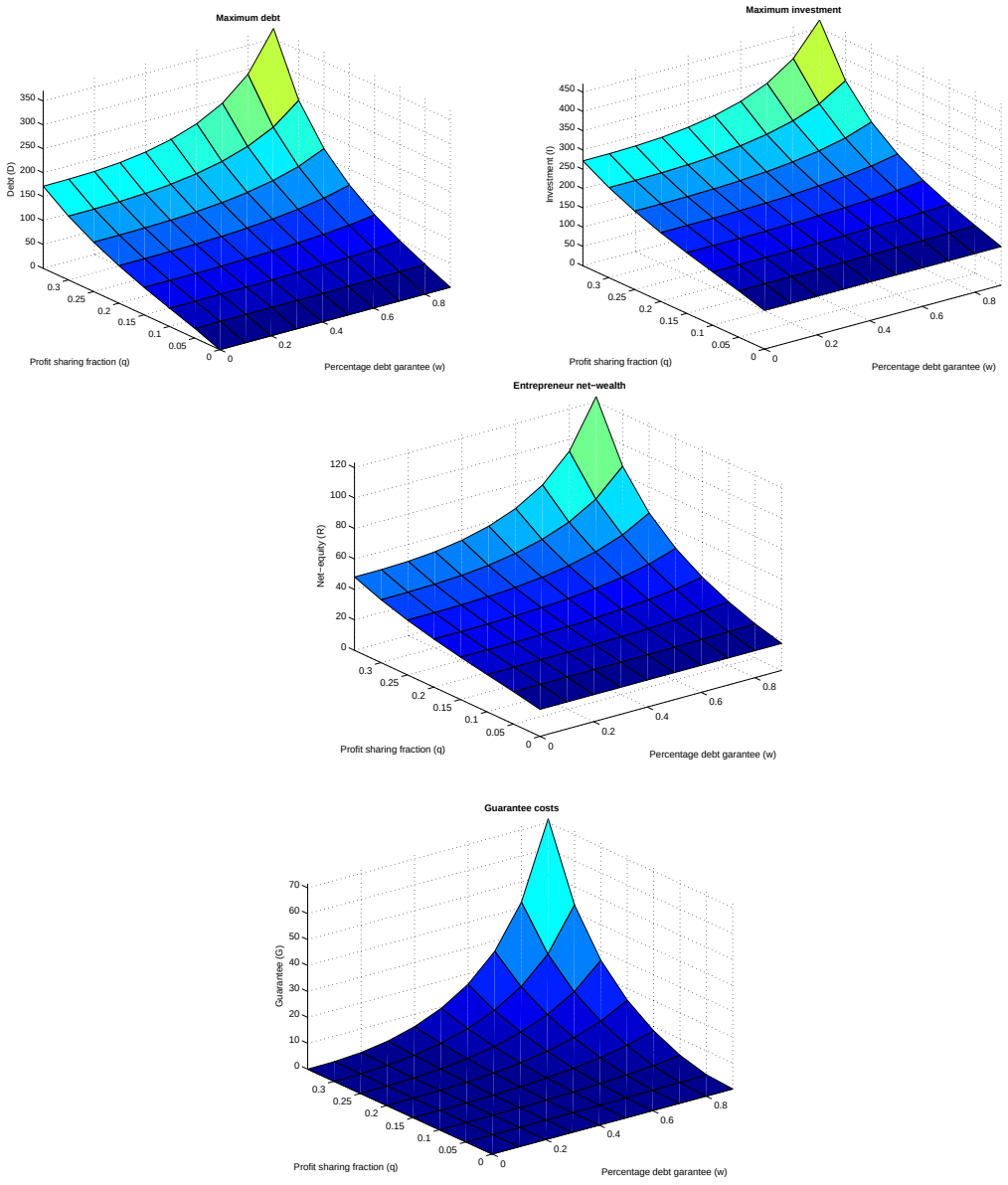


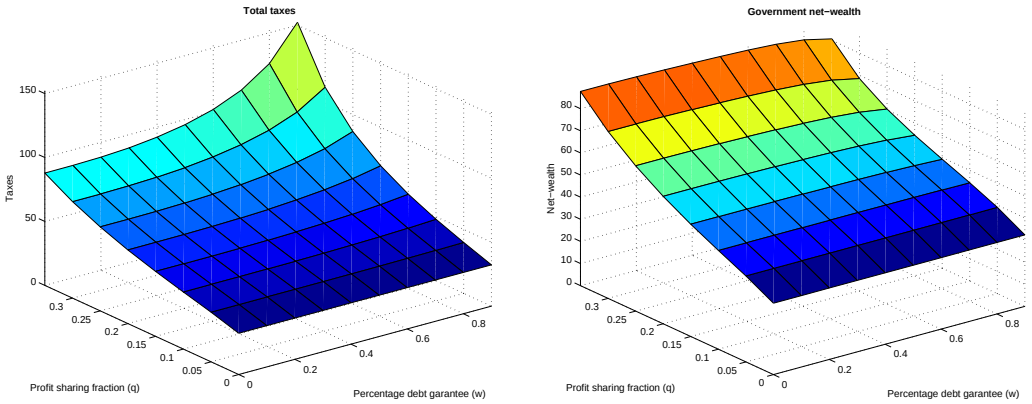
(c) Entrepreneur net-wealth (R) **(d) Debt ratio (D / (R + I))**



This figure is composed of four graphs plotting the optimal policies as a function of the profit sharing fraction (q) for different levels of entrepreneur's capital contribution portion (S / I). The graphs plot respectively, (a) the percentage of loan guarantee by the government (ω), (b) the cost of the financial guarantee to the government, (c) the entrepreneur's net-wealth, and (d) the project debt ratio which is equal to the total debt divided by the sum of the total debt and the total equity. The graphs are computed using the following parameters values: $V_0(I) = 1.5\times I$, $\mu = 0.08$, $\sigma = 0.4$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the total investment level has been normalized to $I = 100$. The capital contribution percentage of the entrepreneur is S/I, which gives the value of S, and the guaranteed debt value is obtained by I - S. The percentage ω is the guarantee percentage need to raise the debt amount I - S.

Figure 5: Optimal Policies as Function of the Sharing Rule and Guarantee Percentage

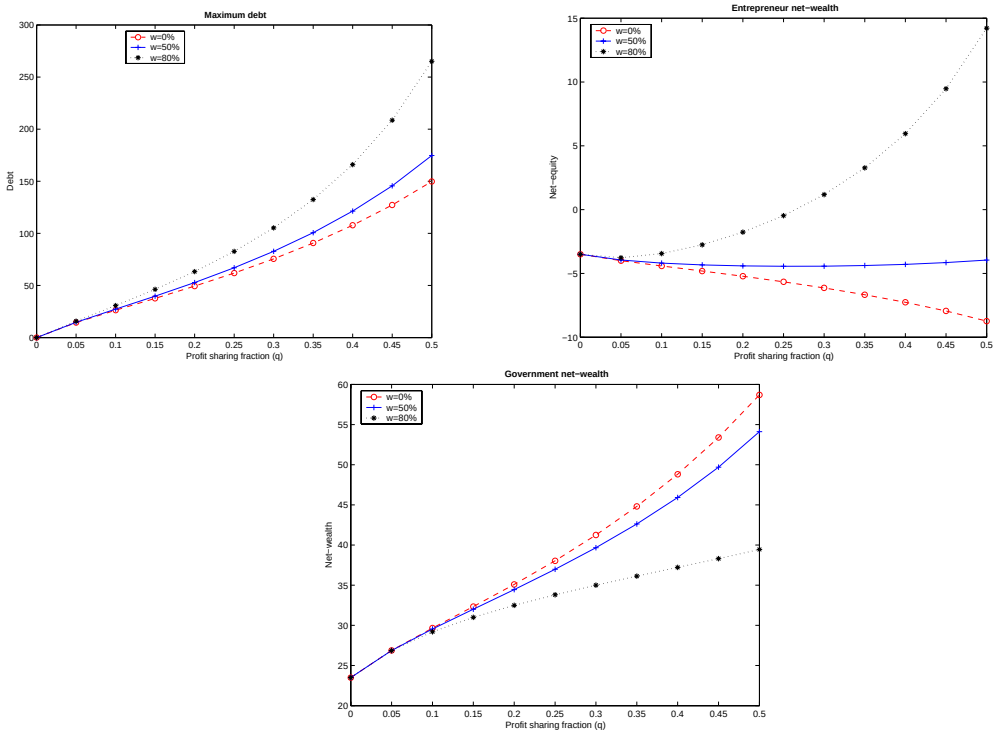




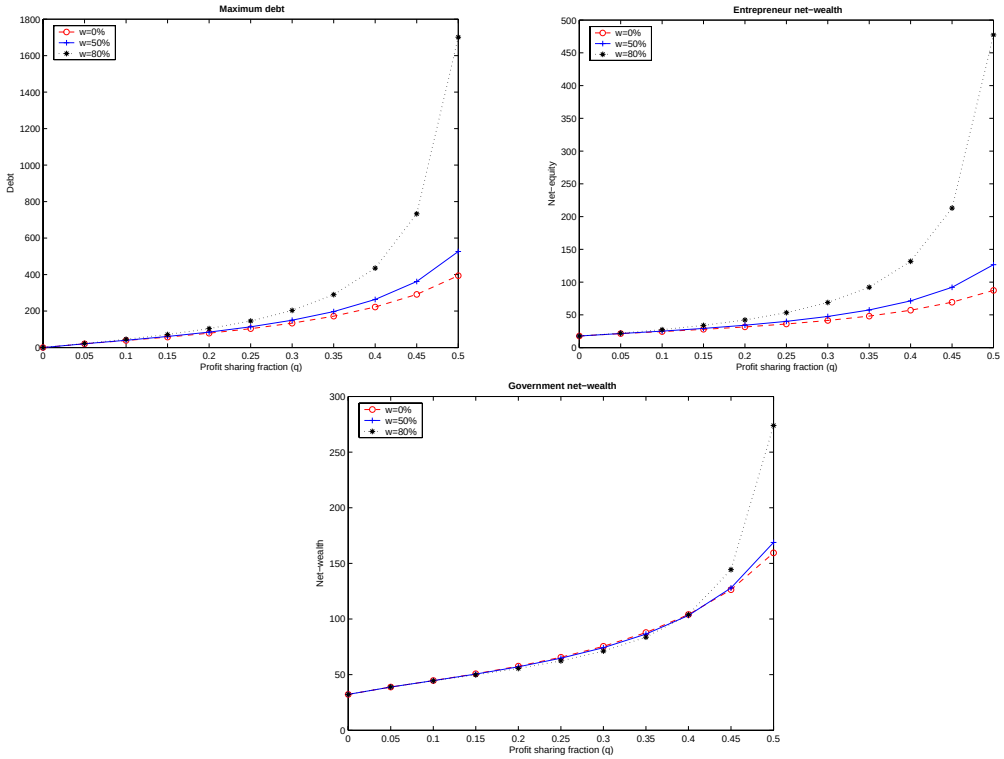
This figure is composed of six graphs. The graphs plot respectively top to bottom and left to right, the maximum debt capacity of the project, the maximum total investment level of project, the net-wealth to the entrepreneur, the cost of the financial guarantee to the government, the total taxes collected by the government, and the net-wealth of the government. The graphs are computed using the following parameters values: $V_0(I) = 1.5\times I$, $\mu = 0.08$, $\sigma = 0.4$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 6: Sensitivity to the Output Price

(a) $V(I) = 1.2 \times I$



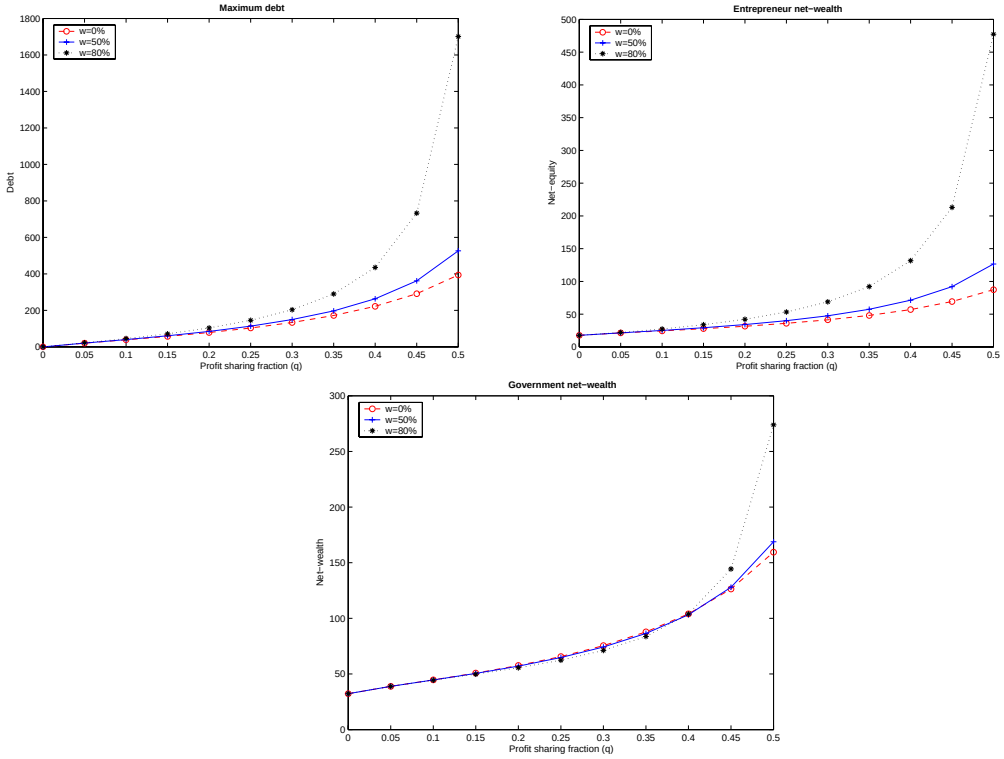
(b) $V(I) = 1.5 \times I$



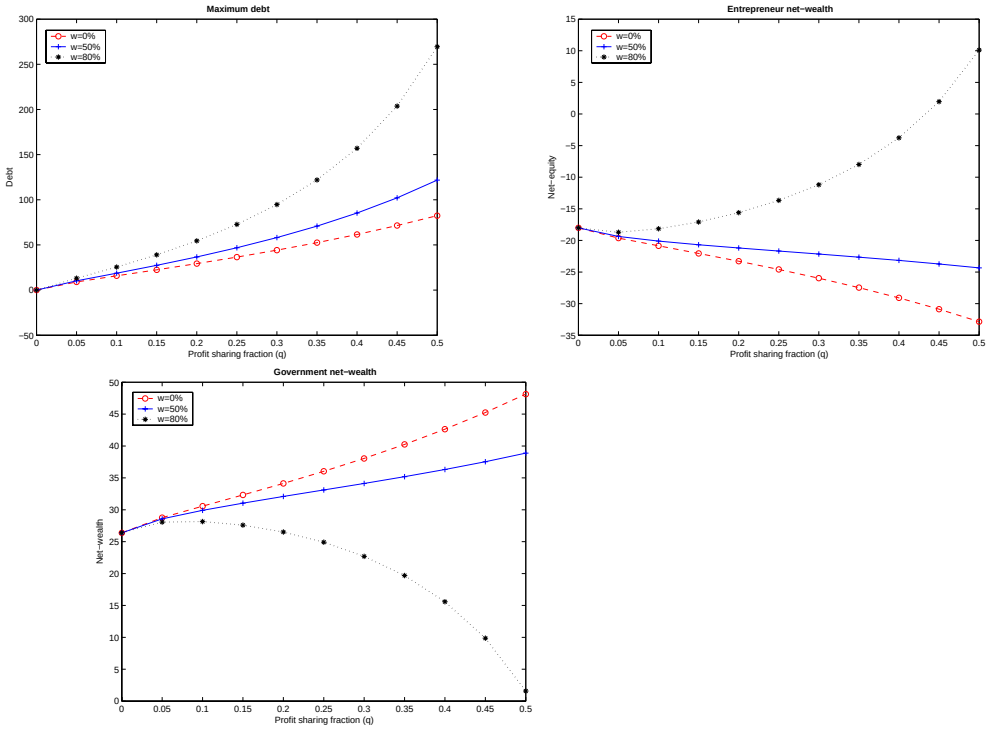
This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the cash flow function defined as $V(I) = 1.2 \times I$. The bottom graphs, panel (b) plot the same variables but for cash flow function $V(I) = 1.5 \times I$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 7: Sensitivity to the Project Risk Level

(a) σ = 0.40



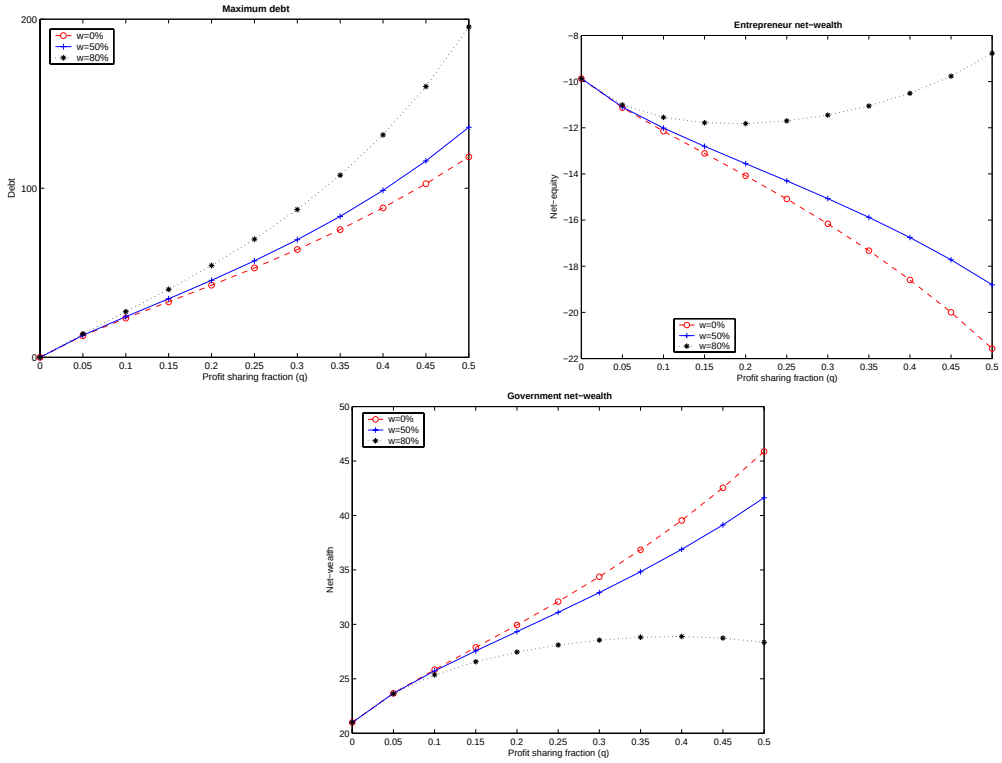
(b) σ = 0.60



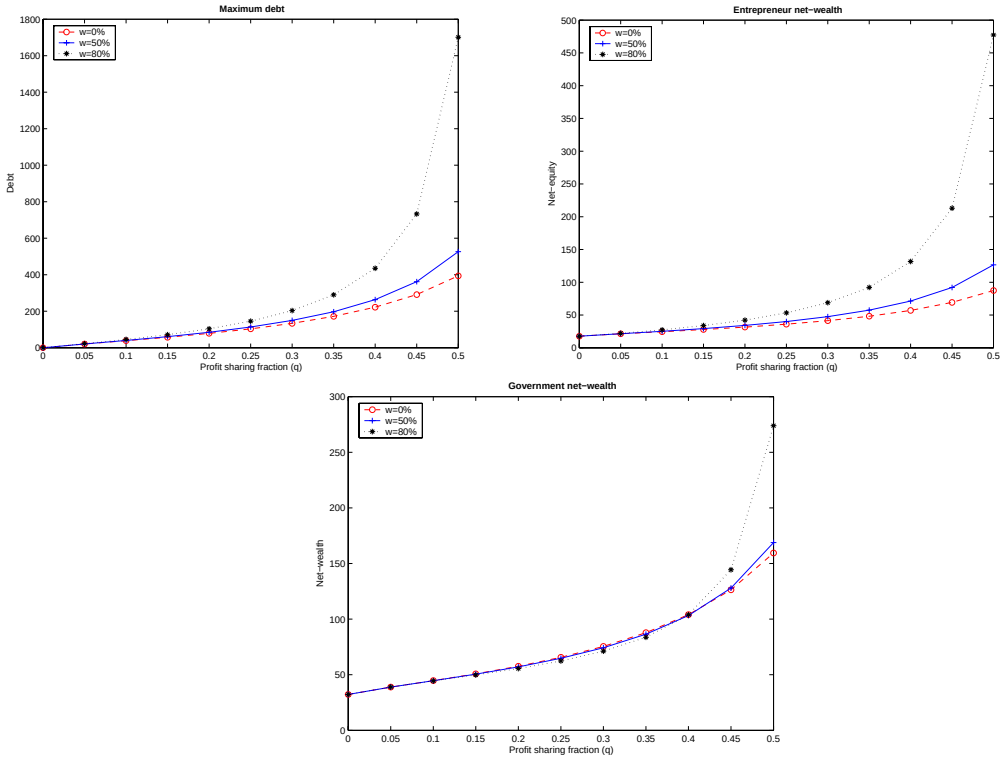
This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the project risk level set at $\sigma = 0.40$. The bottom graphs, panel (b) plot the same variables but for the project risk level set at $\sigma = 0.60$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 8: Sensitivity to the Asset Growth Rate

(a) μ = 0.05



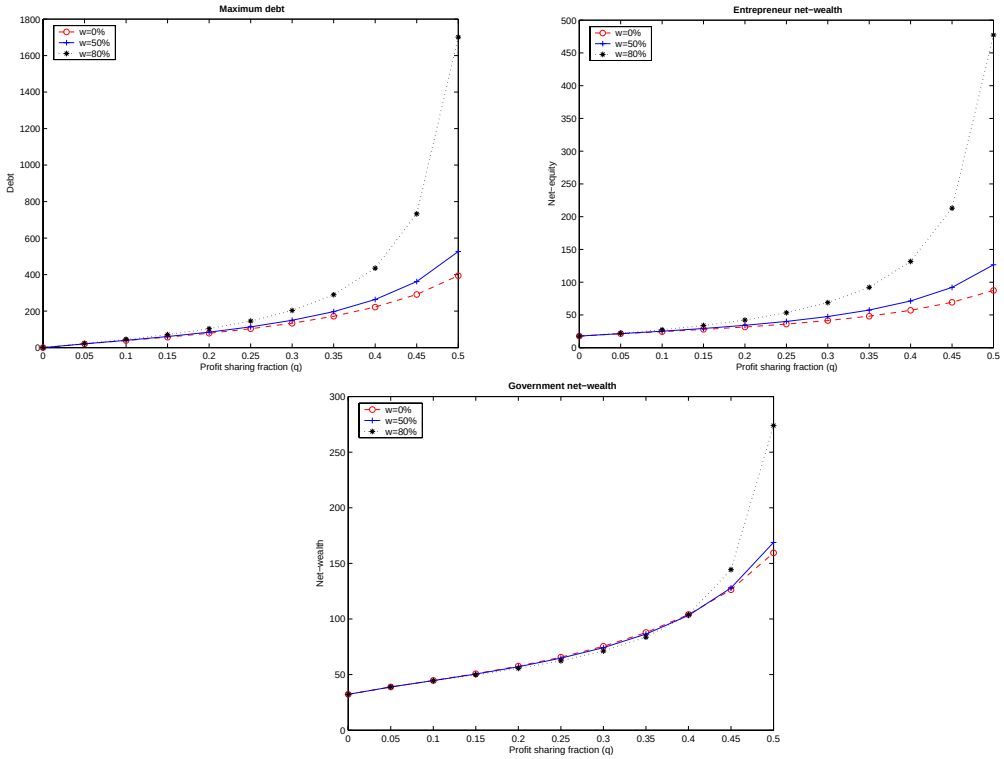
(b) μ = 0.08



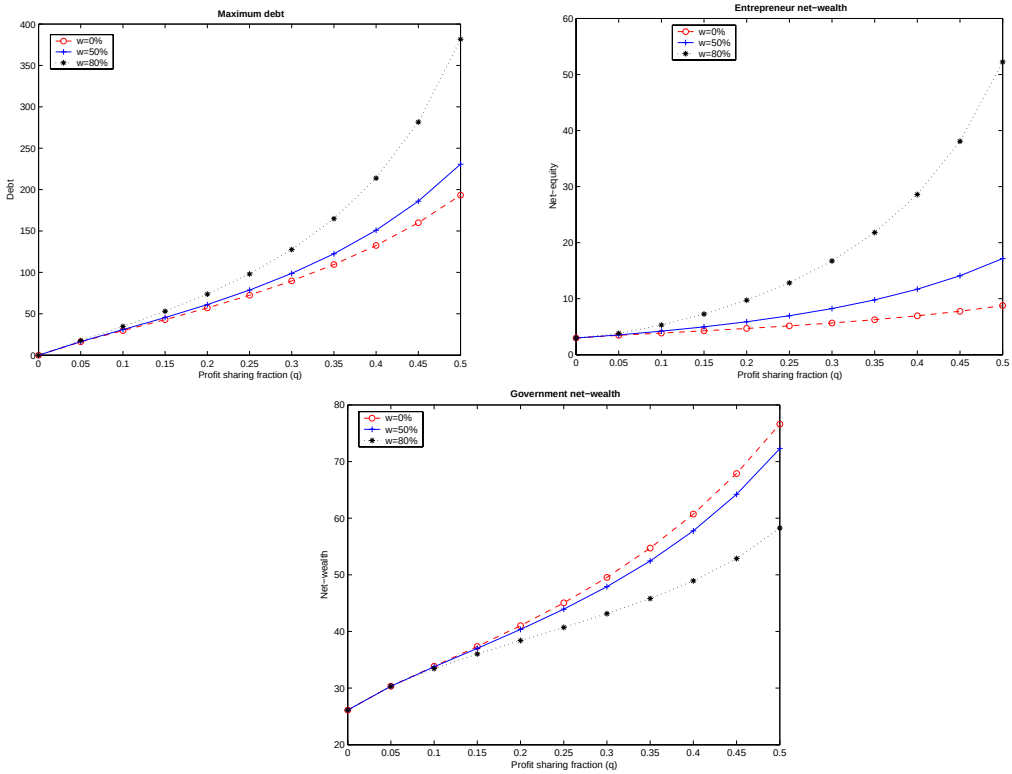
This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the project asset's growth rate set at μ = 0.05. The bottom graphs, panel (b) plot the same variables but for the project asset's growth rate set at μ = 0.08. The graphs are computed using the following parameters values: σ = 0.40, λ = 0.1625, π = 0.015, τ_c = 0.35, T =10; the entrepreneur contributed capital has been normalized to S = 100. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 9: Sensitivity to the Market Price of Risk

(a) $\lambda = 0.1625$



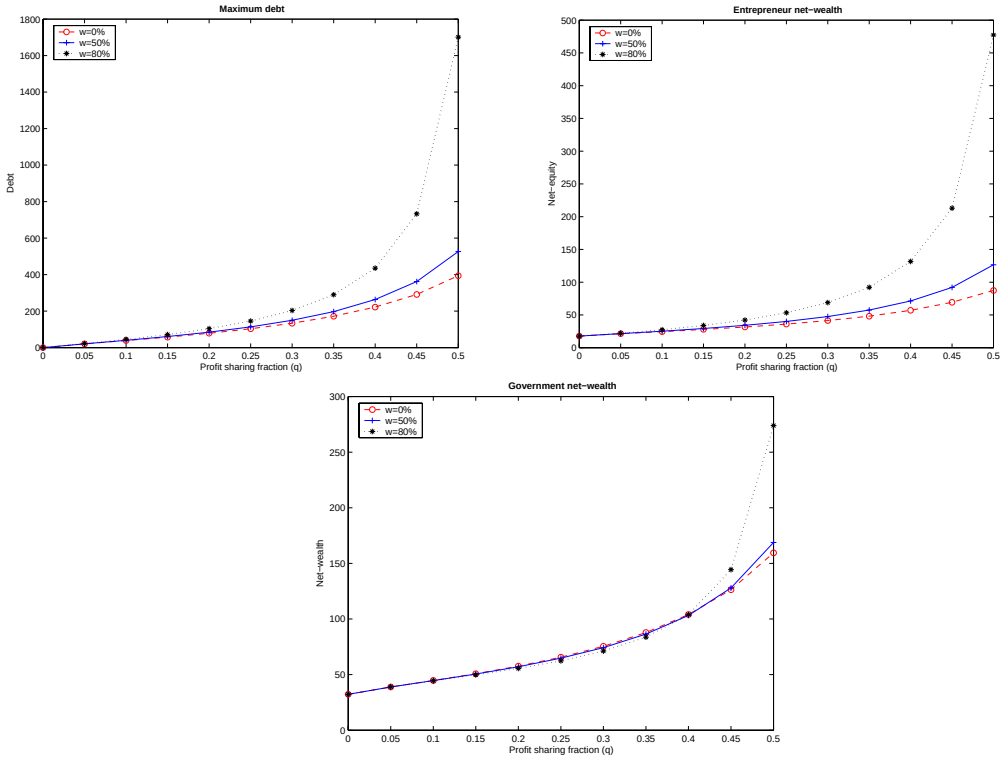
(b) $\lambda = 0.20$



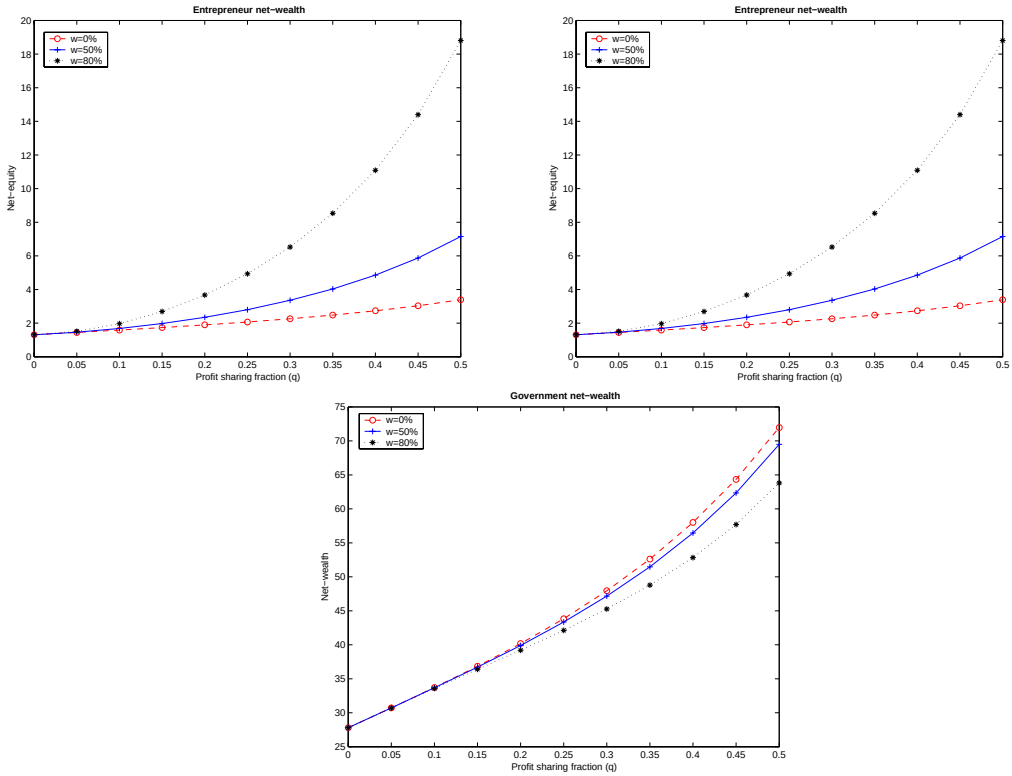
This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the market price of risk set at λ = 0.1625. The bottom graphs, panel (b) plot the same variables but for the market price of risk set at $\lambda = 0.20$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 10: Sensitivity to the Inflation Rate

(a) π = 0.015



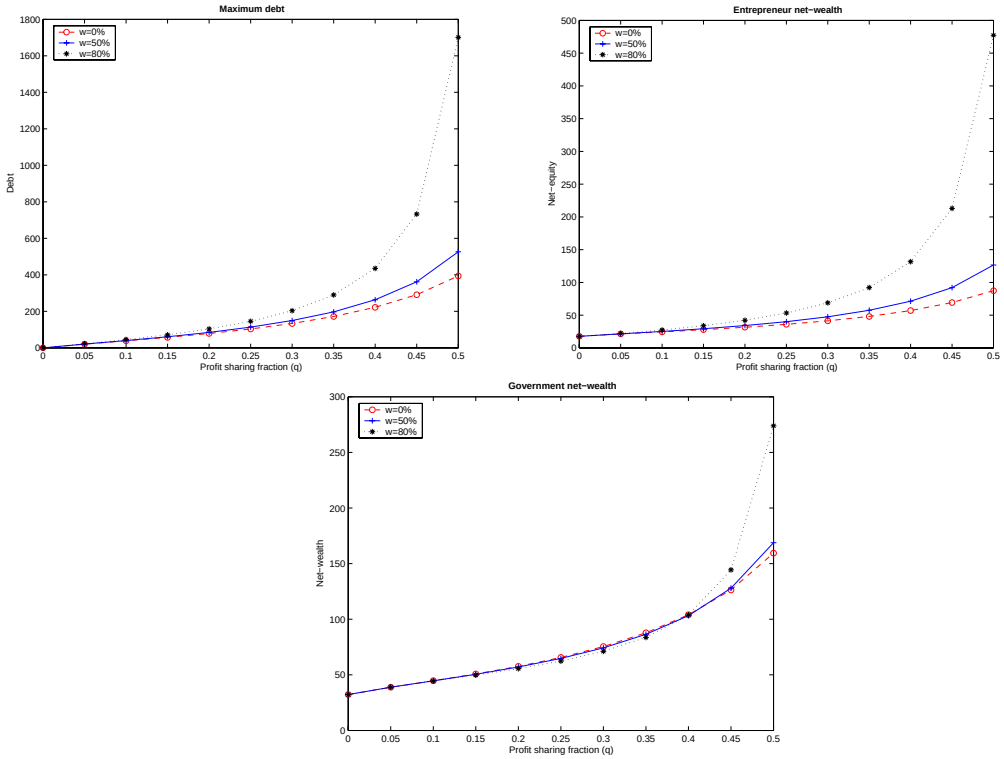
(b) π = 0.030



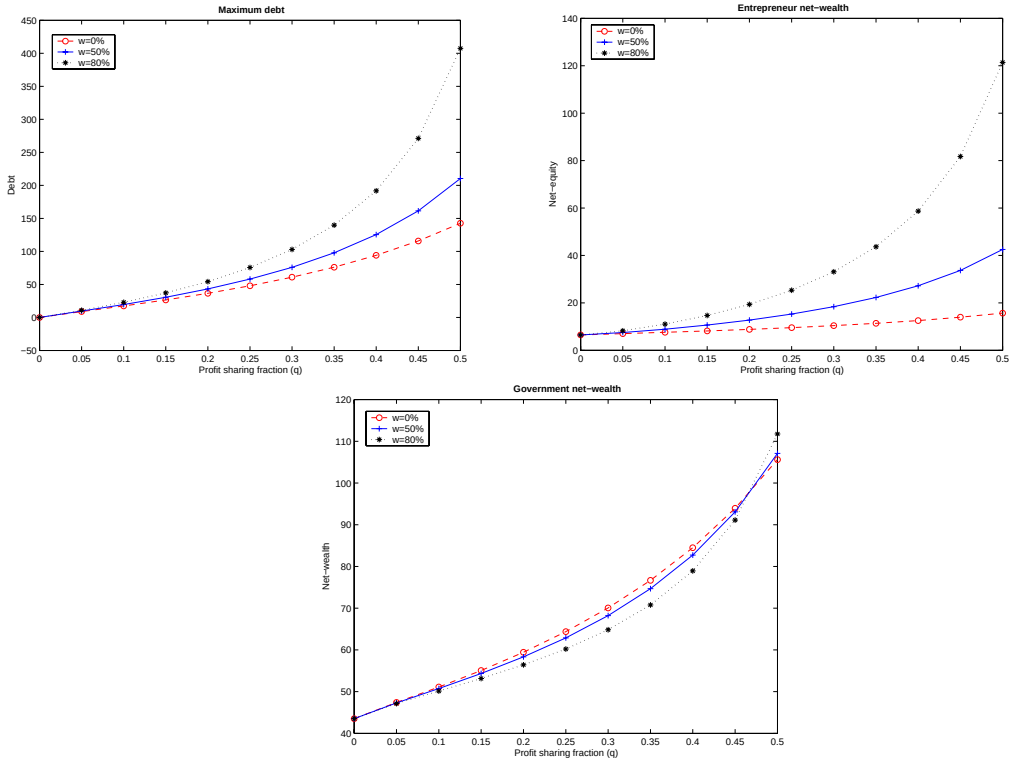
This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the inflation rate set at $\pi = 0.015$. The bottom graphs, panel (b) plot the same variables but for the inflation rate set at $\pi = 0.03$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda = 0.1625$, $\tau_c = 0.35$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 11: Sensitivity to the Maturity of the Project

(a) T = 10 years



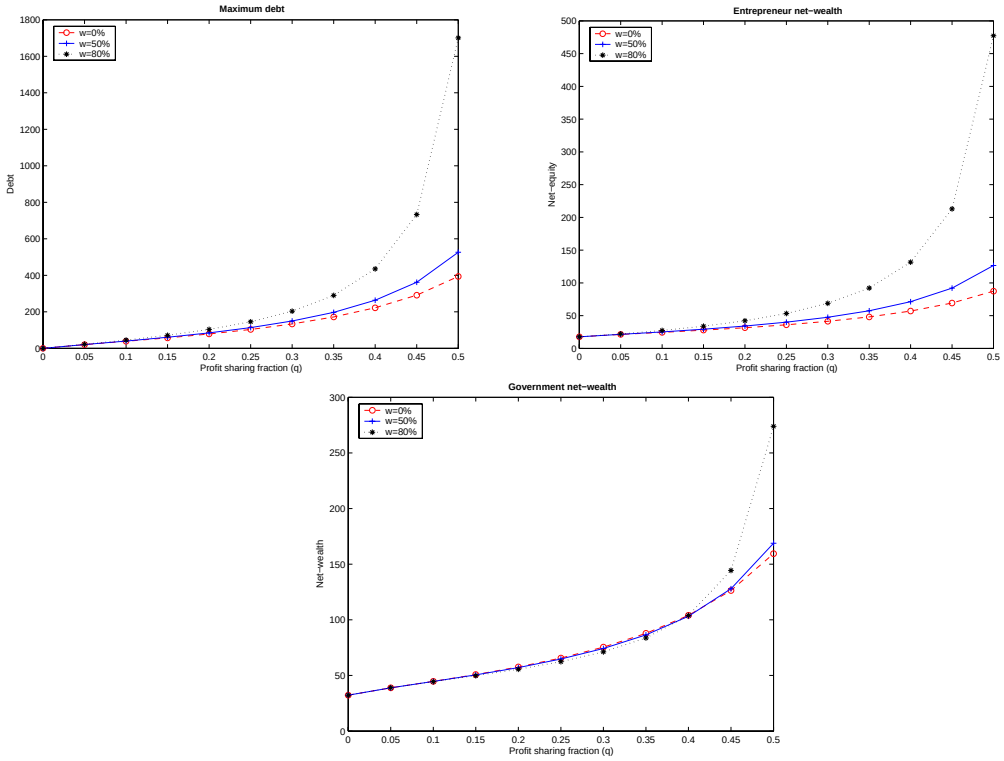
(b) T = 30 years



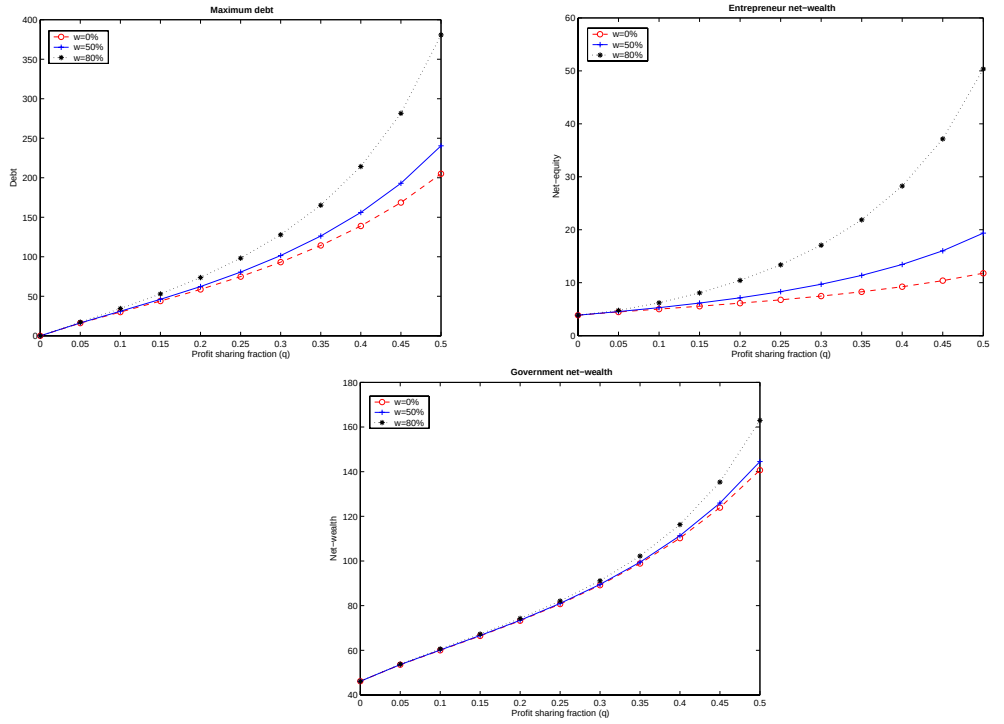
This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the maturity of the project set at T =10 years. The bottom graphs, panel (b) plot the same variables but for the maturity of the project set at T =30 years. The graphs are computed using the following parameters values: μ = 0.08, σ = 0.40, λ = 0.1625, π = 0.015, τ_c = 0.35; the entrepreneur contributed capital has been normalized to S = 100. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 12: Sensitivity to the Tax Rate

(a) $\tau_c = 0.35$



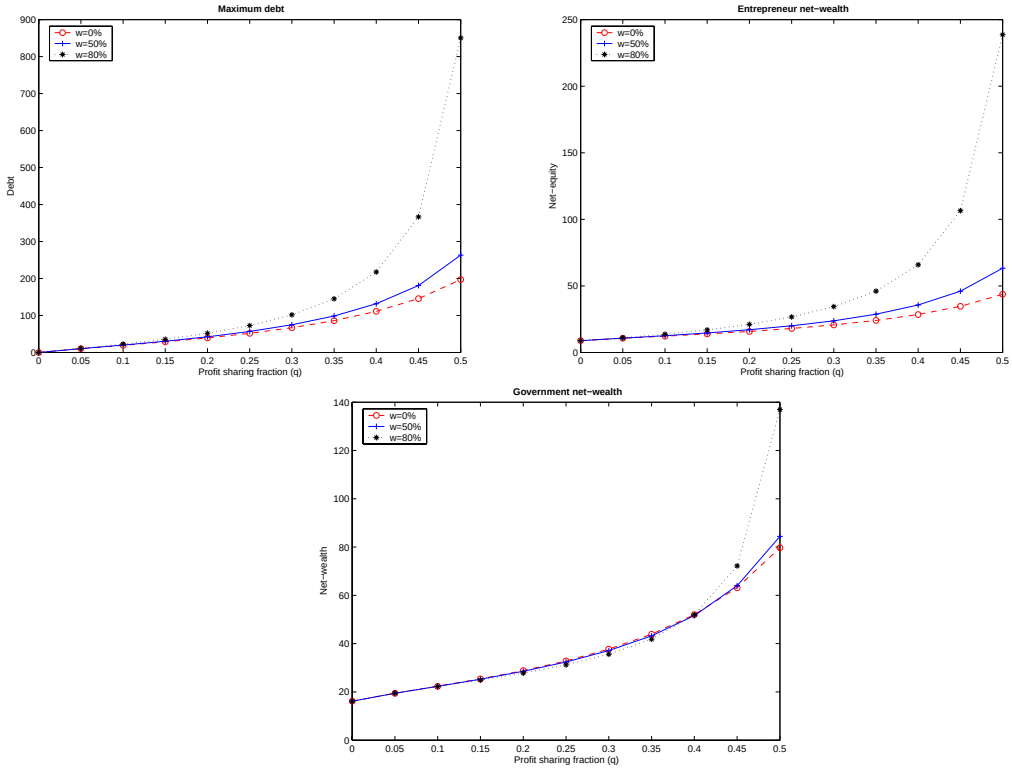
(b) $\tau_c = 0.50$



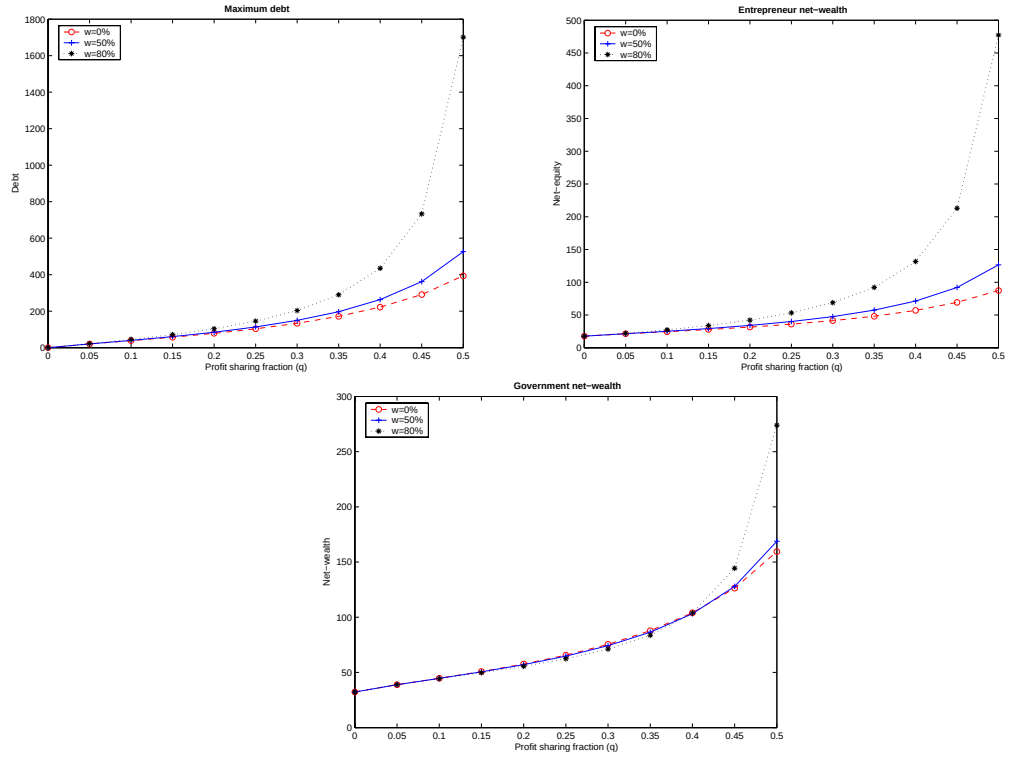
This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the tax rate set at $\tau_c = 0.35$. The bottom graphs, panel (b) plot the same variables but for the tax rate set at $\tau_c = 0.50$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda = 0.1625$, $\pi = 0.015$, $T = 10$; the entrepreneur contributed capital has been normalized to $S = 100$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.

Figure 13: Sensitivity to the Entrepreneur Contributed Capital

(a) S = 50



(b) S = 100



This figure is composed of six graphs. The top graphs, panel (a), are respectively the maximum debt amount, the net-wealth of the entrepreneur and the net-wealth of the government with the entrepreneur contributed capital set to $S = 50$. The bottom graphs, panel (b) plot the same variables but for the entrepreneur contributed capital set to $S = 100$. The graphs are computed using the following parameters values: $\mu = 0.08$, $\sigma = 0.40$, $\lambda = 0.1625$, $\pi = 0.015$, $\tau_c = 0.35$, $T = 10$. The maximum total investment is equal to the entrepreneur contributed capital plus the maximum debt: $I = S + D$.