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Abstract

This paper aims at modelling the density of quarterly inflation based on time-varying conditional variance, skewness and kurtosis model developed by Leon, Rubio, and Serna (2005). They model higher-order moments as GARCH-type processes by applying a Gram-Charlier series expansion of the normal density function. We estimated seven univariate models, including GARCH-M and TARCH-M models, assuming three different distributions for the error term, namely: normal, student t, and GED distributions. Additionally, the model that allows for non-constant higher order moments, GARCHSK-M, has been estimated. Moreover, the paper utilizes two multivariate models, Dynamic Conditional Correlation (DCC) and Diagonal VECH models to isolate the time-varying conditional correlations between inflation and two financial variables, including growth in domestic credit and real exchange rate. Results revealed the significant persistence in conditional variance, skewness and kurtosis, which indicate high asymmetry of inflation. Diagnostic tests indicated that models with invariant volatility, skewness and kurtosis are inferior to the models that permit them to vary over time. Moreover, depending on models of static historic correlation between inflation and the highly financial variables in order to evaluate inflation dynamic behavior is misleading and is a poor informative. Comparing the predictive power of different models showed that basic models are more accurate in forecasting out-of-sample inflation according to some criterions and GARCHSK-M is better for other criterions. By applying Diebold and Mariano's (1995) encompassing test, it was found that all models could be combined together to form a more accurate forecast. We have done the combination of forecasts using equal weights, Bayesian Model Averaging (BMA), and Dynamic Model Averaging (DMA). Results of forecast combination showed that the combined forecasts outperform the projection of best single model.

JEL Classification: C13, E31, E37

Keywords: inflation targeting, conditional volatility, skewness and kurtosis, modelling uncertainty of inflation, multivariate GARCH.

ملخص

تهدف هذه الورقة إلى نمذجة كثافة التضـــخم الفصـــلي المعتمد على الوقت متفاوت التباين المشــروط، الإلتواء ونموذج التفرطح التي وضـعها ليون، روبيو، وسـيرنا (2005). فقد قاموا بخلق نموذج اللحظات العليا كعمليات من نوع GARCH بتطبيق توسـيع سـلسـلة الجرام شـــارلييه من دالة الكثافة العادية. قدرنا ســـبعة نماذج لحيد المتغير، بما في ذلك GARCH-M ونماذج TARCH-M، على افتراض ثلاثة توزيعات مختلفة لمصـــطلح خطأ، وهما: عادي، طالب تي، وتوزيعات GED. بالإضـــافة إلى ذلك، فإن النموذج الذي يســـمح للحظات المرتبة الأعلى الغير ثابتة ، GARCHSK-M. وعلاوة على ذلك، تســـتخدم ورقة نموذجين متعددا المتغيرات، والارتباط الشـرطي الديناميكي (DCC) ونماذج VECH القطري لعزل الارتباطات الوقتية المتفاوتة المشـروطة بين التضـخم واثنين من المتغيرات المالية، بما في ذلك النمو في الائتمان المحلي وســـعر الصـــرف الحقيقي. وكشـــفت النتائج اســـتمرار التباين الكبير في المشروط ، الالتواء والتفرطح، والتي تشير إلى عدم التماثل في ارتفاع التضـخم. وأشـارت الاختبارات التشـخيصـية ان النماذج الثابتة مع التقلب، الالتواء والتفرطح أقل شـــأنا من النماذج التي تســـمح لهم بالتغير مع مرور الوقت. وعلاوة على ذلك، اعتمادا على نماذج الارتباط التاريخي الثابت بين التضـــخم والمتغيرات المالية للغاية من أجل تقييم السـلوك الديناميكي للتضـخم. وبالمقارنة أظهرت القوة التنبؤية من النماذج المختلفة للنماذج الأسـاسـية هي أكثر دقة في التنبؤ بالتضـخم خارج العينة وفقا لبعض المعيارات وGARCHSK-M هو أفضـــل من معيارات أخرى. من خلال تطبيق ديبولد وماريانو (1995) التي تشـــمل الاختبار، فقد وجد أن جميع نماذج يمكن الجمع بينها لتشـكيل توقعات أكثر دقة. قمنا به من مزيج من التوقعات باسـتخدام أوزان متسـاوية، النظرية الافتراضـية نموذج متوسـط (BMA)، وديناميكية نموذج متوسط (DMA). وأظهرت نتائج الجمع بأن التوقعات المجتمعة تتفوق على توقعات نموذج واحد.

1. Introduction

Exploring the relationship between inflation and its higher-order moments is quite important for central banks, especially under inflation targeting regime. That is because the density forecasts allow a much richer setting of anti-inflation policy. Given that the Central Bank of Egypt intended to adopt an inflation targeting framework to anchor its monetary policy when the basic requirements are met (Central Bank of Egypt, 2005), it must have accurate models to predict future inflation with different frequencies. Additionally, the Central Bank of Egypt publishes a quarterly bulletin, including inflation data, which supports the need for forecasting inflation on a quarterly basis. Therefore, the paper aims at exploring the relation between quarterly CPI inflation and its higher-order moments, which is likely helpful in better understanding of the risks involved in inflation.

Historically, the Business and Economic Statistics Section of the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) started publishing the first series of quarterly density forecasts in macroeconomics in 1968 (Tay and Wallis, 2000). In addition, the Bank of England has published a density forecast of inflation in its quarterly Inflation Report since February 1996 (Wallis, 2004).

Research on inflation forecasting is still very limited in Egypt. Noureldin (2005) assessed the robustness of three alternative models to forecast inflation in Egypt. These three models are output gap, money gap, and Vector Autoregressive (VAR) models. However, point forecast does not provide a full description of the uncertainty associated with the forecast. Noureldin (2008) employs GARCH-M model to investigate inflation dynamics in Egypt and found a strong positive relationship between the level and variances of inflation. However, ARCH family models assume that the conditional distribution is time-varying only in the first two moments and ignore the information content in higher-order moments (Chaudhuri, Kim, and Shin, 2011).

However, while modelling the third and fourth moments became popular in analyzing the stock markets, it is not widely used in studying inflation. Roger (2000) found evidence towards right skewness in inflation data. In addition, (Chaudhuri, Kim, and Shin, 2011) found that there is a positive correlation between mean inflation, variance and skewness. Harvey and Siddique (1999) developed an approach to estimate time-varying conditional skewness by modelling conditional volatility and skewness as GARCH (1,1) process assuming that the standardized errors follow noncentral t-distribution. To allow for nonconstant conditional kurtosis, Leon, Rubio, and Serna (2005) developed the methodology of Harvey and Siddique (1999) by jointly modelling time-varying variance, skewness and kurtosis (GARCHSK model) assuming that the error term is derived by Gram-Charlier series expansion of the normal density function. The latter density is easier to estimate than the noncentral t-distribution suggested by Harvey and Siddique (1999). Ahmed (2011) modelled the density of monthly CPI inflation using the GARCHSK in mean (GARCHSK-M)model and presented evidence that models allowing fortime-variant higher order moments outperform models that keep them invariant. Thus, the current paper contributes to the literature by modelling the relationship between quarterly CPI inflation and its second, third, and fourth moments, so the validity of the model to different frequency of data can be examined.

Thus, the current research hypothesizes that models with nonconstant higher order moments are more accurate in explaining the risks involved in inflation compared to models that keep them unchanged. Therefore, the main question is: Does modelling quarterly CPI inflation using models allowing for varying higher order moments helps in better understanding of inflation uncertainty?

To answer this question, nine different models are estimated. These models include 7 univariate models of including GARCH-M and TARCH-M models, assuming three different distributions for the error term. Additionally, GARCH-M model is extended to permit conditional skewness and kurtosis to follow GARCH type structure, assuming a Gram-Charlier series expansion for the normal density function.

In addition to the statistical benefits of accounting for second order moment of inflation, the study models time-varying conditional correlation between inflation and two of financial variables , namely, growth in domestic credit and real exchange rate. This is done by applying two multivariate GARCH models; diagonal VECH (DVECH) and Dynamic Conditional Correlation (DCC). These models allow for better understanding of dynamic co-movements, which improve the decision making process under an inflation targeting regime. This is quite important given the growing debate since the recent financial crisis that suggested the need to pay more attention to include financial variables inside the macroeconomic models (see Borio (2011).

Results indicate that there is a significant persistence in conditional variance, skewness and kurtosis. Moreover, comparing different models shows that GARCHSK-M model is superior to models with time invariant higher order moments in terms of the behavior of standardized residuals and the likelihood ratio test. Additionally, using models of static historic correlation between inflation and the highly financial variables to evaluate inflation dynamic behavior is deceptive and is a poor informative. This is because inflation data for Egypt, like many developing countries, suffers from many structural breaks and changes in adopted policies. Furthermore, forecast evaluation is run recursively for different forecasting horizons ranging 1 quarter to 8 quarters, as inflation in actual policy conduct is likely to be forecasted in a two-year horizon. According to Root Mean Square (RMSE) criterion, the different time-invariant models are more accurate in forecasting power, in comparison with GARCHSK-M. On the other hand, GARCHSK-M outperforms all competing models in terms of Theil Inequality Criterion (TIC) over different forecasting horizons. Applying the encompassing test introduced by Diebold and Mariano (1995) reveals that the all competing models are not encompassed by the best Model according to RMSE over the different forecasting horizons. This implies that these models could be combined together to form single forecast. The combination of forecasts is done using three different combination methods: equal weights, Bayesian Model Averaging (BMA), and Dynamic Model Averaging (DMA). Based on these three approaches, it is evident that the combined forecasts are superior to the projection of the best forecast of individual models.

The paper is structured as follows. Section 2 is devoted to review the existing literature. Section 3 presents the different models employed in the current research, while the preliminary check for the data, analysis of the results and comparison of different models are the core of section 4. Section 5 presents the methodology and results of forecast combinations. Finally, section 6 concludes and draws policy implications.

2. Literature Review

Modelling the relationship between inflation and its higher-order moments is quite important for policymakers to provide a better understanding of the uncertainty of inflation. Friedman (1977) asserts that high inflation leads to more variable inflation. This inflation uncertainty is costly since it distorts relative prices and increases risk in nominal contracts (Berument et al, 2001). From the empirical perspective, Engle (1982) empirically proved that for some kinds of data, including inflation, the variance of the disturbance term is not stable as usually assumed by OLS model. Instead, he used Maximum Likelihood (ML) methodology to study UK inflation and finds that it follows Autoregressive Conditional Heteroscedasticity (ARCH) process. The ML estimator is more efficient than OLS estimate. However, the ARCH model is criticized as there is no clear approach to choose the suitable number of lags of the squared residuals to be included in the model. Additionally, this number of lags may be relatively large leading to non-parsimonious model and a violation of the non-negativity assumptions for the volatility equation. Furthermore, it assumes that the current conditional volatility depends only on the past values of residuals squared, which may be an unrealistic assumption as the volatility response to positive and negative shocks are not similar (Engle,1995; Rachev et al,2007; and Brooks,2002).

Bollerslev (1986) presented a generalized ARCH (GARCH) process by modelling the conditional variance as an ARMA process to allow for a more flexible lag structure without the violation of the non-negativity restrictions. However, the basic GARCH model is limited by assuming that the response of variance to negative and positive shocks is similar. To account for this asymmetry, Nelson (1991) proposed the exponential GARCH (EGARCH) model in which the conditional variance is a function of both the size and the sign of lagged residuals assuming that the residuals follow generalized error distribution (GED). However, this distribution allows shocks of different signs to have a different impact on volatility, but is still symmetric like the normal distribution (Harvey and Siddique, 1999). Glosten, Jagnnnathan and Runkle (1993) introduced a formula that captures the leverage effect of financial time series, namely threshold ARCH (TARCH) or GJR specification[1].

Theoretically, the relationship between inflation and skewness could be examined using two different models. Under a sticky price model, Ball and Mankiw (1995) argue that there a positive correlation between the mean and skewness of the price-change distribution. However, the model assumes that the mean-skewness correlation vanishes in the long-term since this correlation is attributed to short-run considerations. On the other hand, under a flexible price model, Balke and Wynne (1996) show a positive correlation between mean inflation and skewness. Opposite to Ball and Mankiw (1995), they believe that this relation should persist or even it may be strengthened in the long-run. Consequently, modelling the mean-skewness relationship of inflation could highly great important in investigating and forecasting future inflation.

Although ARCH family models are quite suitable in modelling time-varying conditional variance, they assume that skewness and kurtosis are time invariant and ignore the information content in higher-order moments (Chaudhuri, Kim, and Shin, 2011). To fill this gap, Harvey and Siddique (1999) introduced a model to jointly estimate nonconstant conditional variance and skewness. They extended the traditional GARCH (1,1) model by explicitly modelling the conditional variance and skewness using ML framework assuming that the standardized errors follow noncentral t-distribution. To allow for nonconstant conditional kurtosis, Leon, Rubio, and Serna (2005) developed the methodology of Harvey and Siddique (1999) by introducing GARCHSK model assuming that the error term is derived by Gram-Charlier series expansion of the normal density function. This distribution is easier to estimate compared to the noncentral t-distribution suggested by Harvey and Siddique (1999).

Chaudhuri, Kim, and Shin (2011) introduce a semi-parametric functional autoregressive (FAR) model for forecasting a time-varying distribution of the sectoral inflation rates in the UK. Ahmed (2011) modelled the density of monthly CPI inflation using the GARCHSK-M model and found that models that permit higher order moments to vary across time outperform models that keep them constant. This paper contributes to the literature by modelling higher order moments of Egyptian quarterly inflation data. Moreover, it explores the dynamic relationship between inflation and other high volatile financial variables, such as growth in credit and real exchange rate by employing two Dynamic Multivariate GARCH models that have never been applied to study inflation in Egypt. Furthermore, the estimated models are used to calculate the forecasts of inflation on both short-term and medium-term horizons. Finally, the contribution

[1] For more details about the different extensions of ARCH/GARCH models, see Bollerslev (2008).

of the current research is using the forecasts resulting from different individual models to improve the prediction accuracy by providing a combined forecast using two methods of forecast combinations: BMA and DMA, over the different predicting horizons.

3. Empirical Models

This section presents the basic GARCH model briefly as well as the TARCH extension to account for the leverage effect. Then, we present both multivariate GARCH models and GARCHSK-M model that permits conditional skewness and kurtosis to vary across time. The methodology of Leon, Rubio, and Serna (2005), used to estimate the latter model, will be introduced in detail.

3.1 Models of time-varying conditional volatility

Bollerslev (1986) extended the basic ARCH model to relate the conditional variance to both past squared errors and past conditional variances. The GARCH(1,1) model has the following specification of the conditional variance

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}$$

Where h_t is the conditional variance, h_{t-1} is the past volatility which is used as a measure of variance persistence and $\varepsilon_{t-1}^2$is the past squared errors.

In order to ensure that the conditional variance is strictly positive, the following inequality restrictions are to be imposed: $\beta_0 \geq 0,\ \beta_1 \geq 0,\ \beta_2 \geq 0$. Additionally, to insure stationarity, it is also required that $\beta_1 + \beta_2 < 1$ where the persistence of variance becomes higher as β_2 approaches 1.

One of the key restrictions of GARCH (*p*,*q*) models is that they enforce a symmetric response of volatility to positive and negative shocks. GJR specification that captures the leverage effect of financial time series could be written as

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \varepsilon_{t-1}^2 (\varepsilon_{t-1} < 0)$$

According to the TARCH model, the conditions $\beta_0 > 0,\ \beta_1 > 0,\ \beta_1 + \beta_3 > 0,\ \beta_2 \geq 0$ are sufficient to ensure a strictly positive conditional variance. The asymmetry parameter β_3 is allowed to be of either sign to capture the asymmetric effects. This parameter measures the contributions of shocks to both short run persistence $(\beta_1 + \beta_3/2)$ and long run persistence $(\beta_1 + \beta_2 + \beta_3/2)$. Another interpretation of the relation between the mean inflation and its uncertainty allows the conditional variance to be a regressor in the mean equation. This GARCH in mean specification denoted GARCH-M add another term in the equation of the mean as follows

$$\pi_t = \mu h_t + \sum_{i=1}^{n} \alpha_i \pi_{t-i} + \varepsilon_t$$

Where π_t refers to inflation, $h_t$is the conditional volatility. Actually, the relation between inflation, volatility and price dispersion has been investigated using GARCH-M specification (Grier and Perry, 1996). Their results suggest that inflation volatility is superior to trend inflation in investigating price dispersion. Additionally, Wilson (2006) employs an EGARCH-M model to explain the relation between inflation, its volatility and output gap. Their results suggested that higher uncertainty does raise inflation and reduces output, which supports Freidman's (1977) argument.

Multivariate GARCH models are very similar to their univariate counterparts. The main difference between the two versions is that the former also specify equations for how the covariances change over time. Therefore, they are useful in analyzing the dynamic relationship or co-movements between different economic and financial variables. Several different multivariate GARCH formulations have been proposed in the literature; here we present two of them which are diagonal VECH and Dynamic Conditional Correlations (DCC). The VECH model is a the first multivariate GARCH model that was proposed by Bollerslev, Engle and Wooldridge in 1988. The main problem of the unrestricted VECH model is the existence of a very large number of parameter in the conditional variance and covariance matrix. As a result, the estimation of VECH becomes quickly infeasible by increasing the number of variables. The diagonal VECH model avoids this disadvantage by assuming a diagonal conditional variance and covariance matrix. The first order diagonal VECH model can be presented as follows:

$$h_{ij,t} = c_{ij} + \alpha_{ij}\varepsilon_{i,t-1}\,\varepsilon_{j,t-1} + \beta_{ij}h_{ij,t-1} \qquad for\ i,j = 1,2,3$$

where c_{ij}, $\alpha_{ij}$and $\beta_{ij}$are parameter of constants, ARCH and GARCH terms respectively. The covariance matrix can be expressed as$h_t = \begin{pmatrix} h_{11t} & . & . \\ h_{21,t} & h_{22,t} & . \\ h_{31,t} & h_{32,t} & h_{33,t} \end{pmatrix}$, where the elements of the diagonal matrix $h_{11t}, h_{22,t}\ and\ h_{33,t}$ are variances and off diagonal matrix are the time varying correlation.

Concerning Dynamic Conditional Correlations (DCC) models, the constant conditional correlation (CCC)model was proposed by Bollerslev in 1990 by estimating the constant conditional matrix.

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t\varepsilon_t'$$

Where $\varepsilon_{it} = \eta_{it}/\sqrt{h_{it}}$, $\bar{Q}$ is NxN unconditional variance matrix of ε_t. Although, the conditional variances are assumed to vary, the constant correlation seems to be irrelevant to many financial and macro variables where the relationship changes with changing in structural breaks or even in changing in policies.

In 2002, Engle introduced the Dynamic conditional model as nonlinear combination of univariate GARCH models through generalizing the CCC model. The DCC model can be presented in this form:

$$H_t = D_t R_t D_t$$

Where

$D_t = diag(h_{11t}^{\frac{1}{2}}, \dots \dots, h_{NN}^{\frac{1}{2}})$and each h_{iit} is estimated from the traditional univariate GARCH model.

$$R_t = diag\left(q_{11t}^{\frac{1}{2}}, \dots \dots, q_{NN}^{\frac{1}{2}}\right)Q_t diag\left(q_{11t}^{\frac{1}{2}}, \dots \dots, q_{NN}^{\frac{1}{2}}\right)$$

Where$Q_t = (q_{ijt})$ is the NxN symmetric positive definite matrix andQ_t takes the usual GARCH representation:

$$Q_t = (1 - \alpha - B)\bar{Q} + \theta_1\varepsilon_{t-1}\varepsilon_{t-1}' + \theta_2 Q_{t-2}$$

Where $\varepsilon_{it} = \eta_{it}/\sqrt{h_{it}}$, $\bar{Q}$ is *NxN* unconditional variance matrix of ε_t and both θ_1 and θ_1 are non negative coefficients and $\theta_1 + \theta_1 < 1$.

3.2 Modelling conditional variance, skewness and kurtosis[2]:

Leon, Rubio, and Serna (2005) developed a new approach allowing for modelling time-varying variance, skewness and kurtosis jointly as a GARCH process. The employed likelihood

[2] This section is mainly based on LRS (2005) and their development to the GARCH-type model of skewness and kurtosis.

function, based on the series expansion of the normal density function is less complicated to estimate in comparison with the likelihood function proposed by Harvey and Siddique (1999) that assumes non-central t distribution for the model errors.

First, an inflation model is specified as GARCH (1,1) or TARCH (1,1). Then, a GARCH(1,1) specification for both conditional nonconstant skewness and kurtosis is included. Let GARCHSK-M refer to the model when the conditional variance is derived by a GARCH specification while TARCHSK-M when conditional variance is derived by the TARCH (1,1) model. In addition, denote the specification that allows for an asymmetry term in the skewness and kurtosis equation by TARCHTSK. Thus, the different models are specified as follows

Mean equation: $\pi_t = \sum_{i=1}^{n} \alpha_i \pi_{t-i} + \varepsilon_t \varepsilon_{t \approx (0,\sigma_\varepsilon^2)}$ (1)

$\varepsilon_t = \eta_t \sqrt{h_t}$; $\eta_t \approx (0,1)$ $E(\varepsilon_t | I_{t-1}) \approx (0, h_t)$

Variance (GARCH): $h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \varepsilon_{t-2}^2 + \beta_3 h_{t-1}$ (2)

Variance (TARCH): $h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \varepsilon_{t-1}^2 (\varepsilon_{t-1} < 0)$ (3)

Skewness (GARCH): $s_t = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1}$ (4)

Kurtosis (GARCH): $k_t = \delta_0 + \delta_1 \, \eta_{t-1}^4 + \delta_2 k_{t-1}$ (5)

Where $\varepsilon_t$is the error term, $\eta_t$is the standardized residuals, $h_t, s_t,$ and $k_t$are conditional volatility, skewness and kurtosis corresponding to $\eta_t$respectively. They establish that $E_{t-1}(\eta_t) = 0, E_{t-1}(\eta_t^2) = 1, E_{t-1}(\eta_t^3) = s_t$and $E_{t-1}(\eta_t^4) = k_t$. First, two basic models are estimated, a GARCH (1,1)-M (equations (1) and (2)) and a TARCH (1,1) (equations (1) and (3)). This followed by models with nonconstant higher order moments, GARCHSK (equations (1), (2), (4) and (5)).

They employed Gram-Charlier series expansion of the normal density function and truncated at the fourth moment to get the following density function for the standardized errors

$$f(\eta_t | I_{t-1}) = \phi(\eta_t) \left[1 + \frac{s_t}{3!}(\eta_t^3 - 3\eta_3) + \frac{k_t - 3}{4!}(\eta_t^4 - 3\eta_t^2 + 3) \right] = \phi(\eta_t)\psi(\eta_t) \quad (6)$$

Where $\phi(.)$denotes the probability density function (pdf) corresponding to the standard normal distribution. Since some parameter estimates may lead to negative value of $f(.)$ due to the component $\psi(.)$, therefore, $f(.)$ is not a real density function. Additionally, the integral of $f(.)$ on R is not equal to one. Therefore, LRS (2005) introduced a true pdf, by squaring the polynomial part $\psi(.)$, and dividing by the integral of $f(.)$ over R to assure that the density integrates to one. The resulting form of pdf is as follows:

$$w(\eta_t | I_{t-1}) = \phi(\eta_t)\psi^2(\eta_t)/\Gamma_t \quad (7)$$

Where

$$\Gamma_t = 1 + \frac{s_t^2}{3!} + \frac{(k_t - 3)^2}{4!}$$

Thus, the logarithm of the likelihood function for one observation corresponding to the conditional distribution $\varepsilon_t = \eta_t \sqrt{h_t}$, whose pdf is $\sqrt{h_t} w(\eta_t | I_{t-1})$ could be reached after deleting the redundant constants as follows

$$l_t = -\frac{1}{2} ln h_t - \frac{1}{2}\eta_t^2 + ln\left(\psi^2(\eta_t)\right) - ln(\Gamma_t) \quad (8)$$

One advantage of this likelihood function is the similarity with the standard normal density function in addition to two adjustment terms to account for time-varying third and fourth moments. What is more, the aforementioned developed density function in equation (7) nests the normal density function (when s_t= zero andk_t= 3). Thus, the restrictions imposed by the normal density functions (i.e., $\gamma_0 = \gamma_1 = \gamma_2 = \delta_0 = \delta_1 = \delta_2 = 0$) could be tested by conducting a likelihood ratio test.

4. Empirical Results

4.1 Data and preliminary check

The data of quarterly CPI, domestic credit and nominal exchange rate are sourced from International Financial Statistics (IFS) and cover the period 1957:1 to 2015:1. Inflation data is computed as quarterly changes in the logarithm of the CPI. Both the growth rate of domestic credit and real exchange rate for the same period have been calculated over the whole period[3]. The period (1957:1 2007:1) will be used for estimation whereas the period (2007:2 2015:1) will be used for evaluating the forecasting performance of the employed models. The estimation sample is chosen to include the largest number of available observations to provide more accurate results. The selected forecasting period is long enough to allow us to compute the combination of the competing forecasts.

Table no. 1 gives the basic descriptive statistics for the data. It is clear that the data is not likely to be drawn from normal distribution according to Jarque-Bera (JB) test statistic.

Prior to estimate models with time-varying higher order moments, the dynamics structure in the conditional mean is examined by correlogram of inflation as guidance for selecting the appropriate mean specification. According to Brooks (2002), a given autocorrelation coefficient is classified as insignificant if it is within range of $\pm 1.96 \times 1/\sqrt{N}$, where N is the number of observations. In this case, it would imply that a correlation coefficient is classified as significant if it were outside the band of -0.1385 and 0.1385. Exploring the correlogram of the data reveals that autocorrelation coefficients are significant up to the seventeenth lag. Additionally, the coefficients of partial autocorrelation are significant for the first four lags. Therefore, an ARMA process seems appropriate; the information criteria are employed to determine the suitable order. However, by estimating different specification for the mean equation using different orders of AR and MA terms, criteria choose different models. That is while AIC selects an ARMA(1,4) specification of the mean equation, SIC chooses ARMA(1,1). Moreover, the values of both criteria show that many different models provide almost identical values of the information criteria, which indicates that the chosen models do not provide particularly sharp characteristics of the data and other specifications could fit the data almost as well. The selected specification includes first and fourth lags of inflation. Diagnostic checks reveal the absence of serial autocorrelation amongst the residuals while it exists in the sequences of$\varepsilon_t^2, \varepsilon_t^3$ and ε_t^4. Moreover, ARCH LM test indicates the existence of ARCH effects in the residuals. Therefore, a model that assumes nonconstant heteroscedasticity, skewness and kurtosis would be more appropriate in modelling inflation.

As the likelihood function is highly nonlinear, good starting values of the parameters are essential. Thus, the models should be estimated in steps, starting from simpler models that are nested in the complicated ones. In other words, the estimated parameters of the simpler models are used as starting values for more complex ones. Accordingly, this research started modelling inflation using basic GARCH(1,1)-M model and TARCH(1,1)-M model to test the asymmetry of volatility response to the sign of the shock to inflation. It is worth noting that the variance equation is allowed to include two dummies, d74 and d91. The first dummy captures the effects of shifting to the open door policy in 1974 that leads to a high increase in the inflation rate. The second dummy is included to capture the start of Economics Reform and Structural Adjustment Programme (ERSAP) in May 1991. Adding these dummies to the volatility equation allows for exploring their effect on the variability of inflation. Furthermore, both dummies are essential

[3] For details of calculation, see the appendix.

to insure covariance stationarity in the different models. Moreover, I have estimated a GARCH-M and TARCH-M model with GED and t- distribution for the error term. This is done to compare the effect of choosing a non-normal distribution of the error term with models that allow skewness and kurtosis to vary with time.

4.2 Results

Table (2) reports the results of the four models, GARCH-M with normal distribution, t-distribution and GED distribution, and the GARCHSK-M model with time-varying conditional third and fourth moments. Results indicate a significant presence of conditional variance persistence as the parameter of lagged volatility is positive and significant across the different models at different level of significance. Thus, high conditional volatility leads to higher conditional volatility next quarters. Additionally, the coefficient of volatility persistence increases when the error term follows both t-distribution and GED distribution. Also, the variance persistence increases by allowing for nonconstant conditional skewness and kurtosis in GARCHSK-M specification. Concerning the volatility effect in mean equation, the estimated parameters are positive and significant across all models. Allowing the error term to follow non-normal distribution reduces the magnitude of this parameter in comparison with both GARCH-M with GED distribution and GARCHSK-M models. Moreover, allowing the error term to follow a t distribution leads to the highest volatility persistence. Concerning the conditional skewness, it is found that skewness persistence is positive but insignificant while shocks to skewness are negative and significant. Similarly, the conditional kurtosis equation indicates that quarters with high kurtosis are followed by quarters with high kurtosis as concluded from the positivity and significance of lagged kurtosis parameter. Moreover, the coefficient of lagged kurtosis is higher than that of the lagged volatility. Finally, shocks effect to kurtosis are the smallest related to the effects of shocks to volatility and skewness. With respect to dummies effect in the variance equation, d74 is positive and significant in all cases. Additionally, d91 is negative and significant all models except GARCHSK-M.

Results of models that allow for asymmetries are displayed in table (3). First, the asymmetric parameter in the volatility equation, β_3, is found to be negative and significant in all TARCH models with different distributions of the error term. Compared to models without asymmetry term, the inclusion of asymmetry term increases the magnitude of volatility persistence in all cases. Secondly, the shocks to inflation $\beta_1$is found to be significant in the all TARCH-M models where the highest magnitude is in the model that assumes a normal distribution for the error term. Additionally, the persistence parameter in the variance equation is significant in all models with the highest magnitude in TARCH-M with t distribution for the error term. In addition, the parameter of GARCH in mean is significant in all TARCH models.

Concerning the specification of the models, the Ljung-Box Q-statistics for the sequence of $\varepsilon_t, \varepsilon_t^2, \varepsilon_t^3and\varepsilon_t^4$ are insignificant for lag length even larger than 20, which implies the absence of any serial correlation in these series. Furthermore, ARCH LM tests indicate the absence of any further ARCH effects in the standardized residuals. To choose the best model, SIC criterion is set to be equal to $\ln(LML) - (q/2)\ln(N)$, where q is the number of estimated parameters, N is the number of observations, and LML is the value of the log likelihood function using the q estimated parameters. Then, the best model is the one with the highest SIC. According to SIC criterion, the specification in which the third and the fourth moments, GARCHSK-M, are allowed to be time-variant is the best model.

To sum up, these results support Friedman (1977) hypothesis concerning the positive correlation between inflation and its uncertainty, as volatility persistence and GARCH in mean coefficients are significant in all models except GARCH-M with GED distribution. Additionally, the results show the evidence of positive skewness that is consistent with Balke and Wynne (1996) that the mean-skewness correlation could persist even in the long-run.

Finally, the results of the two multivariate GARCH models are presented in table (4)[4]. We employed two financial variables: the growth rate of domestic credit and real exchange rate. These variables are chosen to figure out the dynamic relationship between inflation and those variables that might help the policymakers in conducting their monetary policy given the high importance of the effects of these variables on inflation. Concerning the DCC model, the results of conditional variances of univariate models are presented where both stationarity and nonnegativity assumptions are met. In addition, the bottom part of the table displays the DCC parameters, the effect of past standardized shocks $\theta_1 = 0.073$ and lagged dynamic conditional correlation $\theta_2 = 0.736$. Both parameter estimates are significant, which indicates that the variables are related together in multivariate dynamic relationship. The diagnostic tests revealed that these models are free from autocorrelation and ARCH Effects.

Figure (2) shows the correlation between inflation in one side and both growth of domestic credit and exchange rate on the other according to the DCC model. It is clear that the relation between inflation and the growth of domestic credit is highly dynamic. The correlation between the two variables has decreased in late 1965 after the industrial plan. Also, it declined again with the shift to the open door policy in late 1974. Then, the correlation was strengthened after the implementation of ERSAP in 1991. With the reforms executed by the central bank in 2003, the correlation reached a minimum but the correlation started to increase again after the announcement of the central bank of Egypt regarding its intention to move to inflation targeting regime in mid-2005 after the starting of inflation targeting policy. Overall, the positive sign for the correlation refers to the positive relationship between growth rateof credit and inflation level.

Concerning the correlation between inflation and real exchange rate growth, there is a positive strong relationship between inflation and real exchange rate which implies that more depreciation in the value of Egyptian pound leads to increasing inflation rate. This is especially important since the imports of food and raw materials represent a high portion of Egypt's imports. Additionally, the correlation between inflation and real exchange rate was increased in with the movement to the open door policy and in 1991 with the launch of ERSAP. Finally, this correlation is significantly increased after the float of the Egyptian pound in 2003, which resulted in a high devaluation of the value of the pound.

With respect to the VECH model, the coefficients of ARCH and GARCH terms are significantly differ from zero, which indicates the existence of a strong multivariate GARCH relationship between the three variables. Figure (3) shows the time plot of covariances between inflation and both growth of domestic credit and real exchange rate according to VECH model. The covariance between inflation and growth of domestic credit is highly volatile in most of the time period. Additionally, this covariance relation increased sharply in 1981 with the banking crisis in Egypt but this relation was less volatile starting from late 1990s resulting from the success of monetary policy in controlling it. On the other hand, the figure of covariance between inflation and real exchange rate growth indicates a massive increase in the covariance after ERSAP that witnessed a huge devaluation in the value of the Egyptian pound.

4.3 Diagnostic tests

The first comparison procedure is comparing the behavior of the standardized residuals obtained from different models. The standardized residuals of GARCHSK-M model have the lowest standard deviation of 0.75 in comparison with other model,s which implies that the standardized residuals series from models with time-varying higher order conditional moments have a lower dispersion than those obtained from time-invariant conditional skewness and kurtosis. On the other hand, GARCH-M model with normal distribution has the lowest

[4] For preliminary examination of the included variables, see the appendix.

skewness whereas TARCH-M model with normal distribution has the smallest kurtosis. The second procedure of comparison is to assess the behavior of conditional variances obtained from the different models. The descriptive statistics of these conditional variances are presented in table (6). The volatility of GARCHSK-M model has the lowest standard deviation compared to other models. On the other hand, TARCH-M with GED has the smallest skewness while TARCH-M with t distribution has the lowest kurtosis. Finally, employing the likelihood ratio test to compare GARCH-M and GARCHSK-M, reported in table (7), indicates the rejection of the null hypothesis that the restricted density (i.e., the normal density function) is the correct density.

4.4 Forecasting performance

Table (8) displays the different measures used to assess the predictive power of the employed models. The forecast error statistics RMSE depend on the scale of the dependent variable. Thus, it is a relative measure to compare forecasts across different models. According to this criterion, the smaller the error, the better is the forecasting ability of the related model. With respect to the Theil inequality coefficient, it must lie between zero and one, where zero is a sign of a perfect fit. Additionally, the bias and variance proportion are indications of how far the mean and variation of the forecast are from the mean and the variance of the actual series while the covariance proportion measures the remaining unsystematic forecasting errors. These different proportions must sum up to one where smaller bias and variation proportion refers to a better forecasts. Thus, most of the bias should be concentrated on the covariance proportion.

Another forecasting comparison procedure is to run encompassing tests. The idea behind using the encompassing test is as follows: suppose that we have two alternative sets of forecasts f_1 and f_2 of a variable where the performance of f_1 outperforms f_2 according to some criterion, say RMSE. Then, if the f_2 contains no useful marginal information, than it is said that f_1 encompasses f_2. It follows that if f_2 is not encompassed by f_1, this means that f_2 may provide some marginal information that is not contained in the better forecast. In this case, the two forecasts could be combined together to form a combined forecast. To eliminate the forecasts that are encompassed by the best projection, the models should be ranked according to their predictive power according to RMSE. Then, select the best model with the smallest RMSE and successively test whether the best model forecast encompasses other models using Diebold and Mariano (DM) (1995) test. If the best model encompasses the alternative model at some significance level α, then the encompassed model should be eliminated from the list of models. The test is repeated with all alternative models according to their ranking (Kışınbay, 2007).

The test statistic developed by Diebold and Mariano (1995), abbreviated as DM, is used to test for equal predictive ability of the two competing forecasts. It considers a sample of loss differential series d_t, defined as $d_t = L(e_{1t}) - (e_{2t})$ where L is some arbitrary loss function[5] like RMSE , e_{it} is the t-step ahead forecasts of the model i = 1, 2 and t = 1, 2,…….,T. Equal predictive accuracy amounts to $E(d_t) = 0$, and the test depends on the observed sample mean $\bar{d} = \frac{1}{T}\sum_{t=1}^{T} d_t$. Assuming the covariance stationarity in the loss deferential series, the DM test statistic is asymptotically normally distributed under the null hypothesis of equal predictive accuracy of competing forecasts. The test statistic is as follows

$$DM = \frac{\bar{d}}{\sqrt{\hat{V}(\bar{d})}} \qquad (9)$$

Where $\hat{V}(\bar{d})$ is a consistent estimate of the asymptotic variance of d, and assuming that τ-step-ahead forecasts exhibit dependence up to order τ−1, it is obtained as:

[5]loss function need not be quadratic or even to be symmetric, and forecast errors can be non –Gaussian, nonzero mean, serially correlated and contemporaneously correlated.

$\hat{V}(\bar{d}) \approx \frac{1}{T}(\gamma_0 + 2\sum_{i=1}^{\tau-1}\gamma_i)$ (10)

Where γ_i is the ith autocovariance of $\bar{d}$, estimated by $\hat{\gamma}_t = \sum_{t=i+1}^{T}(d_t - \bar{d})(d_{t-i} - \bar{d})$.

Table (9) presents the results of RMSE for the out-of-sample (2007:2 to 2015:1) period. The forecast evaluation is run recursively for different forecasting horizons ranging from 1 quarter to 8 quarters, as inflation in actual policy conduct is likely to be forecasted in a two-year horizon. According to RMSE, Overall, the performance of different models in forecasting inflation varies significantly with different forecasting horizons. Also, univariate models provide better forecast over all horizon except the very short horizon. That is to say, Multivariate GARCH model with DCC is the best model in the very short horizon (1 step), where its forecasting performances is much lower over longer horizons. On the other hand, TARCH-M model performs badly over short horizon while it is the best model in predicting 8 step ahead forecasts.

On the other hand, the TIC of GARCHSK-M model over all forecast horizons is the lowest and below 0.34 implying a good forecasting power. Also, the variance proportion is the lowest over all horizons except H=3. This indicates that GARCHSK-M model succeeded in tracking the actual variance path in all horizons except the 3 step-ahead forecast. Concerning the bias proportion, results show that in most of horizons, TARCH-M model with normal distribution has the minimum bias implying that the mean of the forecast can moderately track the mean of actual data over the forecasted period. Finally, concerning the covariance proportion, TARCH-M model with normal distribution has the greatest value for horizons 1, 2, 3, 6 and 7, whereas GARCHSK-M model has the highest CP for horizons 4 and 5. This implied that these two models moderately tracked both mean and a variance path in these horizons and the most of bias is due to unsystematic errors. Therefore, however, GARCHSK-M is not selected by RMSE over any forecasting horizon, it is regarded as the best model according to TIC.

The abovementioned results indicate that there is no unique model that performs well at all forecasting horizons. Therefore, at each horizon, the comparisons between the forecasts of the best model and its alternative models are done bilaterally using the DM (1995) forecast encompassing test. Results of the DM test are displayed in table (11), they show that the null hypothesis of equal forecasting accuracy cannot be rejected at 5% level of significance for all models at all forecasting horizons. This implies that all competing models contain marginal information that is not included in the best model according to the RMSE criterion. Consequently, all models could be combined together to produce a single forecast, which is done in section 5.

5. Forecast Combination

As indicated earlier, different parametric models give different forecasts, and choosing the best model according to some criterions will result in discarding some projections, which may have some marginal information that is not contained in the best forecast. Therefore, the inclusion of these predictions to form a combined forecast may provide more accurate results. Especially, it is empirically evidenced that combining forecasts is an efficient approach to improve the accuracy of the forecasting (Clemen, 1989; Armstrong, 1989). Therefore, the current section applies forecast combination techniques to form a combined forecast. The curial issue in combining forecasts is to find the optimal weight that should be assigned to each individual model to minimize a specific loss function. The current paper applies three different procedures of choosing the optimal weights in combining forecasts, namely, simple average, Bayesian Model Averaging (BMA) and Dynamic Model Averaging (DMA).

5.1 Approaches of forecast combinations

Suppose that we have k available forecasts $\hat{y}_{T+h,1}, \hat{y}_{T+1,2}, \dots, \hat{y}_{T+1,k}$, which are coming from k different models to compute a forecast of y_{T+h}. Let$\hat{y}_{T+h} = g(\hat{y}_{T+h,1}, \hat{y}_{T+h,2}, \dots, \hat{y}_{T+h,k}, w_{i,T+h})$ be the combined point forecast as a function of the underlying single forecasts from $\hat{y}_{T+h,1}$ to $\hat{y}_{T+h,K}$, the forecast combination scheme g , and the vector of the parameters of the combination w_{T+h}.

The values of the optimal combination weights $\widehat{w}_{T+1}$ could be obtained by minimising the following loss function:

$$min_{w_{T+h}} \mathrm{E}[L(e_{T+h}(w_{T+h})|\hat{y}_{T+h,1}, \dots, \hat{y}_{T+h,k}] \quad (11)$$

The function$e_{T+\mathrm{h}} = y_{T+\mathrm{h}} - g(\hat{y}_{T+h,1}, \dots, \hat{y}_{T+1,k}, \widehat{w}_{T+h})$ is the combined forecast error, and L is the loss function that is assumed, for simplicity, to be dependent on the forecast error. In most cases there is no closed form solution of equation (11), but analytical results may be computed by imposing restrictions on the loss function and making distributional constraints on the forecast errors. Often it is simply assumed that the objective function is the MSE loss function:

$$L(e_{T+1}(w_{T+1})) = \theta(\hat{y}_{T+h} - y_{T+h})^2 \theta > 0 \quad (12)$$

For this case, the combined forecast chooses a combination of the individual forecasts that best approximates the conditional expectation, $E(ey_{T+1}|\hat{y}_{T+1})$. In the all approaches that we apply we assume the MSE loss function and we fix$\theta = 1$. Different distributional restrictions, for example, assuming a time varying θ, imply different estimation techniques in equation (11). To calculate the optimal weights that should be assigned to competing models, three different approaches have been used. The approach is Equal weight (EQ), which is the simplest method for calculating the combination weights as the mathematical average of all available individual forecasts. Despite its simplicity, many studies have found that it works better than many complicated techniques for calculating combination weights. On the other hand, it can perform worse than even individual competitors in the case of considering many poor forecasting elements. The formula of calculating EQ is given in equation (13)

$$w_i = \frac{1}{k} \quad (13)$$

where w_i is the weights for all models, and k is the number of the considered models.

The second employed technique for combination is Bayesian model averaging (BMA). Assuming k potential models and only one of these models is the true model; firstly, we define the prior probability that associated for each of the available models. Secondly, we estimate the posterior distribution as the weighted average of the conditional predictive densities for the included models. The predictive density of y_{t+h}, given the available observed data till the time t , F_T, is estimated using the weighted average of the conditional predictive densities given the available models with the posterior probabilities by:

$$p(y_{T+h}/F_T) = \sum_{i=1}^{K} p(m_i/F_T)\, p(y_{T+h}/F_{T,}, m_i) \quad (14)$$

where$p(m_i/F_T)$ is the model m_i posterior probability and $p(y_{T+h}/F_{T,}, m_i)$ is the conditional predictive density conditional of the model m_i and the function F_T. The conditional predictive density is calculated given the function F_T and the model m_i as:

$$p(y_{T+h}/F_{T,}, m_i) = \int p(y_{T+h}/\theta_i, F_T, m_i) p(\theta_i/F_i, m_i) d\theta_i \quad (15)$$

where $\int p(y_{T+h}/\theta_i, F_T, m_i)$ is the conditional predictive density of y_{T+h} given θ_i, $F_T$and m_i. Then, the posterior probabilities of model $\mathrm{M_i}$ can be estimated by:

$p(M_i/D) = \ w_i = \frac{P(D/M_i)\,P(M_i)}{\sum_{i=1}^{k} P(D/M_i)\,P(M_i)}$ (16)

where D is specified data set and $P(M_i)$ is the prior probability for the model M_i . Then, the likelihood of the model M_i can be calculated:

$P(D/M_i) = \int P(D/\boldsymbol{\theta}_i, \boldsymbol{M}_i)\, P(\boldsymbol{\theta}_i/\boldsymbol{M}_i) d\boldsymbol{\theta}_i$ (17)

Sinceθ_i is the vector of parameters that associated for model M_i and $P(\theta_i/M_i)$ is the vector of prior density of θ_i under model M_i. we will use the posterior probability for each to get the individuals combination weights w_i. Under the non-informative from, we can assume equal prior probabilities for all models as $\boldsymbol{Prior}_i = \frac{1}{k}$[6].

The final combination method is Dynamic Model Averaging (DMA) which is developed by (Raftery et al., 2010) and it has been applied to forecast inflation by Koop and Korobilis (2012). It combines forecasts from different models based on the predictive likelihood of each model as approximate to the past forecasting performance.

The DMA allows for the weights associated to the different models to vary over time in contrast to the known Bayesian Model Averaging (BMA) approach which yields constant weights for the different models. Koop and Korobilis proved that DMA approach with different constant coefficients models is a good substitute for adopting time varying coefficients models.

In order to illustrate the idea of DMA, consider the cases of n models are available for forecasting. Also, let $M_t \in \{1, \dots\dots, n\}$be one of these available models at time t and the information set available till the point s is $X^s = (X_1, \dots, X_s)'$. Hence, the weight is defined in terms of the probability that this model M engages at time t conditional on information set up to s is:

$w_{t/s,m} = pr(M_t = m/X^s)$ (18)

In addition to, a recursive algorithm that DMA depends on to calculate $w_{t/t,m}$ and $w_{t/t-1,m}$. It also uses a specific approach call "forgetting factor", α. This approach helps to ease the computation burden when there are a large number of available models.The predictive likelihood can be calculated for each model given the predictive density for each model. Then, following updates can be calculated by using the predictive density information, as follows:

$w_{/t,m} = \frac{w_{t/t-1,}p_m(X_t/X^{t-1})}{\sum_{t/t-1}^{k} w_{t/t-1,\iota}p_\iota(X_t/X^{t-1})}$ (19)

In case, we are assuming that $w_{t/t-1,m}$ is known and with assuming some initial starting values $w_{0/0,m}$. We can calculate the other elements in the system: $w_{t/t,m}$ and $w_{t/t-1,m}$ for the models m=1,.......,n.

In regards of the missing quantity$w_{t\backslash t-1,m}$, Raftery et al. (2010) used the following approximation:

$w_{t/t-1,m} = \frac{w_{t-1}^{\alpha}/t-1,m}{\sum_{t=1}^{k} w_{t-1}^{\alpha}/t-1,\iota}$ (20)

The weights assigned to different models at each current period t will be conditional on this model performance in the recent past periods. In which length is the "recent past" is, this is determined by the forgetting factor, λ. We have depends on the benchmark value for the forgetting factor in Raftery et al. (2010), $\lambda = 0.99$ which implies that in case of quarterly data the last 5 years performance receives around 80% in the weighting criteria.

[6]All Bayesian weights and calculations are estimated by using BMS package inside R software.

5.2 Combination results

The main aim for any forecasts combination process is to improve the accuracy of the individual forecasts, hence the good combination scheme should be characterized by two features: the first one should beat all individual models forecasting accuracy and the second should perform well in comparison to the other combination methods. In our analysis, we will compare the forecasting performance between the different forecasting combination schemes and the best model in terms of MSE and RMSE. Table (12) reports MSE and RMSE for all combination methods and table (13) presents the weights associated to the individual models according to the different static combinations schemes where the time varying weights of DMA technique corresponding to three different forecasting horizons, 1 step, 4 steps, and 8 steps, are presented in figure (4 to 6). In general, we can observe that the dynamic combination technique by DMA dominates the best model and all other static combination schemes for all forecasting horizons except the 3 step forecast where EQ is the best combination method. Finally, with exclusion of the third forecasting horizon, we did not face the famous puzzle of combination forecasts that equal weight approach outperforms more complicated combination methods. The reason behind that is we have initial heterogeneous models where each model has its specific information and some specific features

6. Conclusion and Policy Implications

Inflation forecasts are highly important in the actual management of monetary policy, especially under an inflation targeting regime. Therefore, central banks must have accurate inflation forecasts. Additionally, since understanding the risks included in inflation more fully would improve anti-inflation policy settings, a density forecast could help improving inflation forecasting. Therefore, the current paper applied the methodology proposed by Leon, Rubio and Serna (2005) for modelling the relationship between inflation and time-varying conditional heteroscedasticity, skewness and kurtosis.

The estimated univariate models include GARCH-M and TARCH-M models assuming that the error term follows normal, student t, and GED distributions. Additionally, GARCH-M model is extended to allow conditional skewness and kurtosis to follow GARCH type structure assuming a Gram-Charlier series expansion for the normal density function. Moreover, two multivariate GARCH models, diagonal VECH and DCC are estimated. Results indicate the existence of significant persistence in conditional variance, skewness and kurtosis. Additionally, comparing different models through examining the behavior of standardized residuals, and conducting the likelihood ratio test revealed that GARCHSK-M model outperforms other models with time invariant volatility, skewness and kurtosis. Additionally, we assessed the prediction ability of these models for different forecasting horizons ranging 1 quarter to 8 quarters, as inflation in actual policy conduct is likely to be forecasted in a two-year horizon. According to Root Mean Square (RMSE) criterion, GARCHSK-M has lower forecasting accuracy compared to the basic univariate models. In contrast, GARCHSK-M outperforms all competing models in terms of Theil Inequality Criterion (TIC) over different forecasting horizons. Moreover, results of an encompassing test introduced by Diebold and Mariano (1995) showed that the all competing models are not encompassed by the best Model according to RMSE over the different forecasting horizons. This implies that these models could be combined together to form a single forecast. The combination of forecasts are done using three different combination methods: equal weights, BMA, Dynamic combination Average (DMA). Based on these three approaches, there is an evident that the combined forecasts outperform the prediction of the best forecast of individual models.

Based upon conclusions drawn above, the Central bank of Egypt should take into consideration the higher order conditional moments of inflation in constructing their future forecasts. In addition, the use of combined forecast to form the inflation predictions is highly recommended.

Moreover, the positive correlation between inflation and its higher order moments suggests that the Central Bank of Egypt should aim at achieving low average inflation rate to decrease the negative consequences of uncertainty.

Finally, since the likelihood function is highly nonlinear, the employed methodology is limited by the fact that using different optimization algorithms could lead to different estimates and standard errors. Another limitation is that the model is very sensitive to the choice of the starting values. Specifically, setting the initial values of the parameters to zero or close to zero would result in the existence of many local maximum of the likelihood function. Therefore, special care should be taken by setting the initial values away from zero to avoid the possibility of various local maxima. This research could be extended in many ways, such as the inclusion of other financial variables that might help in understanding the behavior of inflation dynamics, such as money supply and interest rate. In addition, applying more recent techniques of forecast combination is encouraged.

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Figure 1: Quarterly Inflating Rate for The Period (1957:1 to 2007:1)

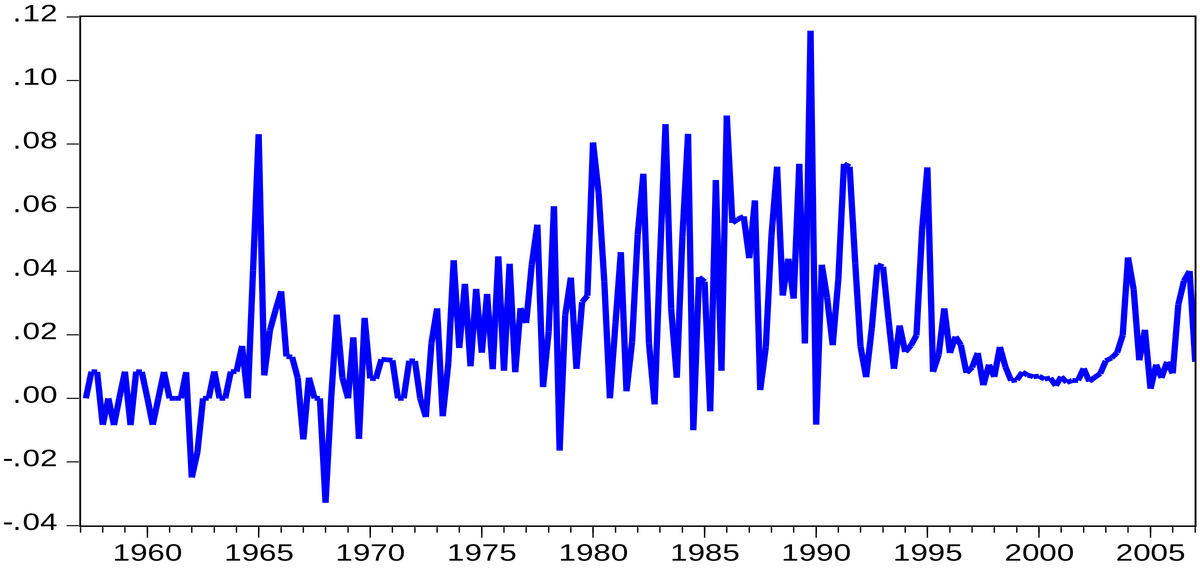

Figure 2: The Dynamic Correlation between Inflation and Both Growth of Domestic Credit and Exchange Rate

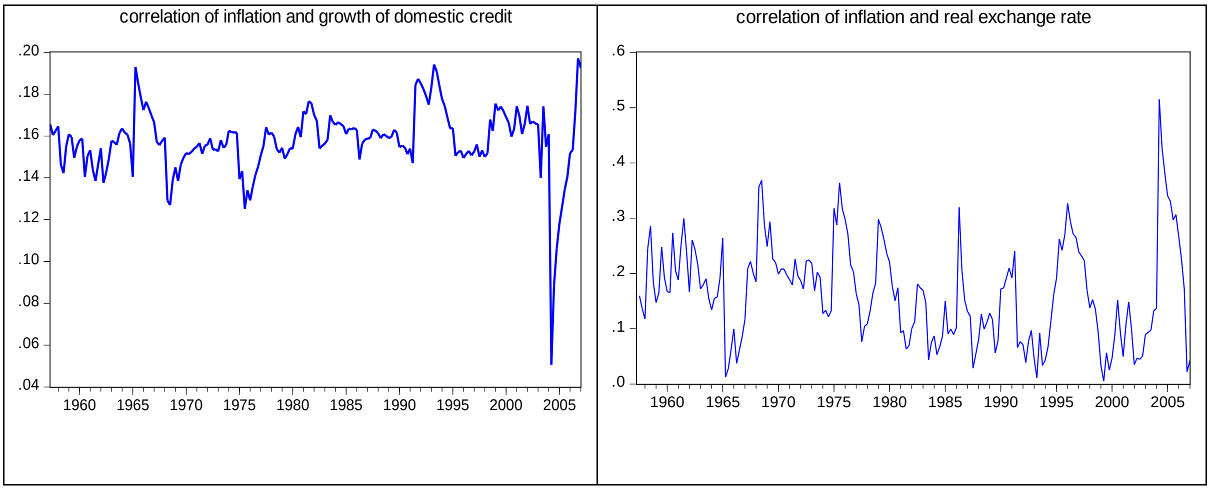

Figure 3: Covariance between Inflation and Both Growth of Domestic Credit and Real Exchange Rate from VECH

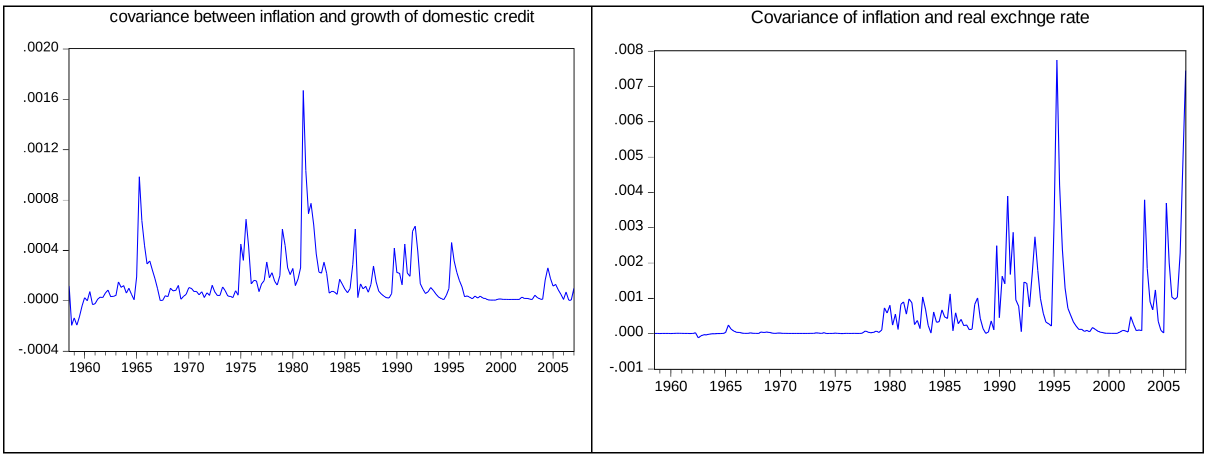

Figure 4: Weights Assigned to Different Models for One Step Ahead Forecast According to DMA

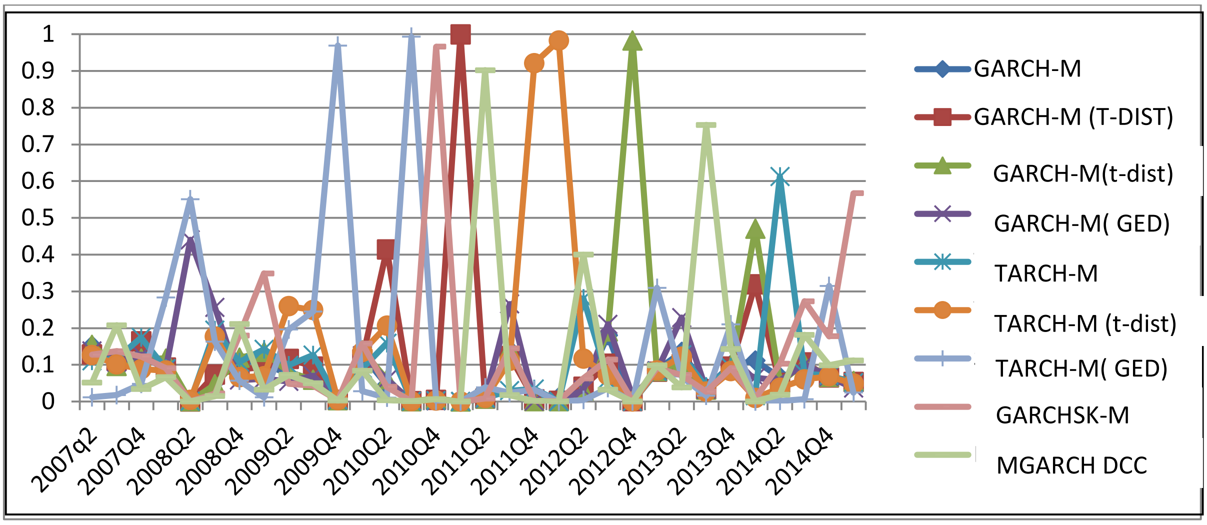

Figure 5: Weights Assigned to Different Models for Four Step Ahead Forecast According to DMA

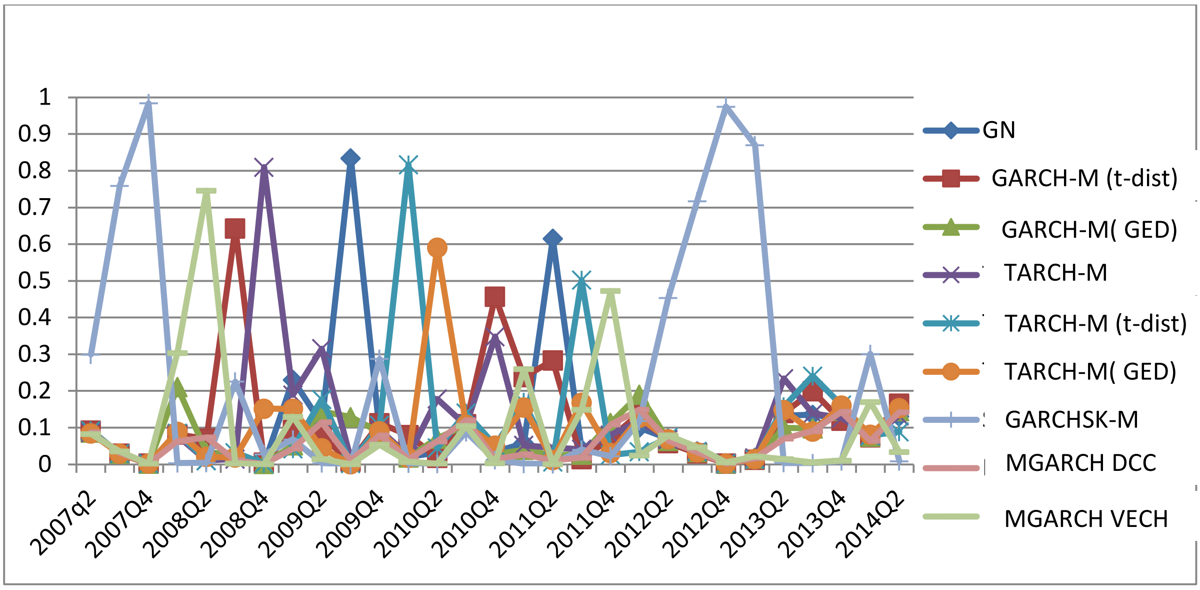

Figure 6: Weights Assigned to Different Models for One Step Ahead Forecast According to DMA

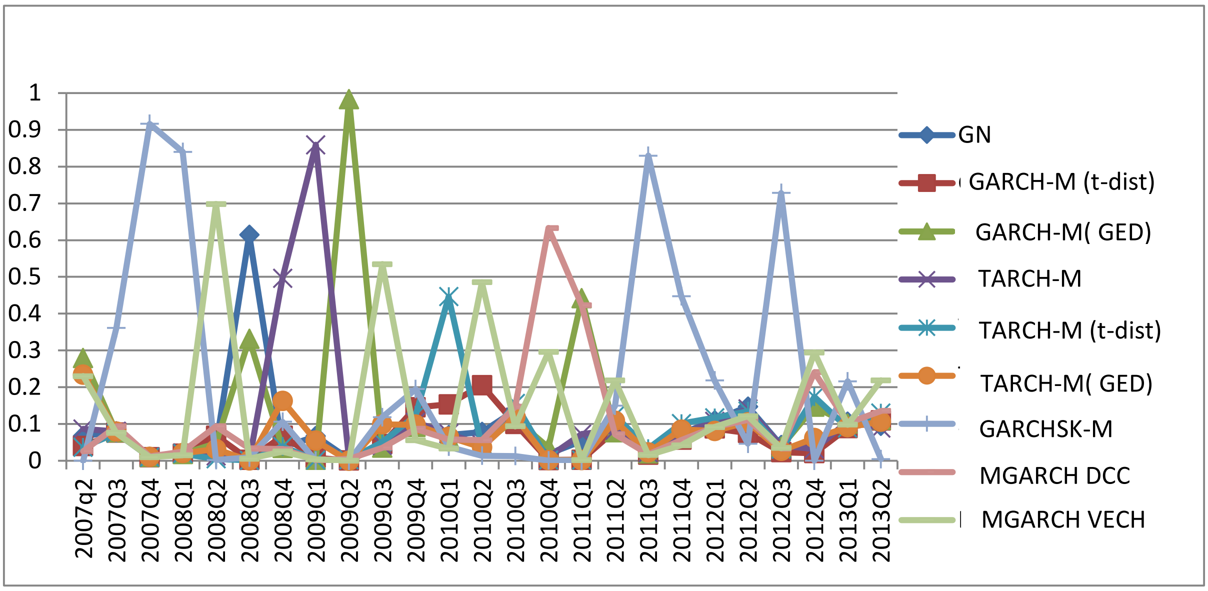

Table 1: Descriptive Statistics of the CPI Inflation 1957:1 to 2007:1)

Mean	0.020112
Median	0.011963
Maximum	0.115602
Minimum	-0.032790
Std. Dev.	0.023657
Skewness	1.159574
Kurtosis	4.437183
JB	62.03284
JB- p value	0.000000

Table 2: Estimates of GARCH-M and GARCHSK-M Model for Inflation (1959:2 2007:1):

Mean equation: $\pi_t = \mu h_t + \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-4} + \varepsilon_t$

Variance equation: $h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \kappa_1 d74 + \kappa_2 d91$

Skewness Equation: $s_t = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1}$

Kurtosis Equation: $k_t = \delta_0 + \delta_1 \eta_{t-1}^4 + \delta_2 k_{t-1}$

Model		GARCH-M		GARCH-M (t-dist)		GARCH-M (GED)		GARCHSK-M	
		estimate	p-value			estimate	p-value	estimate	p-value
	μ	10.70487	**0.0131**	9.962059	**0.0122**	15.46914	**0.0014**	15.84766	**0.0000**
Mean Equation	α_1	0.362465	**0.0002**	0.357985	**0.0000**	0.312845	**0.0003**	0.397144	**0.0000**
	α_2	0.286111	**0.0000**	0.294950	**0.0000**	0.278038	**0.0000**	0.305546	**0.0000**
	β_0	9.61×10^{-5}	**0.0001**	6.52×10^{-5}	**0.0349**	8.52×10^{-5}	**0.0039**	0.000138	**0.0000**
Variance equation	β_1	0.507907	**0.0000**	0.449118	**0.0029**	0.354863	**0.0005**	0.393293	**0.0000**
	β_2	0.186655	**0.0948**	0.418660	**0.0012**	0.294235	**0.0289**	0.255495	**0.0000**
	k_1	0.000376	**0.0321**	0.000383	**0.0861**	0.000389	**0.0268**	0.000474	**0.0000**
	k_2	-0.00043	**0.0166**	-0.000436	**0.0617**	-0.000435	**0.0165**	0.000210	**0.0000**
	t-dist			4.651882	**0.0093**				
	γ_0							-0.147913	**0.0000**
Skewness Equation	γ_1							-0.049117	**0.0000**
	γ_2							0.005199	0.3382
	δ_0							0.972210	**0.0000**
Kurtosis Equation	δ_1							0.010863	**0.0000**
	δ_2							0.588909	**0.0000**
Log-likelihood		511.8632		518.4956		509.9896		928.0167	
SIC		502.73		508.2005		500.8564		912.0178	
Ljung-Box Q-stat.									
ε_t(lag 10)		7.2396	0.612	5.7162	0.839	6.4304	0.778	9.2325	0.510
ε_t^2 (lag 10)		6.0200	0.814	10.070	0.434	7.6228	0.666	6.2244	0.796
ε_t^3 (lag 10)		4.2962	0.933	6.2089	0.797	3.9793	0.948	6.6910	0.754
ε_t^4 (lag 10)		4.3453	0.930	5.0324	0.889	2.9250	0.983	5.6119	0.847

Notes: All models are estimated using ML estimation using Marquardt algorithm. Significant p-values are indicated by bold. t-dist. is the estimation of degrees of freedom of t-distribution, GED parameter is set to equal 1.5.

Table 3: Estimates of TARCH-M Models with Different Distribution (1959:2 2007:1):

Mean equation: $\pi_t = \mu h_t + \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-4} + \varepsilon_t$

Variance equation: $h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \varepsilon_{t-1}^2 (\varepsilon_{t-1} < 0) + \kappa_1 d74 + \kappa_2 d91$

Model		TARCH-M		TARCH-M (t-dist)		TARCH (GED)	
		estimate	p-value	estimate	p-value	estimate	p-value
	μ	19.72773	**0.0002**	8.029947	**0.0561**	26.65659	**0.0002**
Mean equation	α_1	0.254268	**0.0017**	0.390537	**0.0000**	0.171016	**0.0921**
	α_2	0.298124	**0.0000**	0.385944	**0.0000**	0.272075	**0.0000**
	β_0	0.000111	**0.0021**	5.13×10^{-5}	**0.0124**	9.17E-05	**0.0072**
	β_1	0.772270	**0.0016**	0.583225	**0.0002**	0.503393	**0.0030**
Variance equation	β_2	0.197225	**0.0559**	0.587646	**0.0000**	0.324831	**0.0520**
	β_3	-0.702725	**0.0081**	-0.467132	**0.0269**	-0.495986	**0.0063**
	k_1	0.000416	**0.0015**	0.000172	**0.0000**	0.000428	**0.0156**
	k_2	-0.000481	**0.0003**	-0.000222	**0.0000**	-0.000466	**0.0114**
t-dist				1.230947	0.0000		
Log-likelihood		509.0635		511.7202		513.3198	
SIC		498.7886		501.4453		503.0449	
Ljung-Box Q-stat.							
ε_t(lag 10)		9.5904	0.477	6.1276	0.804	6.4377	0.777
ε_t^2 (lag 10)		8.1273	0.616	9.5734	0.479	6.7644	0.747
ε_t^3 (lag 10)		7.9010	0.639	6.1072	0.806	4.5032	0.922
ε_t^4 (lag 10)		7.6536	0.663	4.5840	0.917	5.5914	0.848

Notes: All models are estimated using ML estimation using Marquardt algorithm. Significant p-values are indicated by bold.t-dist. is the estimation of degrees of freedom of t-distribution

Table 4: Variance and Covariance Estimates of MGARCH Models(1959:2 2007:1):

For DCC model: **Mean equation:** each variable is regressed on a constant, AR(1) term and a dummy of ERSAP.
Variance equation:$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \varepsilon_{t-1}^2(\varepsilon_{t-1} < 0)$
For Diagonal VECH: Mean equation: VAR system includes 5 lags as suggested by AIC, LR and FPE criterions
Variance equation$h_{ij,t} = c_{ij} + \alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta_{ij}h_{ij,t-1}$ $\quad for\ i,j = 1,2,3$

	DCC models							VECH model	
	Domestic credit		**inflation**		**Exchange rate**				
	estimate	**p-value**			**estimate**	**p-value**	**coef**	**estimate**	**p-value**
β_0	0.000423	**0.0366**	0.00025	**0.0001**	0.000181	**0.0165**	c_{11}	0.000421	**0.0146**
β_1	0.44730	**0.0250**	0.25704	**0.0253**	0.192352	0.311	c_{22}	0.000140	**0.0072**
β_2	0.55812	**0.000**	0.56328	**0.0003**	0.679877	**0.000**	c_{33}	0.000409	**0.0000**
β_3	-0.43214	**0.0652**	-0.505	**0.0001**	-0.2358	0.240	α_{11}	0.186230	**0.0759**
							α_{12}	0.414618	**0.0005**
							α_{13}	0.526830	**0.0008**
							α_{22}	0.923093	**0.0000**
							α_{23}	0.932919	**0.0000**
							α_{33}	0.940357	**0.0000**
							β_{11}	**0.645619**	**0.0000**
							β_{12}	**0.437165**	**0.0000**
							β_{13}	**0.608988**	**0.0000**
							β_{22}	**0.296015**	**0.0001**
							β_{23}	**0.412361**	**0.0000**
							β_{33}	**0.574435**	**0.0000**
DCC parameters	θ_1	0.073274	**p-value**	**0.0089**					
	θ_2	0.736037	**p-value**	**0.0000**					
Log likelihood			1454.730				1772.009		
Ljung-Box Q-stat.									
ε_t(lag 10)			13.974	**p-value** 0.174					
ε_t^2(lag 10)			10.193	**p-value** 0.424					

Table 5: Descriptive Statistics for Standardized Residuals

Statistic	**GARCH-M**	**GARCH-M (t-dist)**	**GARCH-M (GED)**	**GARCHSK-M**	**TARCH-M**	**TARCH-M (t-dist)**	**TARCH-M (GED)**
Mean	0.144754	0.146564	0.117104	-0.287924	0.081321	0.122737	0.051905
Median	0.160837	0.130266	0.105250	-0.397040	0.112671	0.075155	0.077206
Maximum	3.530883	4.205050	3.847917	2.798154	2.997394	4.360491	3.134593
Minimum	-3.020946	-2.619032	-2.635134	-2.496532	-2.66827	-2.489662	-2.81767
Std. Dev.	0.993460	0.982665	1.010568	**0.757900**	0.997981	1.020606	0.994028
Skwness	**0.153516**	0.589360	0.304888	0.335242	0.114134	0.617398	0.173971
Kurtosis	4.364631	5.573321	4.300026	4.900875	**3.783390**	5.190361	3.896186
Jarque-Bera	15.65190	64.75849	16.49515	32.50300	5.326449	50.57923	7.393699
Probability	0.000399	0.000000	0.000262	0.000000	0.069723	0.000000	0.024802

Table 6: Descriptive Statistics for Conditional Variances

Statistic	**GARCH-M**	**GARCH-M (t-dist)**	**GARCH-M (GED)**	**GARCHSK -M**	**TARCH-M**	**TARCH-M (t-dist)**	**TARCH-M (GED)**
Mean	0.000565	0.000685	0.000537	0.001056	0.000525	0.000639	0.000484
Median	0.000241	0.000281	0.000234	0.001077	0.000226	0.000347	0.000203
Maximum	0.006818	0.006606	0.005019	0.006248	0.005700	0.004914	0.003769
Minimum	5.22×10^{-5}	2.34×10^{-5}	5.61×10^{-5}	0.000187	5.75×10^{-5}	4.28×10^{-6}	8.03×10^{-5}
Std. Dev.	0.000769	0.000859	0.000637	0.000786	**0.000657**	0.000741	0.000494
Skewness	3.991448	2.822785	2.894833	2.386306	3.710096	2.557875	**2.249937**
Kurtosis	27.44630	15.68171	16.67755	14.44898	24.71681	**12.35400**	12.57099
Jarque-Bera	5290.785	1557.643	1764.765	1230.856	4213.434	909.3453	894.8217
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 7: Likelihood Ratio Tests

GARCH-M vs. GARCHSK-M	
Logl(GARCH-M)	928.0167
Logl(GARCHSK-M)	511.8632
LR	839.3032
p-value	0.00000

Table 8: Different Criterions of Predictive Power

1. Root Mean square error

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=T+1}^{T+N}(\hat{\pi}_t - \pi_t)^2}$$

2. Theil inequality coefficient

$$TIC = \frac{\sqrt{\frac{1}{N}\sum_{t=T+1}^{T+N}(\hat{\pi}_t - \pi_t)^2}}{\sqrt{\frac{1}{N}\sum_{t=T+1}^{T+N}\hat{\pi}_t^2} + \sqrt{\frac{1}{N}\sum_{t=T+1}^{T+N}\pi_t^2}}$$

Bias Proportion

$$BP = \frac{\left(\bar{\hat{\pi}} - \pi\right)^2}{\frac{1}{N}\sum_{t=T+1}^{T+N}(\hat{\pi}_t - \pi_t)^2}$$

Variance proportion

$$VP = \frac{(\sigma_{\hat{\pi}} - \sigma_{\pi})^2}{\frac{1}{N}\sum_{t=T+1}^{T+N}(\hat{\pi}_t - \pi_t)^2}$$

Covariance proportion

$$CP = \frac{2(1-r)\sigma_{\hat{\pi}}\sigma_{\pi}}{\frac{1}{N}\sum_{t=T+1}^{T+N}(\hat{\pi}_t - \pi_t)^2}$$

Notes: Where $\sigma_{\hat{\pi}}, \sigma_{\pi}$ are the biased standard deviations of $\hat{\pi}$and π, and r is the correlation between of $\hat{\pi}$and π.4

Table 9: Out-of Sample RMSE Criterion of Different Models for Various Horizons

	GARCH-M	**GARCH -M (t-dist)**	**GARCH -M (GED)**	**TARCH-M**	**TARCH-M (t-dist)**	**TARCH-M (GED)**	**GARCHSK-M**	**MGARCH DCC**	**MGARCH BEKK**
H=1	0.018695	0.019713	0.018248	0.019334	0.018071	0.018625	0.020791	0.01750	0.022134
	[5]	[7]	[3]	[6]	[2]	[4]	[8]	**[1]**	[9]
H=2	0.02041	0.021128	0.020406	0.021683	0.020726	0.020805	0.020857	0.021217	0.021176
	[2]	[6]	**[1]**	[9]	[3]	[4]	[5]	[8]	[7]
H=3	0.021139	0.021129	0.02178	0.022864	0.021812	0.022194	0.02192	0.021316	0.021998
	[2]	**[1]**	[4]	[9]	[5]	[8]	[6]	[3]	[7]
H=4	0.021184	0.020837	0.021637	0.021525	0.021928	0.021321	0.022012	0.021965	0.023438
	[2]	**[1]**	[5]	[4]	[6]	[3]	[8]	[7]	[9]
H=5	0.020644	0.02035	0.020729	0.019819	0.019552	0.019679	0.022367	0.021112	0.021485
	[5]	[4]	[6]	[3]	**[1]**	[2]	[9]	[8]	[7]
H=6	0.019494	0.01991	0.019738	0.020348	0.018462	0.019457	0.02218	0.020196	0.019662
	[3]	[6]	[5]	[8]	**[1]**	[2]	[9]	[7]	[4]
H=7	0.017371	0.019151	0.019022	0.019598	0.019278	0.019264	0.021941	0.019223	0.0211
	[1]	[3]	[2]	[7]	[6]	[5]	[9]	[4]	[8]
H=8	0.01917	0.019593	0.019933	0.018917	0.020937	0.019201	0.023198	0.019605	0.020815
	[2]	[4]	[6]	[1]	[8]	[3]	[9]	[5]	[7]

Notes: The numbers in the square brackets indicate rankings of the models where [1] indicates the best models. H refers to the forecasting horizon.

Table 10: Evaluation of Out-of Sample Forecasts Power of Different Models Using TIC and Its Components Criterions

		GARCH-M	GARCH-M (t-dist)	GARCH-M (GED)	TARCH-M	TARCH-M (t-dist)	TARCH-M (GED)	GARCHSK -M	MGARCH DCC	MGARCH BEKK
H=1	TIC	0.363854	0.982952	0.358889	0.35254	0.338452	0.34295	**0.302587**	0.36222	0.4265
	BP	0.099504	0.6776532	0.11804631	0.04446	0.0507977	**0.0426786**	0.285806	0.21863	0.05725
	VP	0.249546	0.314733	0.29127	0.07196	0.219993	0.133012	**0.068299**	0.4727	0.1829
	CP	0.65095	0.0076135	0.59068339	**0.88357**	0.7292090	0.8243092	0.645895	0.30865	0.7597
H=2	TIC	0.420821	0.426045	0.427583	0.41340	0.406722	0.39857	**0.30065**	0.46036	0.4065
	BP	0.204543	0.1628098	0.23261685	**0.10701**	0.1166622	0.1043012	0.247444	0.27745	**0.0874**
	VP	0.321693	0.2386596	0.35816646	0.09506	0.2460877	0.1453142	**0.044533**	0.40619	0.19189
	CP	0.473765	0.5985304	0.40921668	**0.79792**	0.6372499	0.7503844	0.708024	0.32805	0.71257
H=3	TIC	0.438909	0.426471	0.462651	0.43563	0.428863	0.426805	**0.31443**	0.46508	0.43011
	BP	0.230208	0.200475	0.259972	**0.11802**	0.136794	0.122085	0.224175	0.31078	0.13033
	VP	0.166406	0.407493	**0.048039**	0.11245	0.184938	0.076312	0.301212	0.20231	0.29212
	CP	0.477671	0.59721	0.438816	**0.80566**	0.678268	0.76546	0.727786	0.28172	0.70325
H=4	IC	0.444631	0.423783	0.463127	0.41491	0.427826	0.414069	**0.318861**	0.47891	0.47612
	BP	0.24956	0.227199	0.285687	0.15648	**0.14781**	0.157556	0.196942	0.30125	0.18063
	VP	0.335454	0.225844	0.321924	0.10172	0.141726	0.129848	**0.066071**	0.37716	0.17135
	CP	0.414986	0.546956	0.392389	0.74178	0.710464	0.712596	**0.736988**	0.31866	0.64801
H=5	TIC	0.457764	0.451798	0.484275	0.40788	0.410668	0.409337	**0.329224**	0.50776	0.46485
	BP	0.286904	0.280964	0.342037	**0.19451**	0.204605	0.194762	0.244933	0.38045	0.22828
	VP	0.377908	0.256781	0.403163	0.11354	0.187152	0.156592	**0.021517**	0.48775	0.16628
	CP	0.314396	0.462256	0.2548	0.69192	0.608243	0.648645	**0.733549**	0.1318	0.60543
H=6	TIC	0.475717	0.474442	0.501826	0.44549	0.419869	0.433237	**0.336506**	0.52101	0.45689
	BP	0.306922	0.273113	0.374432	**0.16879**	0.229835	0.186327	0.304728	0.38417	0.27288
	VP	0.344957	0.240325	0.438324	0.07858	0.201703	0.133976	**0.006801**	0.47155	0.15050
	CP	0.348121	0.486562	0.187244	**0.75263**	0.568462	0.679697	0.688471	0.14107	0.57661
H=7	TIC	0.443149	0.480644	0.511155	0.44375	0.460163	0.446261	**0.343699**	0.52328	0.50612
	BP	0.351619	0.286373	0.382762	**0.15516**	0.198514	0.17115	0.341344	0.39295	0.19653
	VP	0.349239	0.208344	0.418577	0.05369	0.150501	0.09663	**0.00643**	0.49988	0.09661
	CP	0.299142	0.505283	0.198661	**0.79114**	0.650985	0.732219	0.652226	0.09960	0.67733
H=8	TIC	0.451423	0.497017	0.529578	0.45877	0.433467	0.467717	**0.293675**	0.52490	0.53386
	BP	0.367816	0.293385	0.392099	**0.16197**	0.21902	0.179934	0.420356	0.41277	0.21872
	VP	0.344212	0.185285	0.395252	0.04975	0.1278	0.084253	**0.008512**	0.50333	0.08537
	CP	0.257243	0.500927	0.18389	0.78003	**0.833853**	0.726696	0.649427	0.08133	0.68157

Notes: The best model corresponding to each criterion over different horizons is written in bold. H refers to the forecasting horizon.

Table 11: Diebold and Mariano (DM) Test

	GARCH-M	GARCH-M (t-dist)	GARCH-M (GED)	TARCH-M	TARCH-M (t-dist)	TARCH-M (GED)	GARCHSK -M	MGARCH DCC	MGARCH BEKK
H=1	1.458992	1.492113	1.45896	1.44863	1.45668	1.452802	1.402642	***	1.47174
	(0.07228)	(0.06783)	(0.07228)	(0.0737)	(0.0726)	(0.0731)	0.080362		(0.0705)
H=2	-0.15979	-0.718046	***	-0.1653	0.049276	-0.067375	-0.13379	0.07662	-0.3046
	(0.4365)	(0.2363)		(0.4343)	(0.4803)	(0.4731)	(0.44678)	(0.4694)	(0.3803)
H=3	-0.00316	***	-0.209487	-0.2860	-0.178037	-0.30003	-0.03094	-0.0408	-0.1276
	(0.4987)		(0.4170)	(0.3874)	(0.4293)	(0.3821)	0.487659	(0.4837)	(0.4492)
H=4	-0.15002	***	-0.366954	-0.1491	-0.24206	-0.1917	-0.0451	-0.3204	-0.2720
	(0.4403)		(0.3568)	(0.4407)	(0.4043)	(0.4239)	(0.4819)	0.3743	(0.3927)
H=5	-0.10364	-0.227974	-0.32143	-0.0979	***	-0.049223	-0.113005	-0.2868	-0.1649
	(0.4587)	(0.4098)	(0.3739)	(0.461)		(0.48037)	(0.45501)	(0.3871)	(0.4345)
H=6	-0.27653	-0.271964	-0.310418	-0.0615	***	-0.1720	-0.15768	-0.2981	-0.0911
	0.39107	0.392825	0.378121	(0.4754)		(0.4317)	(0.4373)	(0.3828)	(0.4636)
H=7	***	-0.47247	-0.2873	-0.2411	-0.23165	-0.28025	-0.19892	-0.2585	-0.5165
		(0.31829)	(0.3869)	(0.4047)	(0.4084)	(0.3896)	(0.42112)	(0.398)	(0.3029)
H=8	0.263896	0.059182	0.05778	***	0.046206	0.033042	0.06991	0.06076	-0.0829
	(0.395)	(0.4764)	0.476962		(0.4815)	(0.4868)	(0.4721)	(0.4757)	(0.4669)

Notes: ***indicates that the model is the best one at the corresponding forecasting horizon. The number between brackets are the probability of the test statistic.

Table 12: Out-of Sample Forecasts Power of Different Combination Methods

Criterion	H=1	H=2	H=3	H=4	H=5	H=6	H=7	H=8
Best model								
MSE	0.000306	0.000416	0.000446	0.000434	0.000382	0.000341	0.000302	0.000358
RMSE	0.017503	0.020406	0.021129	0.020837	0.019552	0.018462	0.017371	0.018917
Equal Weights (EQ)								
MSE	0.000314	0.000371	**0.000404**	0.000398	0.000341	0.000307	0.000292	0.000318
RMSE	0.01771	0.019272	**0.020103**	0.01995	0.018456	0.01753	0.017094	0.017823
Bayesian model averaging (BMA)								
MSE	0.000295	0.000355	0.000406	0.000183	0.000338	0.000257	0.000285	0.000329
RMSE	0.017172	0.018854	0.020148	0.01353	0.018391	0.016017	0.016867	0.018148
Dynamic Model Averaging (DMA)								
MSE	**0.000133**	**0.000344**	0.000432	**0.000181**	**0.000303**	**0.000217**	**0.000237**	**0.000154**
RMSE	**0.011549**	**0.018534**	0.020785	**0.013465**	**0.017413**	**0.014737**	**0.015392**	**0.012402**

Table 13: Bayessian Model Average Weights

	GARCH-M	GARCH-M (t-dist)	GARCH-M (GED)	TARCH-M	TARCH-M (t-dist)	TARCH-M (GED)	GARCHSK-M	DCC	BEKK
H=1	0.067724	0.067724	0.137949	0.067724	0.135448	0.067724	0.075477	0.374001	0.006228
H=2	0.101858	0.101893	0.156022	0.10181	0.102002	0.101624	0.200494	0.072962	0.061336
H=3	0.105897	0.121279	0.113756	0.107715	0.115231	0.109746	0.106074	0.109762	0.110541
H=4	0.110024	0.114588	0.112444	0.10716	0.112002	0.108586	0.110807	0.114097	0.110292
H=5	0.111115	0.110642	0.110088	0.110362	0.110226	0.110459	0.117276	0.109987	0.109844
H=6	0.083204	0.079913	0.078737	0.076998	0.115099	0.072726	0.277065	0.151469	0.06479
H=7	0.146743	0.089894	0.092044	0.087965	0.088377	0.087661	0.127142	0.145691	0.134483
H=8	0.075629	0.096739	0.207075	0.158825	0.099929	0.10046	0.079033	0.08909	0.09322

Appendix

1. Calculation of Both Growth Rate of Domestic Credit and Real Exchange Rate

The credit growth rate reflects the rate of increase in the domestic credit level, and is computed using the domestic credit volume, which measures the money amount lent to private agents inside the economy. The domestic credit growth is calculated as:

$$CD_t = \frac{CL_t - CL_{t-1}}{CL_{t-1}}$$

Where CD_t is the credit Growth in period t, CL_t is the level of domestic credit.

Real Exchange Rate is calculated as:

$$REX_t = nominal\ exchange\ rate\ for\ dollar\ per\ Egyptian\ pounds_t * \frac{Egypt\ CPI_t}{US\ CPI_t}$$

This implies that the higher value for REX_t means depreciation in the Egyptian pound value.

Finally, the change in real exchange rate is calculated as

$$x_t = \frac{REX_t - REX_{t-1}}{REX_{t-1}} 100\%$$

2. Preliminary Examination of Data of Multivariate GARCH Model

Table A1: Phillips-Perron unit root tests

variable	test statistic	5% critical value	Result
inflation	-12.63888	-2.874086*	I (0)
Growth of domestic credit	-15.02129	-2.874086*	I (0)
Rel exchange rate	-8.957970	-2.874086*	I (0)

Notes: * indicate the rejection of the null hypothesis of the existence of unit root I each series